2023	12	25

TS-P6

SEAT NUMBER

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(4 Pages)

calculator is not allowed.
5. All symbols having their usual meanings unless otherwise stated.
6. For each MCQ, correct answer must be written along with its alphabet.
7. Evaluation of each MCQ would be done for the first attempt only.

Physical Constants:
(1) $\pi = 3.142$ (2) $g = 10 m/s^2$ (3) $h = 6.63 \times 10^{-34} J.s$ (4) $c = 3 \times 10^8 m/s$
(5) $e = 1.6 \times 10^{-19} C$ (6) $\varepsilon_0 = 8.85 \times 10^{-12} C^2/N.m^2$ (7) $\mu_0 = 4\pi \times 10^7 T.m/A$,
(8) $\sigma = 5.7 \times 10^{-8} W/m^2 K^4$

SECTION-A

Q.1 Select and write the correct answers to the following questions: **[10]**

1) **The first law of thermodynamics is concerned with the conservation of** **(1)**
Ans:
b) energy
2) **The average value of an alternating current over a full cycle is always**
[I_0= peak value of current] **(1)**
Ans:
a) zero
3) **The angle at which maximum torque is exerted by an external electric field on an electric dipole is** **(1)**
Ans:
d) 90^0
4) **The property of light which does not change when it travels from one medium to another is** **(1)**
Ans:
c) frequency
5) **The root mean square speed of the molecules of a gas is proportional to**
[T = absolute temperature of the gas] **(1)**
Ans:

a) $\sqrt{T}$

6) **The unit Wb. m^{-2}** **(1)**

Ans:

d) the tesla

7) **When a bob performs a vertical circular motion, the string remaining in a vertical plane, the difference in the tensions in the string at the horizontal position and the uppermost position is** **(1)**

Ans:

c) 3mg

8) **A liquid rises in a glass capillary tube to a height of 2.5 cm at room temperature. If another glass capillary tube of radius half that of the former is immersed in the same liquid, the rise of the liquid in it will be** **(1)**

Ans:

c) 5 cm

9) **In young's double-slit experiment, the two coherent sources have different amplitudes. If the ratio of the maximum intensity to the minimum intensity is16:1, then the ratio of the amplitudes of the two source is** **(1)**

Ans:

b) 5:3

10) **The equation of a simple harmonic progressive wave travelling on a string is y = 8 sin (0.02x - 4) cm. The speed of the wave is** **(1)**

Ans:

d) 200 cm/s

Q.2 Answer the following questions in one sentence: **[8]**

(1) Define potential gradient of a potentiometer wire. **(1)**

Ans:

Potential gradient along a potentiometer wire is the fall of potential per unit length of the wire and is given by the potential difference between the ends of the potentiometer wire divided by the length of the wire.

(2) State the formula for critical velocity in terms of Reynolds number for a fluid flow **(1)**

Ans:

The critical velocity of a fluid flow which marks the transition of a steady to turbulent flow is given by

$$V_{critical} = \frac{\eta \Re}{\rho d}$$

Where, Re is the critical Reynolds number, ρ is the density of the fluid and η its coefficient of viscosity. d is some characteristic dimension of the system. For a sphere held stationary in a fluids stream, d is its diameter; for a fluid flow in a pipe, d is the pipe diameter.

(3) Is it always necessary to use red light to get photoelectric effect? **(1)**

Ans:

Red light cannot be used for photoelectric effect.

For photoelectric effect, the wavelength of the incident radiation must be less than the threshold wavelength of the emitter. But wavelength of red light are much longer than the threshold wavelength of a metal with the lowest work function.

(4) Write the Boolean expression for the exclusive – OR (XOR) gate **(1)**

Ans:

The Boolean expression for the output Y of a XOR gate is

$Y = A \oplus B$

Where A and B are the two inputs.

(5) Write the differential equation for angular SHM **(1)**

Ans:

The differential equation of angular SHM is

$$I = \frac{d^2\theta}{dt^2} + c\theta = O$$

Where I is the moment of inertia of the body in angular SHM, $\frac{d^2\theta}{dt^2}$ is the angular acceleration of the body when its angular displacement from the equilibrium position is θ and c is the torque per unit angular displacement acting on the body.

(6) What is the mathematical formula for the third postulate of Bohr's atomic model? **(1)**

Ans:

By Bohr's third postulate, when the electron in an H-atom makes a transition to a lower energy state, the energy of the photon emitted is equal to the energy differences between the two states:

$h\nu = E_m - E_n$

where E_m and E_n are the energies of the electron in the mth and nth orbits $(m > n)$ and hv is the energy of the emitted photon of frequency v.

(7) Two inductor coils with inductance 10 mH and 20 mH are connected in series. What is the resultant inductance of the combination of the two coils? **(1)**

Ans:

For the series combination of the inductors,

$L = L_1 + L_2 = 10 + 20 = 30\, mH$.

(8) Calculate the moment of inertia of a uniform disc of mass and 10 kg and radius 60 cm about an axis perpendicular to its plane and passing through its centre **(1)**

Ans:

The moment of inertia of the disc,

$$I = \frac{1}{2} MR^2 = \frac{1}{2}(10)(0.6)^2 = 10 \times \frac{0.36}{2} = 1.8\, kg \cdot m^2.$$

SECTION-B

Attempt any eight of the following questions: **[16]**

Q.3 Define moment of inertia of a rotating rigid body. State its SI unit and

dimensions. **(2)**

Ans:

Moment of inertia of a rigid body about an axis of rotation is defined as the sum of products of mass of each particle and the square of its perpendicular distance from the axis of rotation.

OR

A quantity that measured the inertia of rotational motion of the body is called “Rotational Inertia” or “Moment of Inertia” of the body.

It’s denoted by ‘I’ or ‘MI’ and is given by, (If body is considered to be made up of discrete particle)

$M.I=m_1 r_1^2+m_2 r_2^2+\ldots+m_n r_n^2$

$I=\sum_{i=1}^{n} m_i r_i^2$

Where,

$m_i=$¿Masses of the particles

$r_i=$¿ Perpendicular distances from the axis of rotation

If the rigid body is not considered to be made up of discrete particle but a continuous distribution of matter then the summation is replaced by integration. The mass ‘m’ of the particle is replaced by ‘dm’, which represent infinitesimally small part of mass of the whole body, situated at a perpendicular distance ‘r’ from the axis of rotation.

