2023	12	23

TS-P4

SEAT NUMBER

IIT INSPIRE
ACADEMY OF SCIENCE
A Place for Excellence & Perfection

XI & XII Science (CBSE/state)
IIT- JEE (Mains + Advance)
NEET, MH-CET , NDA
Mo. No. 9595445177/9021445177

Branches : Chhatrapati Sq., Mangalmurti Sq.

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(4 Pages)

calculator is not allowed.
5. All symbols having their usual meanings unless otherwise stated.
6. For each MCQ, correct answer must be written along with its alphabet.
7. Evaluation of each MCQ would be done for the first attempt only.

Physical Constants:
(1) $\pi=3.142$ (2) $g=10\, m/s^2$ (3) $h=6.63\times10^{-34}\, J.s$ (4) $c=3\times10^{8}\, m/s$
(5) $e=1.6\times10^{-19}\, C$ (6) $\varepsilon_0=8.85\times10^{-12}\, C^2/N.m^2$ (7) $\mu_0=4\pi\times10^{7}\, T.m/A$,
(8) $\sigma=5.7\times10^{-8}\, W/m^2 K^4$

SECTION-A

Q.1 Select and write the correct answers to the following questions: [10]

1) Water rises to a height 'h' in a capillary of the surface of the earth. On the surface of the moon, the height of water column in the same capillary will be ___________________
a) 6h **(1)**

2) Reversible thermodynamics process is ______
c) bidirectional process **(1)**

3) Kirchhoff's voltage law and current law are respectively in a accordance with the conservation of __________
c) energy and charge **(1)**

4) Weber-m^{-2} is equal to ____
a) tesla **(1)**

5) The core transformer is laminated to reduce the loses due to ____________
a) eddy currents **(1)**

6) At resonance, the voltage and current are
d) in phase **(1)**

7) A LED emits visible light when its
c) holes and electrons recombine **(1)**

8) A body is acted upon by a constant torque. In 4 second its angular momentum changes from L to 4L. The magnitude of the torque is __________

b) 3L/4 **(1)**

9) The potentiometer wire 10 m long and 20 ohm resistance is connected to 3 volt e.m.f battery and 10 ohm resistance. The value of potential gradient in volt/m of the wire will be

b) 0.2 **(1)**

10) An electron in an atom revolves around the nucleus in an orbit of radius 0.7 $\dot{A}$. Calculate the equivalent magnetic moment, if the frequency of revolution of electron $2 \times 10^{10} MHz$.

a) $4.92 \times 10^{-23} Am^2$ **(1)**

Q.2 Answer the following questions in one sentence: **[8]**

(1) **What is conical pendulum?** **(1)**

Ans:

A tiny mass (assumed to be a point object and called a bob) connected to a long, flexible, massless, inextensible string, and suspended to a rigid support revolves in such a way that the string moves along the surface of a right circular cone of vertical axis and the point object performs a uniform horizontal circular motion. Such a system is called a conical pendulum.

(2) **Why is the surface tension of the paints and lubricating oil kept low?** **(1)**

Ans:

Surface tension of lubricating oils and paints is kept low in order to help them spread over a large area.

(3) **Give an example of some familiar process in which heat is added to an object, without changing its temperature?** **(1)**

Ans:

During boiling of water although heat is continuously added to the water, its temperature does not change.

(4) **What is wavefront?** **(1)**

Ans:

The locus of all points having the same phase at a given instant of time is called a wavefront.

(5) **How galvanometer be converted into an ammeter?** **(1)**

Ans:

A galvanometer can be converted into an ammeter by connecting a low value resistance in parallel (shunt) with galvanometer.

(6) **Define magnetization.**
(1)

Ans:

The ratio of magnetic moment to the volume of the material is called magnetization.

(7) **Find kinetic energy of 5 litre of a gas at S.T.P. Given standard pressure is** $1.013 \times 10^5 N/m^2$**.** **(1)**

Ans:

Given: $V = 5\ L = 5 \times 10^{-3}\ m^3$
$P = 1.013 \times 10^5\ N/m^2$, $T = 273\ K$

To find: Kinetic energy of the gas (K.E.)

Formula: $K.E. = \frac{3}{2} PV$

Calculation:

From formula,

$$K.E. = \frac{3}{2} \times 1.013 \times 10^5 \times 5 \times 10^{-3}$$

$$= \mathbf{7.5975 \times 10^2\ J}$$

Ans: The kinetic energy of the gas is **7.5975×10^2 J.**

(8) **At what distance from the mean position is the kinetic energy of particle performing S.H.M. of amplitude 8 cm, three times its potential energy? (1)**

Ans:

Solution:

Given: A = 8 cm, K.E. = 3 P.E.

To find: Distance (x)

Formulae: i. $KE = \frac{1}{2} m\omega^2 (A^2 - x^2)$

ii. $PE = \frac{1}{2} m\omega^2 x^2$

Calculation:

Given: KE = 3 PE

$$\frac{1}{2} m\omega^2 (A^2 - x^2) = 3 \times \frac{1}{2} m\omega^2 x^2$$

.... [From formula (i) and (ii)]

$\therefore \quad 4x^2 = A^2$

$\therefore \quad x = \frac{A}{2} = \frac{8\ cm}{2} = \mathbf{4\ cm}$

Ans: Distance from the mean position where the kinetic energy is thrice of potential energy is **4 cm.**

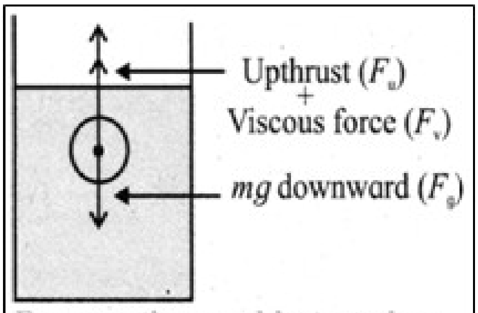
SECTION-B

Attempt any eight of the following questions: [16]

Q.3 Define terminal velocity. Obtain an expression for the terminal velocity of a small sphere falling under gravity through a viscous fluid. (2)

Ans:

Terminal velocity: *The constant maximum velocity acquired by a body falling through a viscous liquid is called as terminal velocity.*



Forces acting on object moving through a viscous medium.

Consider a spherical object falling through a viscous fluid. Forces experienced by it during its downward motion are,

1. Viscous force (F_v), directed upwards. Its magnitude goes on increasing with increase in its velocity.
2. Gravitational force, or its weight (F_g), directed downwards, and
3. Buoyant force or up thrust (F_u), directed upwards.

Net downwards force given by $f = F_g - (F_v + F_u)$, is responsible for initial increase in the velocity. Among the given the forces, F_g and F_u are constant while F_v increase with increase in velocity. Thus, a stage is

reached when the net force f becomes zero. At this stage, $F_g = F_v + F_u$. After that, the downward velocity remains constant. This constant downward velocity is called terminal velocity. Obviously, now onwards, the viscous force F_v is also constant. The entire discussion necessarily applies to streamline flow only.
Consider a spherical object falling under gravity through a viscous medium. Let the radius of the sphere be r, its mass m and density ρ. Let the density of the medium be σ and its coefficient of viscosity be η. When the sphere attains the terminal velocity, the total downward force acting on the sphere is balanced by the total upward force acting on the sphere.
Total downward force ¿ Total upward force weight of sphere (mg) ¿
Viscous force +¿ by out ant to due to the medium

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v + \frac{4}{3}\pi r^3 \sigma g$$

$$6\pi\eta r v = \left(\frac{4}{3}\pi r^3 \rho g\right) - \left(\frac{4}{3}\pi r^3 \sigma g\right)$$

$$6\pi\eta r v = \left(\frac{4}{3}\right)\pi r^3 g(\rho - \sigma)$$

$$v = \left(\frac{4}{3}\right)\pi r^3 g(\rho - \sigma) \times \frac{1}{6\pi\eta r}$$

$$v = \left(\frac{2}{9}\right)\frac{r^2 g(\rho - \sigma)}{\eta}$$

This is the expression for the terminal velocity of the sphere. We can also write,