$I=\int dm.r^2$

$dm=$¿ An infinitesimally small mass of the body

$r=$¿ Distance of mass dm from the axis of rotation

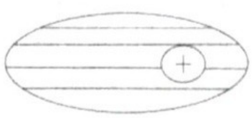
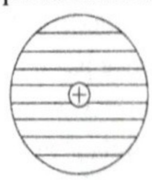
S.I. units: $kg.m^2$

C.G.S. Units: $g.cm^2$

Dimensions: $[M^1 L^2 T^0]$

Q.4 What are polar and nonpolar dielectrics **(2)**

Ans:

Sr. No.	Polar dielectric	Non-polar dielectric
i.	A dielectric molecule in which the centre of mass of positive charges (protons) does not coincide with the centre of mass of negative charges (electrons), because of the asymmetric shape of the molecules is called polar dielectric.	A dielectric in which the centre of mass of the positive charges coincides with the centre of mass of the negative charges is called a non-polar dielectric.
ii.	Representation: 	Representation: 

iii.	They have permanent dipole moments of the order of 10^{-30} Cm. They act as tiny electric dipoles, as the charges are separated by a small distance.	These have symmetrical shapes and have zero dipole moment in the normal state.
iv.	Examples: HCl, water, alcohol, NH_3	Examples: H_2, N_2, O_2, CO_2, benzene, methane

Q.5 What is a thermodynamic process? Give any two types of it **(2)**

Ans:

A thermodynamic process is a process in which the initial thermodynamic state of a system changes to its final state. During the process, there may be

1. addition of heat to the system
2. removal of heat from the system
3. change in the temperature of the system
4. change in the volume of the system.
5. change in the pressure of the system.

Thermodynamic processes can be broadly classified into two types:
1) reversible processes and 2) irreversible processes.

Or

A thermodynamic process may be (i) isobaric (constant pressure) (ii) isochoric (constant volume) (iii) isothermal (constant temperature) (iv) adiabatic (no addition of heat to the system or no removal of heat from the system).

Q.6 Derive an expression for the radius of the *n*th Bohr orbit of the electron in a hydrogen atom. **(2)**

Ans:

Consider an electron revolving in the nth orbit around the nucleus of an atom with the atomic number Z. let m and – e be the mass and charge of the electron, r the radius of the orbit and v the linear speed of the electron.

According to Bohr's first postulate,

Centripetal force on the electron = electrostatics force of attraction exerted on the electron by the nucleus.

$$\therefore \frac{mv^2}{r} = \frac{1}{4_{\pi\varepsilon 0}} \cdot \frac{Ze^2}{r^2}$$

Where ε_0 is the permittivity of free space.

$$\therefore 4^2 = \frac{Ze^2}{4_{\pi\varepsilon 0} mr}$$

According to Bohr's second postulate,

$$mvr = \frac{nh}{2\pi}$$

where h Planck's constant and n is the principal quantum number which takes integral values 1, 2, 3…., etc.

$$\therefore = \frac{nh}{2\pi mr}$$

$$\therefore v^2 = \frac{n^2 h^2}{4_\pi^2 m^2 r^2}$$

Equating the right-hand sides of Eqs. (2) and (4),

$$\frac{Ze^2}{4\pi\varepsilon_0 mr} = \frac{n^2h^2}{4_\pi^{\ 2} m^2 r^2}$$

$$\therefore r = \left(\frac{\varepsilon_0 h^2}{\pi m Z e^2}\right) n^2$$ Equation (5) gives the required expression.

Q.7 What are harmonics and overtones? (Any two points) **(2)**

Ans:

1. A stationary wave is set up in bounded medium in which the boundary could be a rigid support (i.e., a fixed end, as for instance a string stretched between two rigid supports) or a free end (as for instance an air column in a cylindrical tube with one or both ends open). The boundary conditions limit the possible stationary waves and only a discrete set of frequency is allowed.
2. The lowest allowed frequency (n_1), is called the fundamental frequency of vibration.
3. Integral multiples of the fundamental frequency is called the harmonics.
4. $(n_1, n_2, n_3 \ldots\ldots)$ the fundamental frequency being called the first harmonic. The second harmonic is twice the fundamental or $2n_1$, the third harmonic is $3n_1$, and so on.
5. The harmonics represent the fundamental and all its integral multiples. They may be present in a given sound or not.
6. The higher allowed frequencies are called the overtones. Above the fundamental, the first allowed frequency is called the first overtone; the next higher frequency is the second overtone, and so on.
7. Overtones are always present in sound.
8. The relation between overtones and allowed harmonics depends on the system under consideration.

Q.8 Distinguish between a potentiometer and voltmeter. **(2)**

Ans:

Potentiometer	Voltmeter
1. A potentiometer is used to determine the emf of a cell, potential difference and internal resistance.	1. A voltmeter can be used to measure the potential difference and terminal voltage of a cell. But it cannot be used to measure the emf of a cell.
2. Its accuracy and sensitivity are very high.	2. Its accuracy and sensitivity are less as compared to a potentiometer.
3. It is neither portable nor a direct-reading instrument.	3. It is both portable and a direct-reading instrument.

Q.9 What are mechanical equilibrium and thermal equilibrium? **(2)**

Ans:

A system is said to be in mechanical equilibrium when there are no unbalanced forces within the system and between the system and its surroundings.

A system is said to be in thermal equilibrium when its temperature is uniform throughout the system and does not change with time.

Q.10 An electron in an atom is revolving round the nucleus in a circular orbit or radius $5.3\,x\,10^{-11}$ m with the speed of 3.6 $x\,10^{-6}$ m/s. Find the angular momentum of the electron. (2)

Ans:

Data :

$r=5.3\times 10^{-11}m,\ v=3\times 10^{6}m/s$

$e=1.6\times 10^{-19}C,\ m_e=9.1\times 10^{-31}kg$

The angular momentum of the electron is

$L_0=m_e vr$

$¿(9.1\times 10^{-31})(3\times 10^{6})(5.3\times 10^{-11})$

$¿27.3\times 5.3\times 10^{-36}$

$¿1.447\times 10^{-34}kg.m^2/s$

Q.11 Plane wavefronts of light of wavelength 6000 A^0 is incident on two slits on a screen perpendicular to the direction of light rays. If the total separation of 10 bright fringes on a screen 2m away is 2cm, find the distance between the slits. (2)

Ans:

Data $:\lambda=6000\,Å=6\times 10^{-7}m,\ D=2m,\ 10W=2\times 10^{-2}m$

$\therefore The\ fringe\ width\ W=\frac{2\,cm}{10}=0.2\,cm=2\times 10^{-3}m$

$W=\frac{\lambda D}{d}$

$\therefore d=\frac{\lambda D}{d}=\frac{(6\times 10^{-7})(2)}{2\times 10^{-3}}$

$¿6\times 10^{-4}m$

Q.12 Eight droplets of water, each of radius 0.2 mm, coalesce into a single drop. Find the decrease in the surface area. (2)

Ans:

Data $:r=0.2\,mm=2\times 10^{-4}m$

Let R be the radius of the single drop formed due to the coalescence of 8 droplets of mercury.

Volume of 8 droplets = volume of the single drop

As the volume of the liquid remains constant.