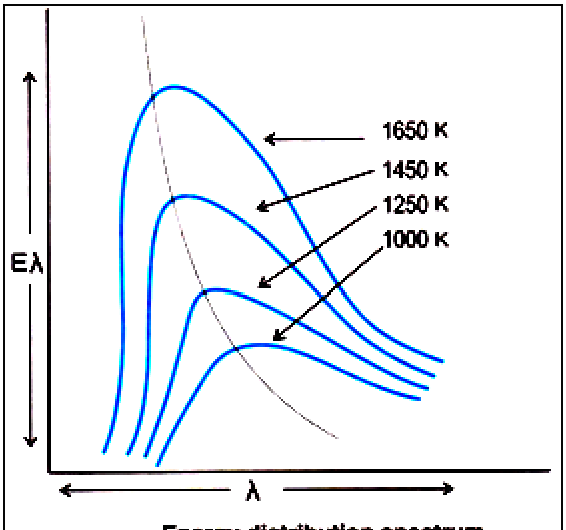
$$\eta = \frac{2}{9}\frac{r^2(\rho - \sigma) g}{v}$$

The above equation gives coefficient of viscosity of a fluid.

Q.4 Explain spherical distribution of black body radiation. (2)

Ans:

The study of spectrum of black body radiation in terms of wavelength was carried out by Lummer and Pringsheim maintaining the black body at different temperature. They kept the source at different fixed temperatures and measured the intensity of radiation corresponding to the different wavelengths. The measurements were represented graphically in the form of curves showing the variation of intensity of radiation (E_λ) with the wavelength (λ) at different constant temperatures.



Energy distribution spectrum of a black body.

Observation:

From the experimental curves it is observed that

1. Intensity of radiations emitted increases with increase of wavelength.
2. For particular wavelength (λ_{max}), the intensity of radiation emitted is maximum and then decreases with further increase in wavelength.
3. Area under the curve (E_λ) versus (λ) represents total energy emitted per second per unit area by the black body including all the wavelengths.

Conclusion:

When temperature of black body is increased

1. Energy distribution curve continues to be non-uniform.

2. Peak of E_λ versus λ curve shifts towards the left, it means as temperature increases, value of λ_{max} decreases.
3. At higher temperature total energy emitted per second per unit area corresponding to all wavelengths increases.

Q.5 What are harmonics and overtones? **(2)**

Ans:

1. A stationary wave is set up in bounded medium in which the boundary could be a rigid support (i.e., a fixed end, as for instance a string stretched between two rigid supports) or a free end (as for instance an air column in a cylindrical tube with one or both ends open). The boundary conditions limit the possible stationary waves and only a discrete set of frequency is allowed.
2. The lowest allowed frequency (n_1), is called the fundamental frequency of vibration.
3. Integral multiples of the fundamental frequency is called the harmonics.
4. $(n_1, n_2, n_3 \ldots\ldots)$ the fundamental frequency being called the first harmonic. The second harmonic is twice the fundamental or $2n_1$, the third harmonic is $3n_1$, and so on.
5. The harmonics represent the fundamental and all its integral multiples. They may be present in a given sound or not.
6. The higher allowed frequencies are called the overtones. Above the fundamental, the first allowed frequency is called the first overtone; the next higher frequency is the second overtone, and so on.
7. Overtones are always present in sound.
8. The relation between overtones and allowed harmonics depends on the system under consideration.

Q.6 Distinguish between a potentiometer and a voltmeter. **(2)**

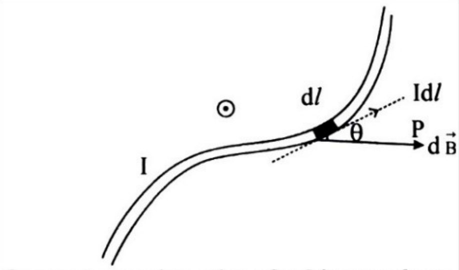
Ans:

No.	Potentiometer	Voltmeter
i.	Its resistance is infinite.	Its resistance is high but finite.
ii.	It does not draw any current from the source of known e.m.f.	It draws some current from the source of e.m.f.
iii.	The potential difference measured by it is equal to actual potential difference (p.d.).	The potential difference measured by it is less than the actual potential difference (p.d.).
iv.	It has high sensitivity.	It has low sensitivity.
v.	It measures e.m.f as well as p.d.	It measures only p.d.
vi.	It is used to measure internal resistance of a cell.	It cannot be used to measure the internal resistance of a cell.
vii.	It is more accurate.	It is less accurate.
viii.	It does not give direct reading.	It gives direct reading.
ix.	It is not portable.	It is portable.
x.	It is used to measure lower voltage values only.	It is used to measure lower as well as higher voltage values.

Q.7 Explain Biot-Savart law. **(2)**

Ans:

1. Consider an arbitrarily shaped wire carrying a current I.
2. Let dl be a length element along the wire. The current in this element is in the direction of the length vector $\vec{d}\,l$ which produces differential magnetic field $\overrightarrow{dB}$ directed into the plane of paper as shown in figure below:



Current carrying wire of arbitrary shape

3. Consider point P at distance r from element dl. Net magnetic field at the point P can be obtained by integrating i.e., summing up of magnetic field $\overrightarrow{dB}$ from these length elements.
4. Experimentally, the magnetic field $\overrightarrow{dB}$ produced by current I in the length element $\overrightarrow{dl}$ is

   $$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2} \text{ .... (1)}$$

   Where, $\theta$is the angle between the direction of $\vec{d}\,l$ and $\vec{r}$ and μ_0 (permeability of free space)

   ¿$4\pi \times 10^{-7} T\,m/A \approx 1.26 \times 10^{-6}\,Tm/A$
5. The direction of $d\vec{B}$ is dictated by the cross product $\vec{d}\,l \times \vec{r}$.

   Vectorially, $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}$ .... (2)

   Equation (1) and (2) are known as the Biot-Savart law.
6. Biot-Savart law can be used to calculated magnetic field produced by various distributions such has current carrying straight wire, current carrying circular arc etc.

Q.8 Explain what is self-inductance? **(2)**

Ans:

1. For parallel combination of two coils, the current through each parallel inductor is a fraction of the total current and the voltage across each parallel inductor is same.
2. As a result, a change in total current will result in less voltage dropped across the parallel array than for any one of the individual inductors.
3. There will be less voltage drop across parallel inductor for a given rate of change in current than for any of the individual inductors.
4. Less voltage for the same rate of change in current results in less inductance.
5. Thus, the total inductance of two coils is less than the inductance of either coil.

Q.9 What do you understand by the term wave-particle duality? Where does it apply? (2)

Ans:

1. From the experimental observations made by scientists, it was realized that some phenomena, like interference and diffraction, can be explained by considering light (or electromagnetic radiation in general) as a wave.

2. On the other hand, some other observations like photoelectric effect and black body radiation can be explained only if we consider light as consisting of photon with definite quantum of energy and momentum.
3. Also, there are some phenomena which can be explained by both the theories.
4. It is therefore necessary to keep both the physical models of the light to explain the careful experimental observations; one dominates in some situations and the other works in rest of the situations.
5. There is thus a need to hypothesize the dual character of light. This phenomenon is termed as wave-particle duality.
6. It applies not only for light but for the whole electromagnetic spectrum.

Q.10 During a stunt, a cyclist (considered to be a particle) is undertaking horizontal circles inside a cylindrical well of radius 6.05 m. If the necessary friction coefficient is 0.5, how much minimum speed should be the stunt artist maintain? Mass of the artist is 50 kg. If she/he increases the speed by 20%, how much will the force of friction be? (2)

Ans:

Given: $r = 6.05$ m, $\mu_s = 0.5$, $M = 50$ kg,
For case B, $v_2 = v_1 + 20\,\%$ of $v_1 = 1.2\, v_1$

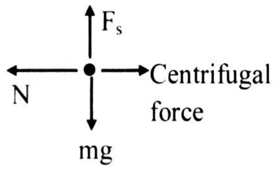
To find:
i. Minimum speed maintained by stunt-artist (v_{min})
ii. Force of friction if velocity is increased to 20%.

Formulae:
i. v_{min} for the horizontal position of rod $= \sqrt{2rg}$
ii. $F_s = mg$

Calculation:

i. From formula (i),

$$v_{min} = \sqrt{2rg} = \sqrt{2 \times 6.05 \times 10} = \sqrt{121} = \mathbf{11\ m/s}$$



ii. Force of friction,

$F_s = mg = 50 \times 10 = \mathbf{500\ N}$

Ans: i. The minimum speed maintained by stunt artist (v_{min}) is **11m/s**.
ii. The force of friction is **500N**.