$\therefore 8\times \frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3$

$\therefore 8r^3=R^3$

$\therefore 2r=R$

Surface area of 8 droplets $¿\times 4\pi r^2$

Surface area of single drop $¿4\pi R^2$

$\therefore Descrease \in surface\ area$

$¿8\times 4\pi r^2-4\pi R^2$

$¿4\pi ¿$

$¿4\pi[8r^2-(2r)^2]$

$¿4\pi\times 4r^2\ 16\pi r^2$

$¿16(3.142)(2\times 10^{-4})^2$

$¿64\times 3.142\times 10^{-8}$

$¿ 2.011 \times 10^{-6} m^2$

Q.13 A 0.1 H inductor a $25 x 10^{-6} F$ capacitor and a 15Ω resistor are connected in series to a 120 V/50 Hz ac source. Calculate the resonant frequency. (2)

Ans:

$Data : L = 0.1\, H, C = 25 \times 10^{-6} F$

Resonant frequency, $f_r = \frac{1}{2\pi\sqrt{LC}}$

$\therefore f_r = \frac{1}{2.(3.142)\sqrt{0.1 \times 25 \times 10^{-6}}}$

$¿ \frac{10_3}{(6.248)\sqrt{2.5}} \quad ¿ 100.7\, Hz$

Q.14 The difference between the two molar specific heats of a gas is 9000.K. If the ratio of the two specific heats is 1.5, calculate the two molar specific heats (2)

Ans:

$Data : S_p - S_v = 9000 \frac{J}{kg} \cdot K, \frac{S_p}{S_v} = 1.5$

$\therefore S_p = 1.55_v$

$\therefore 1.55_v - S_v = 9000$

$\therefore S_v = \frac{9000}{0.5} = 18000\, J/kg \cdot K$

$\therefore S_p = S_v + 9000 = 18000 + 9000 = 27000\, J/kg \cdot K$

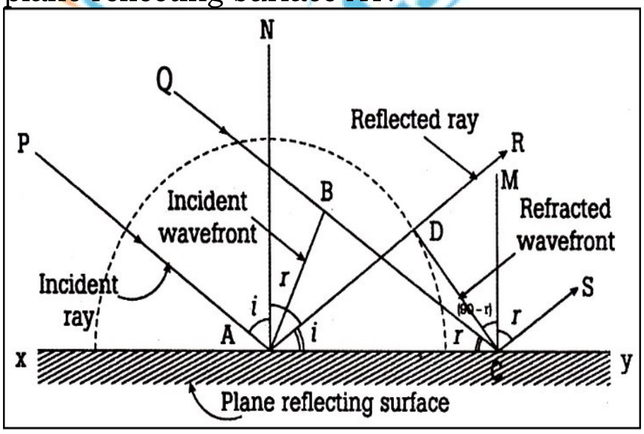
SECTION-C

Attempt any eight of the following questions:
[24]

Q.15 With the help of a neat diagram, explain the reflection of light on a place reflecting surface. (3)

Ans:

1. Consider a plane wave front AB bounded by the rays PA and QB incident obliquely on plane reflecting surface XY.



2. According to Huygens' Principle, the point A becomes secondary source sending secondary wave back in the same medium.
3. Suppose the secondary wave from the point B reaches the point C on xy in time t covering the distance $Bc = ct$.
4. During the time t, secondary wave from A covers the distance $Bc = AD = ct$.
5. Construct a hemisphere with A as its centre & AD as its radius, represent wave front of A.

6. Now tangent CD to hemisphere through the point D, represents reflected wave front because C and D are in the same phase.
7. The reflected wave front CD is bounded by the rays AR and CS.
8. NA and MC are normal to xy at the points A and C respectively.
9. $\angle PAN = \angle i$ –angle of incident and $\angle NAR = \angle r$- angle of reflection.
10. By geometry angle of incidence= angle of reflection.
    In ΔABC and ΔADC,
    $\angle ABC = \angle ADC = 90^0$, BC=AD, AC is common
    Then above triangles are congruent
    $\angle BCA = \angle DAC$, $\angle PAX = \angle DAC$,
    $90^0 - \angle i = 90^0 - \angle r$, $\angle i = \angle r$
    Laws of Reflection:
1. Angle of incidence is equal to angle of reflection.
2. Incident ray normal and reflected ray lie in the same plane.
3. Incident ray and reflected ray are opposite to normal.

Q.16 What are magnetization, magnetic intensity and magnetic susceptibility? (3)

Ans:

Magnetization:

1. The net magnetic dipole moment per unit volume, in the magnetic material is called as magnetization.
   It is denoted by $\overrightarrow{M_Z}$.
   If magnetic specimen of volume 'V' acquires net magnetic dipole moment M_{net} due to the magnetizing field, then $\overrightarrow{M_Z} = \frac{M_{net}}{V}$

   Magnetic intensity:
   The ratio of the strength of magnetizing field to the permeability of free space is called as magnetic intensity.
   The strength of magnetic field at a point can be given in terms of vector quantity called as magnetic intensity (H).
   Magnetic intensity is a quantity used in describing magnetic phenomenon in terms of their magnetic field.
   $H = \frac{B_0}{\mu_0}$ or $B_0 = \mu_0 H$

   Magnetic susceptibility:
   The ratio of magnitude of intensity of magnetization to that of magnetic intensity is called as magnetic susceptibility.
   It is given by,
   $\vec{x} = \frac{\overrightarrow{M_z}}{H}$

a. Magnetic Susceptibility (χ) is the indicator of measure of the response of a given material to the external applied magnetic field.
b. In other words, it indicates as to how much magnetization will be produced in a given substance when kept in an external magnetic field.
c. When the substance is kept in a magnetic field, the atomic dipole moments either align or oppose the external magnetic field.
d. If the atomic dipole moments of the substance are opposing the field, $\chi$is observed to be negative, and if the atomic dipole moments align themselves in the direction of field, $\chi$is observed to be positive.
e. The number of atomic dipole moments of getting aligned in the direction of the applied magnetic field is proportional to χ. It is large for soft iron $(\chi > 1000)$.

Q.17 Prove that the frequency of the beats is equal to the difference between the frequencies of the two sound notes giving rise to beats. **(3)**

Ans:

Production of Beats: The alternate waxing and waning of sound after definite intervals of time, due to superposition of two waves of nearly equal frequencies, is called production of beats.