Q.11 A gas contained in a cylinder fitted with a frictionless piston expands against a constant external pressure of 1 atm from the volume of 5 litres to a volume of 10 litres. In doing so it absorb 400 J of thermal energy from its surroundings. Determine the change in internal energy of the system. (2)

Ans:

Solution:

Given: $p = 1 \text{ atm} = 1.013 \times 10^5$ Pa, $Q = 400$ J,
$V_i = 5 \text{ L} = 5 \times 10^{-3} \text{ m}^3$,
$V_f = 10 \text{ L} = 10 \times 10^{-3} \text{ m}^3$

To find: Change in Internal energy of system.

Formulae: i. $W = p\Delta V = p(V_f - V_i)$

ii. $\Delta U = |Q| - |W|$

Calculation:

From formula (i),

$W = 1.013 \times 10^5 (10 \times 10^{-3} - 5 \times 10^{-3})$
$= 1.013 \times 10^5 \times 5 \times 10^{-3}$
$= 5.065 \times 10^2$
$= 506.5$ J

From formula (ii),

$\Delta U = |400| - |506.5|$
$= \mathbf{-106.5\ J}$

Ans: The change in the internal energy of the system is **– 106.5 J.**

Q.12 In a biprism experiment, the fringes are observed in the focal plane of the eyepieces at a distance of 1.2 m from the slits. The distance between the central bright band and the 20^{th} bright band is 0.4 cm. When a convex lens is placed between the biprism and the eyepiece, 90 cm from the eyepiece, the distance between the two virtual magnified images is found to be 0.9 cm. Determine the wavelength of the light used. (2)

Ans:

Given: $y_{20} = 0.4 \text{ cm} = 0.4 \times 10^{-2}$ m, $D = 1.2$ m,
$n = 20$
Magnified image,
$d_1 = 0.9 \text{ cm} = 0.9 \times 10^{-2}$ m.
$v = 90 \text{ cm} = 0.9$ m

$\therefore$ $u = D - v = 1.2 - 0.9 = 0.3$ m

To find: Wavelength (λ).

Formulae: i. $d = \dfrac{d_1 u}{v}$ ii. $y_n = \dfrac{n\lambda D}{d}$

Calculation:

From formula (i),

$$d = \frac{0.9 \times 10^{-2} \times 0.3}{0.9} = 0.3 \times 10^{-2} \text{ m.}$$

From formula (ii),

$$\lambda = \frac{y_n d}{nD}$$

$$\therefore \quad \lambda = \frac{0.4 \times 10^{-2} \times 0.3 \times 10^{-2}}{20 \times 1.2} = 5000 \times 10^{-10} \text{ m}$$

$= \mathbf{5000\ Å}$

Ans: Wavelength of the light used is **5000 Å.**

Q.13 A 6 μF capacitor is charge by 300 V supply. It is then disconnected from the supply and is connected to another uncharged 3 μF capacitor. How much electrostatic energy of the first capacitor is loss in the form of heat and electromagnetic radiation? (2)

Ans:

When capacitor $C_1 = 6\ \mu F$ is connected to 300 V supply, charge acquired by capacitor,

$Q = C_1V = 6 \times 10^{-6} \times 300$

$= 1800 \times 10^{-6}$ C

$= 18 \times 10^{-4}$ C

Energy stored in Capacitor,

$U_1 = \frac{1}{2}QV = \frac{1}{2}\times 18\times 10^{-4}\times 300$

$= 2700 \times 10^{-4}$

$= 27 \times 10^{-2}$ J

Now Capacitor C_1 is disconnected and connected to uncharged capacitor C_2.