Analytical treatment of beats frequency:

1. Consider two sound waves of equal amplitude but of slightly different frequencies n_1 and n_2 $(n_1 > n_2)$, passing simultaneously through a given point in space.
2. Suppose that the two waves start in phase, the displacement y_1 and y_2 of the waves at an instant 't' can be represent by,
   $y_1 = A\sin\omega_1 t = A\sin 2\pi n_1 t$ .... (1)
   $y_2 = A\sin\omega_2 t = A\sin 2\pi n_2 t$ .... (2)
3. By the principle of superposition of waves, the resultant displacement 'y' is given by,
   $y = y_1 + y_2$
   ¿ $A\sin 2\pi n_1 t + A\sin 2\pi n_2 t$
   $\therefore\ y = A(\sin 2\pi n_1 t + \sin 2\pi n_2 t)$
   By using,
   $$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$$
   $$\therefore\ y = 2A\sin 2\pi\left(\frac{n_1+n_2}{2}\right)t\cos 2\pi\left(\frac{n_1-n_2}{2}\right)t$$
   $$\therefore\ y = 2A\cos 2\pi\left(\frac{n_1-n_2}{2}\right)t\sin 2\pi\left(\frac{n_1+n_2}{2}\right)t \text{ ..... (3)}$$
4. Let us substitute,
   $$R = 2A\cos 2\pi\left(\frac{n_1-n_2}{2}\right)t \text{ .... (4)}$$
   And $n = \dfrac{n_1+n_2}{2}$
   $\therefore\ y = R\sin(2\pi nt)$ .... (5)
   This is the equation of simple harmonic motion. Whose frequency 'n' is the mean of the frequency of the two waves and amplitude R is variable.

Waxing:

Waxing will be possible, if R is maximum.

I.e. $R = \pm 2A$

From equation (4),

$$R = 2A\cos 2\pi\left(\frac{n_1-n_2}{2}\right)t = \pm 2A$$

$$\therefore \cos 2\pi\left(\frac{n_1-n_2}{2}\right)t = \pm 1$$

$$\therefore 2\pi\left(\frac{n_1-n_2}{2}\right)t = 0, \pi, 2\pi, 3\pi \text{ ....}$$

$$\therefore \pi(n_1-n_2)t = 0, \pi, 2\pi, 3\pi \text{ ....}$$

Time at which maximum intensity is produces, is

$$t = 0, \frac{1}{n_1-n_2}, \frac{2}{n_1-n_2}, \frac{3}{n_1-n_2}$$

Time interval between two successive waxing is produced, is

$$T = \frac{1}{n_1-n_2} - 0 = \frac{2}{n_1-n_2} - \frac{1}{n_1-n_2}$$

$¿ \frac{3}{n_1 - n_2} - \frac{2}{n_1 - n_2}$ and so on.

$\therefore T = \frac{1}{n_1 - n_2}$

Frequency of waxing $¿ \frac{1}{Period} = \frac{1}{T} = n_1 - n_2$

Waning:

Waning will be possible, if R is minimum.

I.e. $R = 0$

From equation (4),

$2A\cos 2\pi \left(\frac{n_1 - n_2}{2} \right) t = 0$

$\therefore \cos 2\pi \left(\frac{n_1 - n_2}{2} \right) t = 0$

$\therefore 2\pi \left(\frac{n_1 - n_2}{2} \right) t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots\ldots$

Time at which minimum intensity is produces, is

$t = \frac{1}{2(n_1 - n_2)}, \frac{3}{2(n_1 - n_2)}, \frac{5}{2(n_1 - n_2)}, \ldots\ldots\ldots\ldots$

The time interval between two successive waning is,

$T = \frac{3}{2(n_1 - n_2)} - \frac{1}{2(n_1 - n_2)}$

$¿ \frac{5}{2(n_1 - n_2)} - \frac{3}{2(n_1 - n_2)}$

$\therefore T = \frac{1}{n_1 - n_2}$

Period of waxing ¿ Period of waning ¿ Period of beats ¿ T

∴ Frequency of beats,

$N = \frac{1}{T} = n_1 - n_2$

Q.18 Define (a) inductive reactance (b) capacitive reactance (c) impedance (3)

Ans:

(a) Inductive reactance

The opposing nature of an inductor to the flow of alternating current is called inductive reactance.

(b) Capacitive reactance

1. The peak value of alternating current through a capacitor is given by,

   $i_0 = \frac{e_0}{(1/\omega C)}$ …… (1)
2. For a capacitive a.c. circuit, According to Ohm's law,

   $i = \frac{V}{R}$ …… (2)

   Where, R¿ resistance of the circuit
3. Comparing equation (1) and (2), we conclude that $\left(\frac{1}{\omega C} \right)$ plays a similar role in capacitive AC circuit as resistance in a purely resistances circuit.
4. Hence the effective resistance offered by the capacitor called the capacitive reactance denoted by X_C.

$\therefore X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

(c) impedance

The ratio of rms voltage to the rms value of current is called impedance. The SI unit of impedance is ohm (Ω).

Q.19 Derive an expression for the kinetic energy of a body rotating with a uniform angular speed. **(3)**

Ans:

Consider a rigid body rotating about given axis of rotation with constant angular velocity 'ω'.

When rigid body rotates about given axis of rotation, every particle of this body performs uniform circular motion with constant angular velocity 'ω'.

Let m_1, m_2, m_3,..... m_n be mass of particles situated at a distance r_1, r_2, r_3,... r_n from given axis of rotation.

Then linear velocity of particles in circular motion is given by,

$v_1 = r_1 \omega$

$v_2 = r_2 \omega$

$v_3 = r_3 \omega$

.

.

$v_n = r_n \omega$

$\therefore$ Kinetic energy of different particles in circular motion is given by,

$KE_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 r_1^2 \omega^2$

$KE_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 r_2^2 \omega^2$

$KE_3 = \frac{1}{2} m_3 v_3^2 = \frac{1}{2} m_3 r_3^2 \omega^2$

.

.

$KE_n = \frac{1}{2} m_n v_n^2 = \frac{1}{2} m_n r_n^2 \omega^2$

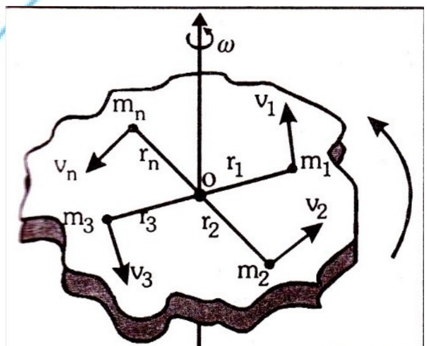


Fig. 3.2

$\therefore$ Total kinetic energy of rotating body

$KE_{rot} = KE_1 + KE_2 + KE_3 + \ldots + KE_n$

¿ $\frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 + \ldots + \frac{1}{2} m_n r_n^2 \omega^2$

$KE_{rot} = \frac{1}{2} \left(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots + m_n r_n^2 \right) \omega^2$

$\therefore KE = \frac{1}{2} \left[\sum_{i=1}^{n} m_i r_i^2 \right] \omega^2$ .... (1)

$\because \sum_{i=1}^{n} m_i r_i^2 = I$

$\therefore$ KE of rotation (KE) ¿ $\frac{1}{2} I \omega^2$ ...... (2)

Where, $I = MI = m_1 r_1^2 + m_2 r_2^2 + \ldots + m_n r_n^2$

To show $KE \propto n^2$

$\because KE = \frac{1}{2} I \omega^2$

But, $\omega = 2\pi n$

Where, n is the frequency of revolution.

$\therefore KE = \frac{1}{2} I (2\pi n)^2 = \frac{1}{2} \times 4\pi^2 n^2 I = 2\pi^2 n^2 I$

$\therefore KE = 2\pi^2 I n^2$ .... (3)

In the equation (3), $2\pi^2 I =$ ¿ Constant

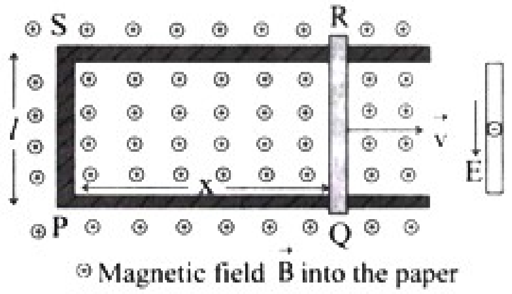
$\therefore KE = constant . n^2$

$\therefore KE \propto n^2$

Q.20 Derive an expression for the emf *e* generated in a conductor of length *l* moving in a uniform magnetic field B with uniform velocity *v* along x-axis. (3)

Ans:

1. Consider a rectangular frame of wire ABCD of area $(l x)$ is situated in a constant magnetic field $(\vec{B})$.
2. As the wire BC of length l is moved out with velocity $\vec{v}$ to increase x the area of the loop ABCD increases. Thus, the flux of $\vec{B}$ through the loop increases with time.
3. According to the 'Flux Rule' the induced emf will be equal to the rate at which the magnetic flux through a conducting circuit is changes.
4. The induced emf will cause a current in the loop. It is assumed that there is enough resistance in the wire so that the induced currents are very small producing negligible magnetic field.



A frame of wire PQRS in magnetic field $\vec{B}$ and wire BC is moving with velocity $\vec{v}$ along x-axis

5. As the flux ϕ through the frame ABCD is Blx, magnitude of the induced emf can be written as
   $|e| = \frac{d\phi}{dt} = \frac{d}{dt}(B lx) = Bl\frac{dx}{dt} = B l v$, ... (1)
   Where, v is the velocity of wire BC increasing the length x of wires AB and CD.
6. A charge q which is carried along by the moving wire BC, experiences Lorentz force $\vec{F} = q(\vec{v} \times \vec{B})$; which is perpendicular to both $\vec{v}$ and $\vec{B}$ and hence is parallel to wire BC.
7. The force $\vec{F}$ is constant along the length l of the wire BC (as v and B are constant) and zero elsewhere ($\because v = 0$ for stationary part CDAB of wire frame).
8. When a charge q moves a distance l along the wire, the work done by the Lorentz force is $W = F \cdot l = q v B \sin\theta . l$, where θ is the angel between $\vec{B}$ and $\vec{v}$.
9. The emf generated is work/charge i.e.,
   $e = \frac{W}{q} = v B \sin\theta \cdot l$ .... (2)
10. For maximum induced emf, $\sin\theta = 1$
    $e_{max} = B l v$ .... (3)
11. Thus, from Eq. (1) & (2) for any circuit whose parts move in a fixed magnetic field, the induced emf is the time derivative of flux (ϕ) regardless of the shape of the circuit.

Q.21 Derive an expression for the terminal velocity of a spherical objects falling under gravity through a viscous medium. (3)

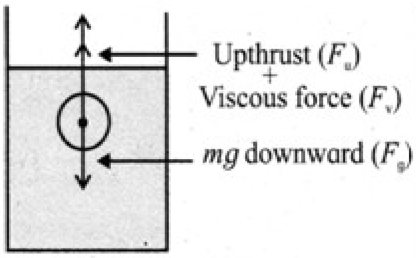
Ans:

Terminal velocity:

The constant maximum velocity acquired by a body falling through a viscous liquid is called as terminal velocity.

Consider a spherical object falling through a viscous fluid. Forces experienced by it during its downward motion are,

1. Viscous force (F_v), directed upwards. Its magnitude goes on increasing with increase in its velocity.
2. Gravitational force, or its weight (F_g), directed downwards, and
3. Buoyant force or up thrust (F_u), directed upwards.



Forces acting on object moving through a viscous medium.

Net downwards force given by $f = F_g - (F_v + F_u)$, is responsible for initial increase in the velocity. Among the given the forces, F_g and F_u are constant while F_v increase with increase in velocity. Thus, a stage is reached when the net force f becomes zero. At this stage, $F_g = F_v + F_u$. After that, the downward velocity remains constant. This constant downward velocity is called terminal velocity. Obviously, now onwards, the viscous force F_v is also constant. The entire discussion necessarily applies to streamline flow only.

Consider a spherical object falling under gravity through a viscous medium. Let the radius of the sphere be r, its mass m and density ρ. Let the density of the medium be σ and its coefficient of viscosity be η. When the sphere attains the terminal velocity, the total downward force acting on the sphere is balanced by the total upward force acting on the sphere.

Total downward force ¿ Total upward force weight of sphere (mg) ¿ Viscous force +¿ by out ant to due to the medium

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v + \frac{4}{3}\pi r^3 \sigma g$$

$$6\pi\eta r v = \left(\frac{4}{3}\pi r^3 \rho g\right) - \left(\frac{4}{3}\pi r^3 \sigma g\right)$$

$$6\pi\eta r v = \left(\frac{4}{3}\right)\pi r^3 g(\rho - \sigma)$$

$$v = \left(\frac{4}{3}\right)\pi r^3 g(\rho - \sigma) \times \frac{1}{6\pi\eta r}$$

$$v = \left(\frac{2}{9}\right)\frac{r^2 g(\rho - \sigma)}{\eta}$$

This is the expression for the terminal velocity of the sphere. We can also write,

$$\eta = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{v}$$

The above equation gives coefficient of viscosity of a fluid.

Q.22 Determine the shortest wavelengths of the Balmer and Paschen series, given the limit for Lyman series is 912 A^0. **(3)**

Ans:

Data : $\lambda_{L\infty} = 912\,Å$

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

For the shortest wavelength (series limit) of a spectral series, $m = \infty$

$$\therefore \frac{1}{\lambda_\infty} = R\left(\frac{1}{n^2} \frac{-1}{\infty}\right) = R\left(\frac{1}{n^2} - 0\right) = \frac{R}{n^2}$$

For the lyman series, $n = 1$*. Then,*

$$\frac{1}{\lambda_{L\infty}} = \frac{R}{1} = R$$

$\therefore \lambda_{L\infty} = \frac{1}{R} = 912\ \mathring{A}$

For the balmer seris , cn=2. Then ,

$\frac{1}{\lambda_{B\infty}} = \frac{R}{2^2} = \frac{R}{4}$

$\therefore \lambda_{B\infty} = \frac{4}{R} 4 \times 912 = 3648\ \mathring{A}$

For the paschen series , n=3. Then ,

$\frac{1}{\lambda_{P\infty}} = \frac{R}{3^2} = \frac{R}{9}$

$\therefore \lambda_{P\infty} = \frac{9}{R} = 9 \times 912 = 8208\ \mathring{A}$

Q.23 Calculate the value of the magnetic field at a distance of 3 cm from a very long straight wire carrying a current of 6 A. **(3)**