The system of two capacitors will acquire common potential V′ and charge Q′

From charge conservation,

$Q = Q'$

$VC_1 = V'(C_1 + C_2)$

$V' = \frac{VC_1}{(C_1+C_2)} = \frac{300\times 6\times 10^{-6}}{(6+3)\times 10^{-6}} = 200$ V

Energy stored in system,

$U_2 = \frac{1}{2}QV' = \frac{1}{2}\times 18\times 10^{-4}\times 200$

$= 1800 \times 10^{-4}$

$= 18 \times 10^{-2}$ J

$\therefore$ Loss in energy $= U_1 - U_2$

$= 27 \times 10^{-2} - 18 \times 10^{-2}$

$= \mathbf{9 \times 10^{-2}}$ **J**

Q.14 A 100 μF capacitor is charge with a 50 V source supply. Then source supply is removed and the capacitor is connected across an inductance, as a result of which 5 A current flows through the inductance. Calculate the value of inductance. (2)

Ans:

Given: $C = 100\ \mu F = 100 \times 10^{-6}\ F = 10^{-4}\ F$

$V = 50$ V, $I = 5$ A

To find: Inductance (L)

Formulae: i. $U_E = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}CV^2$

ii. $U_B = \frac{1}{2}LI^2$

Calculation: From energy conservation,

Energy stored in capacitor

= Energy stored in inductor

From formula (i) and (ii)

$\frac{1}{2}LI^2 = \frac{1}{2}CV^2$

$\therefore$ $L = \frac{CV^2}{I^2} = \frac{10^{-4}\times(50)^2}{5^2}$

$= \frac{10^{-4}\times 5^2\times 10^2}{5^2} = 10^{-2} = \mathbf{0.01\ H}$

Ans: The value of Inductance is **0.01 H**.

SECTION-C

Attempt any eight of the following questions:
[24]

Q.15 State and prove theorem of parallel axes about moment of inertia. **(3)**

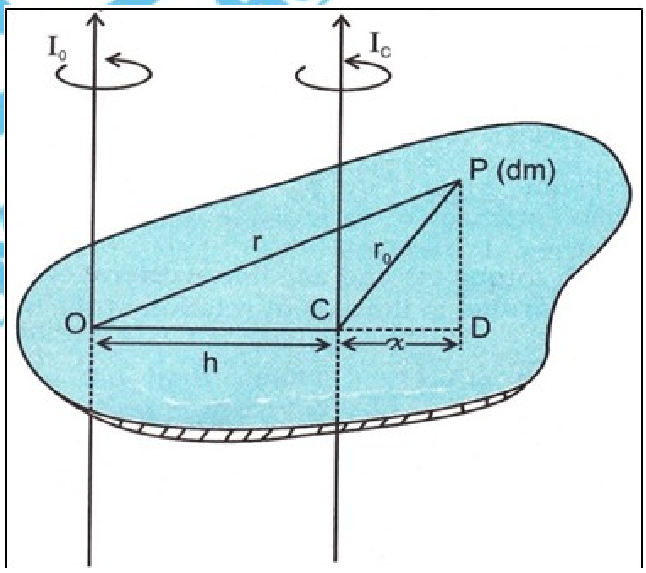
Ans:

Principle of parallel axes:

The moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis passing through its centre of mass and the product of the mass of the body and the square of the distance between the two parallel axes.

Proof:

1. Consider a rigid body of mass 'M' rotating about an axis passing through a point 'O' and perpendicular to plane of the figure.
2. Let 'I_0' be the moment of inertia of the body about an axis of rotation passing through point 'C', called the centre of mass of the body. Let 'I_c' be the moment of inertia of the body about point 'C'. Let, $OC=h$, the distance between the two parallel axes. $OP=r$ and $CP=r_0$
3. Take a small element of the body of mass 'dm' situated at a point 'P'. Join OP and CP then,
   $I_0=\int OP^2 dm=\int r^2 dm$ And
   $I_c=\int CP^2 dm=\int r_0^2 dm$
4. Now from point 'P', draw PD perpendicular to OC produced. Let distance $CD=x$
   From the geometry of the figure,
   $OP^2=OD^2+PD^2$
   $\therefore OP^2=(h+CD)^2+PD^2$
   ¿ $h^2+2h.CD+CP^2+PD^2$
   ¿ $h^2+2h.CD+CP^2$
   $(\because CD^2+PD^2=CP^2)$
   $\therefore OP^2=CP^2+h^2+2hCD$
   $\therefore r^2=r_0^2+h^2+2hx$ …………… (1)



5. Multiplying the above equation (1) by 'dm' on both the sides and integrating, we get
   $\int r^2 dm=\int r_0^2 dm+$¿ $\int h^2 dm+\int 2hx\, dm$¿
   $\therefore \int r^2 dm=\int r_0^2 dm+$¿ $\int h^2 dm+2h\int x dm$¿
6. $\int x\, dm=0$ as 'C' is centre of mass and algebraic sum of moments of all the particles about centre of mass is always zero, for body in equilibrium.
   $\therefore \int r^2 dm=\int r_o^2 dm+h^2\int dm+0$ ……… (2)
   But $\int dm=M=$¿¿ Mass of the body
   And $\int r^2 dm=I_0 \wedge \int r_0^2 dm=I_c$
   $\therefore$ Equation (2) becomes $I_0=I_c+Mh^2$
   This proves the theorem of parallel axes about moment of inertia.