Ans:

$Data : a = 3\,cm = 0.03\,m , I = 6\,A , \frac{\mu_0}{4\pi} = 10^{-7}\ Wb/A{\cdot}m$

The magnetic induction,

$B = \frac{\mu_0 I}{2_{\pi a}} = \frac{\mu_0}{4_{\pi}} \frac{2I}{a}$

¿ $(10^{-7}) \frac{2(6)}{3 \times 10^{-2}}$

¿ $4 \times 10^{-5}\ T = 40\ \mu T$

Q.24 A parallel-plate capacitor filled with air has plates of area 6 cm^2 and plate separation 3mm. Calculate its capacitance. **(3)**

Ans:

$Data : k = 1 (air) , A = 6\,cm^2 = 6 \times 10^{-4}\,m^2 , d = 3\,mm = 3 \times 10^{-3}\,m ,$

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N{\cdot}m^2$

The capacitance of the air capacitior , $C_0 = \frac{\varepsilon_0 A}{d}$

¿ $\frac{(8.85 \times 10^{-12})(6 \times 10^{-4})}{3 \times 10^{-3}} = 8.85 \times 2 \times 10^{-13}$

¿ $1.77 \times 10^{-12}\ F = 1.77\ pF$

Q.25 An emf of 91 mV is induced in the winding of a coil when the current in a nearby coil is increasing at the rate of 1.3 A/s. What is the mutual inductance M of the two coil in mH? **(3)**

Ans:

$Data : \vee e_2 \vee$ ¿ $91\,mV = 91 \times 10^{-3}\,V , \frac{dI_1}{dt} = 1.3\ A/s$

¿ $e_2 \vee$ ¿ $M \frac{dI_1}{dt}$

¿ $\frac{91 \times 10^{-3}}{1.3}$

¿ $70 \times 10^{-3}\,H$

¿ $70\,mH$

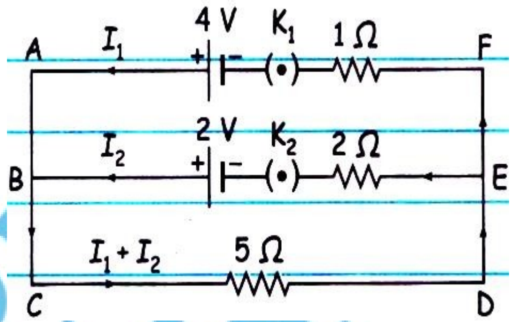
Q.26 Two cells of emfs 4 v and 2 v having respective internal resistance 1 Ω and 2 Ω are connected in parallel so as to send a current through the external resistance od 5 Ω. Find the current through the external resistance. (3)

Ans:

Let $I_1 \wedge I_2$ be the currents through the two branches as shown in the figure. The current through the 5 Ω resistor will be $I = I_1 + I_2$

[Kirchhoff's current law].

Applying Kirchhoff's voltage law o loop ABCDEFA, we get



$-5(I_1+I_2) - I_1 + 4 = 0$

$\therefore 6I_1 + 5I_2 = 4$

Applying Kirchhoff's voltage law to loop BCDEB, we get

$-5(I_1+I_2) - 2I_2 + 2 = 0$

$\therefore 5I_1 + 7I_2 = 2$

Multiplying Eq. (1) by 5 and Eq. (2) by 6, we get

$30I_1 + 25I_2 = 20$

¿$30I_1 + 42I_2 = 12$

Subtracting Eq. (3) from Eq. (4), we get

$17I_2 = -8$

$\therefore I_2 = \frac{-8}{17} A$

Substituting this value of I_2 in Eq. (1), we get,

$6I_1 + 5\left(\frac{-8}{17}\right) = 4$

$\therefore 6I_1 = 4 + \frac{40}{17} = \frac{68+40}{17} = \frac{108}{17}$

$\therefore I_1 = \frac{18}{17} A$

$\therefore I + I_1 + I_2 = \frac{18}{17} - \frac{8}{17} = \frac{10}{17} A$

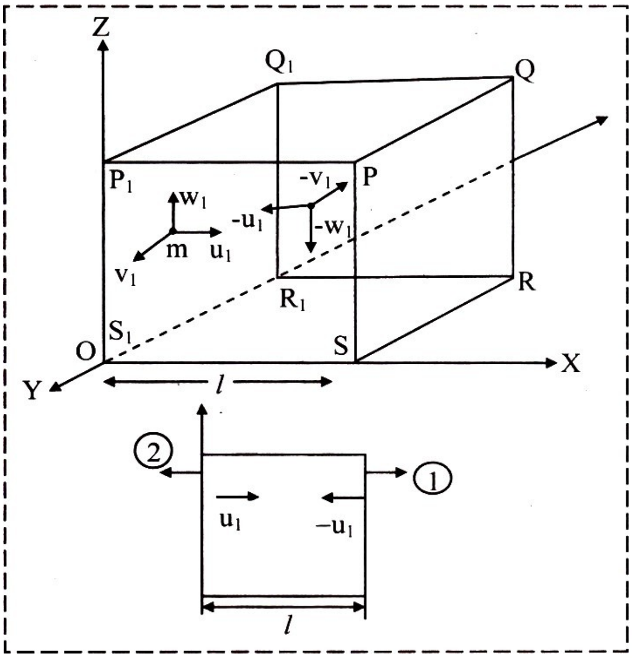
SECTION-D

Attempt any three of the following question: [12]

Q.27 Derive an expression for the pressure exerted by a gas on the basis of kinetic theory of gases. (4)

Ans:

1. Consider a fixed mass of a perfect gas enclosed in a cubical vessel of each side l and with perfectly elastic walls. The gas molecule is constantly moving in all directions with all possible velocities. When a gas molecule collides with other molecule, there is change in magnitude and direction of velocity. Therefore, the gas molecule possess momentum, hence momentum is imparted to wall of vessel. Thus, pressure is exerted by gas molecules on the walls of vessel.



2. $N=$ ¿ Total number of molecules of the gas,
   $m=$ ¿ Mass of each molecule,
   $M=Nm=$ ¿ Total mass of gas
   $A=l^2=$ ¿ Area of each face,
   $V=l^3=$ ¿ Volume of the cube,
   $Q=\frac{M}{V}=\frac{Nm}{l^3}=$ ¿ Density of the gas
   Let velocities of the molecules are
   $\overline{C_1}, \overline{C_2}$ ……… $\overline{C_N}$.
   Let each velocity is resolved into components parallel to X, Y and Z-axis.
   Let the components of $\overline{C_1}$ be $\overline{u_1}$, $\overline{v_1}$, and $\overline{w_1}$
   Let the components of $\overline{C_2}$ be $\overline{u_2}$, $\overline{v_2}$ and $\overline{w_2}$
   .
   .
   Let the components of $\overline{C_N}$ be $\overline{u_N}$, $\overline{v_N}$ and $\overline{w_N}$
   $\therefore C_1^2=u_1^2+v_1^2+w_1^2$
   $C_2^2=u_2^2+v_2^2+w_2^2$
   .
   .
   $C_N^2=u_N^2+v_N^2+w_N^2$ …………….. (1)
   Consider a molecule of mass 'm' moves with velocity C_1, moving towards PQRS with velocity u_1 along x-axis.
   Initial momentum of the molecule ¿ mu_1
   After colliding with PQRS, the molecule rebounds with velocity $-u_1$
   $\therefore$ Momentum after collision ¿ $-mu_1$
   $\therefore$ Change in momentum of molecule in one collision ¿ $-mu_1-mu_1=-2mu_1$
   Negative sign shows that the molecule has lost momentum during collision but momentum is conserved.
   $\therefore$ The change in momentum per collision of the surface
   $PQRS=+2mu_1$ ……….. (2)
   The distance travelled by this molecule, between two successive collisions with the wall PQRS ¿ $2l$.
   $\therefore$ Time interval (t) between two successive collisions with the wall PQRS is given by

$$t=\frac{Distance}{Speed}=\frac{2l}{u_1}$$

∴ Rate of change of momentum of the wall

$$PQRS=\frac{Change\in momentum}{time}=\frac{2mu_1}{\frac{2l}{u_1}}=\frac{mu_1^2}{l}$$

∴ According to Newton's second law rate of change of momentum is equal to force exerted on the wall, let the force is f_1

$$\therefore f_1=\frac{mu_1^2}{l}$$

Similarly, the forces exerted by the molecules moving along X-axis with velocities u_2, u_3 ...... u_N is given by

$$f_2=\frac{mu_2^2}{l},\ f_3=\frac{mu_3^2}{l}\ ....\frac{mu_N^2}{l}$$

$$\therefore f_x=f_1+f_2+¿......f_N$$

$$f_x=\frac{mu_1^2}{l}+\frac{mu_2^2}{l}+¿.......\frac{mu_N^2}{l}$$

$$f_x=\frac{m}{l}\left[u_1^2+u_2^2+.........+u_N^2\right]$$

Similarly resultant force exerted perpendicular to Y and Z axis i.e., f_y+f_x are given by

$$f_Y=\frac{m}{l}\left[V_1^2+V_2^2+.........V_N^2\right]$$

$$f_Z=\frac{m}{l}\left[w_1^2+w_2^2+.........+w_N^2\right]$$

Pressure exerted by a gas molecule on the walls of cubical container,

$$P_x=\frac{f_x}{A}=\frac{f_x}{l^2}$$

$$P_x=\frac{m}{l^3}\left[u_1^2+u_2^2+.........+u_N^2\right]$$

Similarly, $P_y=\frac{m}{l^3}\left[V_1^2+V_2^2+.........V_N^2\right]$

$$P_z=\frac{m}{l^3}\left[w_1^2+w_2^2+.........+w_N^2\right]$$

∵ The pressure exerted by gas molecules is same in all directions

$$P_x=P_y=P_z=P$$

$$\therefore P_x+P_y+P_z=3P,$$

$$\therefore \frac{P_{x+P_y+P_z}}{3}=P$$

$$P=\frac{1}{3}\left[\frac{m}{l^3}\left(u_1^2+u_2^2+.........+u_N^2\right)+\frac{m}{l^3}\left(v_1^2+v_2^2+.........+v_N^2\right)+\frac{m}{l^3}\left(w_1^2+w_2^2+.........+w_N^2\right)\right]$$

$$P=\frac{1}{3}\cdot\frac{m}{l^3}\left[\left(u_1^2+v_1^2+w_1^2\right)+\left(u_2^2+v_2^2+w_2^2\right)+\left(u_N^2+v_N^2+w_N^2\right)\right] \qquad .... (2)$$

$$P=\frac{1}{3}\cdot\frac{m}{l^3}\left[V_1^2+V_2^2+.........V_N^2\right] \ ...... (3)$$

[From (1) and (2)]

By the definition,

$\because V_{rms}=\sqrt{\frac{V_1^2+V_2^2+V_3^2}{N}}$

$V_{rms}^2=\frac{V_1^2+V_2^2+........+V_3^2}{N}$

$\therefore V_1^2+V_2^2+........+V_N^2=N\,V_{rms}^2$

Putting the value in equation (3)

$P=\frac{1}{3}.\frac{m}{l^3}N\,V_{rms}^2$ .... (4)

But $Nm=M$

$P=\frac{1}{3}\frac{m}{V}V_{rms}$.............. (5)

But $\frac{M}{V}=¿\varrho¿$ density of gas

$\therefore P=\frac{1}{3}\varrho V_{rms}^2$ .... (6)

The expression given by equation (6) represents the pressure exerted by the gas.

Q.28 What is a rectifier? With the help of a neat circuit diagram, explain the working of a halfwave rectifier? (4)

Ans:

An electronic circuit, which rectifies AC voltage, is called rectifier.

Half wave rectifier:

1. Diode as half wave rectifier is as shown in diagram.

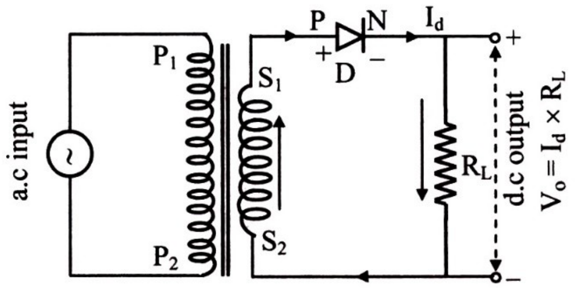
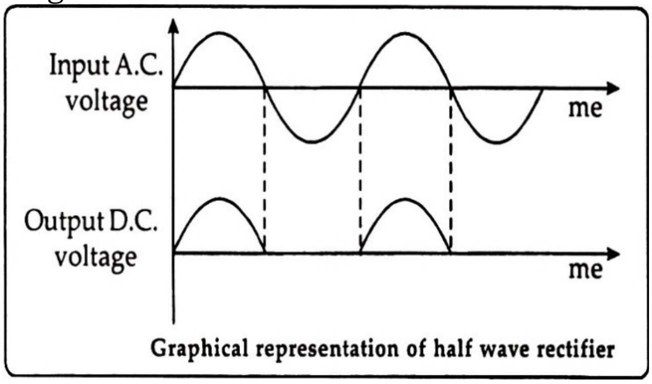


Figure (a): Half wave rectifier

P_1P_2	**:**	**Primary coil of step down transformer**
S_1S_2	**:**	**Secondary coil of step down transformer**
D	**:**	**P-N junction diode**
R_L	**:**	**Load resistance**

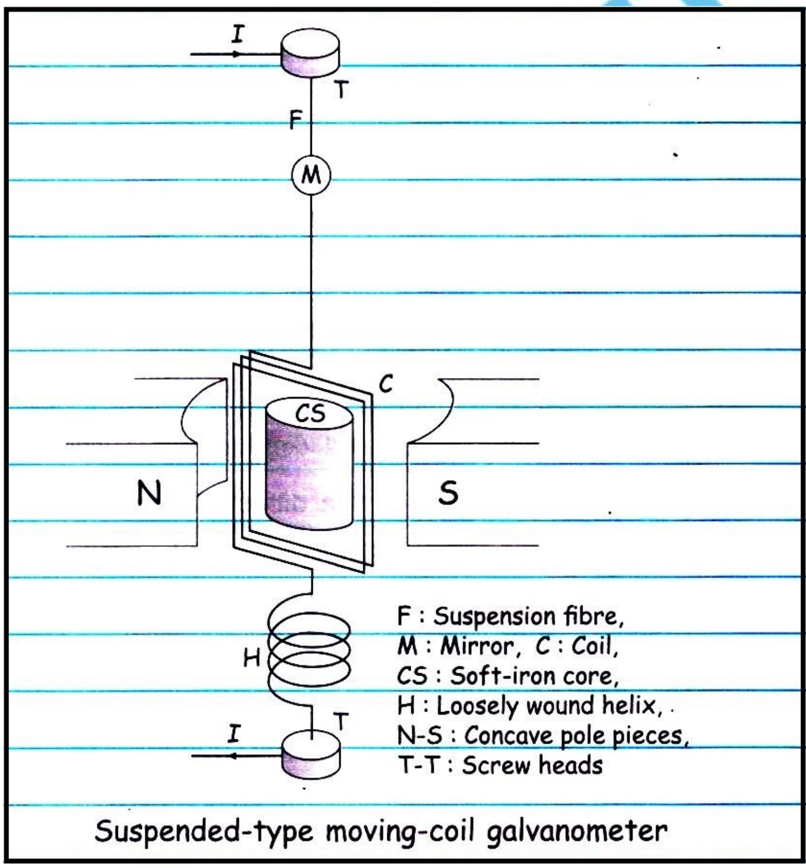
2. In this circuit, AC voltage is applied to the primary of the transformer. In series, with secondary a diode and a load resistance is connected. Output is taken across load resistance RL. Same AC is also generated across secondary.
3. When S_1 terminal of secondary is +ve with respect to S_2 terminal, diode is in forward bias and it conducts. Current flow through load resistance from A to B. output voltage is obtained across load resistance, which varies in the same manner as that of load current. When S_1 terminal is –ve with respect to S_2 diode will be in reverse bias. No current flows through the load resistance. We get zero output across load resistance. The same action repeats after every half cycle.

4. The output voltage is obtained only for half cycle so this circuit is called as half wave rectifier. The O/P voltage with respect to I/P voltage are as show in diagram.



Graphical representation of half wave rectifier

Q.29 Draw a neat, labelled diagram of a suspended-coil type moving-coil galvanometer. (2)
The initial pressure and volume of a gas enclosed in a cylinder are $2\times10^5\ N/m^2$ and $6\times10^{-3} m^3$, respectively. If the work done in compressing the gas at constant pressure is 150 J find the final volume of the gas. (2)
Ans:



Suspended-type moving-coil galvanometer

Data : $p=2\times10^5\ Pa, V_i=6\times10^{-3} m^3, W=-150\,J$

For an isobaric process, the work done is

$W=P(V_f-V_i)$

$$V_f=V_i+\frac{W}{p}=(6\times10^{-3})+\frac{(-1.50\times10^2)}{2\times10^5}$$

$¿(6\times10^{-3})-(0.75\times10^{-3})=(6-0.75)\times10^{-3}$

$¿5.25\times10^{-3} m^3$

Q.30 Define seconds pendulum. Derive a formula for the length of a seconds pendulum. A particle performing linear SHM has maximum velocity of 25 cm/s and maximum acceleration of $100\, cm/s^2$. Find the period of oscillations. **(4)**

Ans:

Definition: A simple pendulum of period two seconds is called a seconds pendulum.
The period pf a simple pendulum is

$$T=2\pi\sqrt{\frac{L}{g}}$$

For a second pendulum, T = 2 s.

$$\therefore\ 2=2\pi\sqrt{\frac{L}{g}}\ \therefore\ L=\frac{9}{\pi^2}$$

This expression gives the length of the seconds pendulum at a place where the acceleration due to gravity is g.

$$data: V_{max}=25\, cm/s, \vee a_{max} \vee ¿\, 100\, cm/s^2$$

$$V_{max}=\omega A \wedge ¿\, a_{max} \vee ¿\, \omega^2 A$$

$$\therefore \vee \frac{a_{max}}{V_{max}} \vee ¿\, \frac{\omega^2 A}{\omega A}=\omega$$

$$\therefore\ \omega=\frac{100}{25}=4\, rad/s$$

$$\therefore\ The\ period\ of\ oscillations\ of\ the\ particle,$$

$$T=\frac{2\pi}{\omega}=\frac{2\pi}{4}=\frac{\pi}{2}=\frac{3.142}{2}=1.571\, s$$

Q.31 Explain de Broglie wavelength. Obtain an expression for the de Broglie wavelength of wave associated with material particles. **(2)**
The photoelectric work function for a metal is 4.2 eV. Find the threshold wavelength. **(2)**

Ans:

1. De Broglie proposed that a moving material particle of total energy E and momentum p has a wave associated with it (analogous to a photon).
2. He suggested a relation between properties of wave, like frequency and wavelength, with that of a particle, like energy and momentum.
   $$p=\frac{E}{c}=\frac{hv}{c}=\frac{h}{\lambda}$$
3. Thus, frequency and wavelength of a wave associated with a material particle, of mass m moving with a velocity v, are given as
   $$v=\frac{E}{h} \text{ and } \lambda=\frac{h}{p}=\frac{h}{mv}$$
4. De Broglie referred to these waves associated with material particles as matter waves. The wavelength of the matter waves, given by equation (10, is now known as de Broglie wavelength and the equation is known as de Broglie relation.

$$Data: \varnothing=4.2eV, e=1.6\times10^{-19}C, h=6.63\times10^{-34}J\cdot s, c=3\times10^{8}m/s$$

$$\varnothing=4.2\, eV=|4.2\, eV|\left(1.6\times10^{-19}\frac{J}{eV}\right)=6.72\times10^{-19}J$$

$$\varnothing=hv_0=hc/\lambda_0 ¿$$

$$\therefore\ \lambda_0=\frac{hc}{\varnothing}$$

¿¿¿

¿ $\frac{19.89}{6.72} \times 10^{-7}$

¿ $2.960 \times 10^{-7} m = 296.0\, nm$.