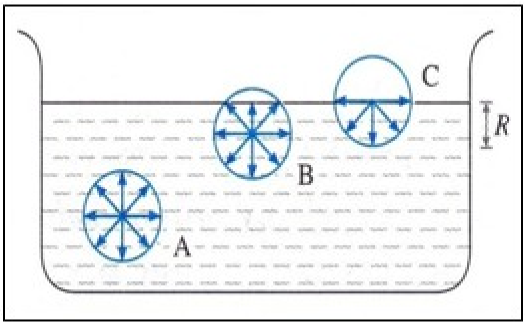
Q.16 Explain the surface tension on the basis of molecular theory. **(3)**

Ans:

The phenomenon of surface tension arises due to the cohesive forces between the molecules of a liquid.

The net cohesive force on the liquid molecules within the surface film differs from that on molecules deep in the interior of the liquid.

Consider three molecules of a liquid: A molecule A well inside the liquid, and molecules B and C lying within the surface film. The figure also shows their sphere of influence of radius R.



1. The sphere of influence of molecule A is entirely inside the liquid and the molecule is surrounded by its nearest neighbours on all side. Hence, molecule A is equally attracted from all sides, so that the resultant cohesive force acting on it is zero. Hence, it is free to move anywhere within the liquid.
2. For molecule B, large part of its sphere of influence lies inside the liquid. It's just below free surface at a distance less than range of molecular attraction (R). A small part of its sphere of influence is outside the liquid surface. This part contains air molecules whose number is negligible compared to the number of molecules in an equal volume of the liquid. Therefore, molecules B experience a net downward resultant unbalanced cohesive force, trying to pull it inside the liquid.
3. For molecule C, the upper half of its sphere of influence is outside the liquid surface and lower half lying inside the liquid but no of air molecule in a sphere of influence is very small as compared to liquid molecule. Therefore, the net downward resultant unbalanced cohesive forces act on molecule C as a result the molecule 'C' gets attracted inside the liquid molecule within liquid.
4. Thus, all molecules lying within a surface film of thickness equal to R experience a net unbalanced downward cohesive force directed into the liquid. Therefore, they all are pulled inside the liquid.
5. These minimize the total no of molecule in surface film. Hence surface film shrinks, become horizontal (except near wall). Since it cannot shrink any further, beyond a limit, it remains under tension. Thus, the surface film of liquid behaves like a stretched elastic membrane under tension. This is known as surface tension.
6. The surface area is proportional to the number of molecules on the surface. To increase the surface area, molecules must be brought to the surface from within the liquid. For this, work must be done against the cohesive forces. This work is stored in the liquid surface in the form of potential energy. With a tendency to have minimum potential energy, the liquid tries to reduce the number of molecules on the surface so as to have minimum surface area. This is why the surface of a liquid behaves like a stressed elastic membrane and gives rise to the phenomenon of the surface tension.

Q.17 Determine the expression for the work done and heat transferred for an isothermal process. (3)

Ans:

1. Consider the isothermal expansion of an ideal gas.
2. Let its initial volume be V_i and the final volume be V_f.
3. The work done in an infinitesimally small isothermal expansion is given by, $dW = pdV$.
4. The total work done in bringing out the expansion from the initial volume V_i to the find volume V_f is given by,

   $$W = \int_{V_i}^{V_f} pdV \quad \ldots\ldots\ldots\ldots\ (1)$$
5. But for an ideal gas, $pV = nRT$. Using this in the equation (1) we get,

$$W = nRT\int_{V_i}^{V_f} \frac{dV}{V}$$

$$\therefore W = nRTln\frac{V_f}{V_i} \text{ .... (2)}$$

6. For an ideal gas, its internal energy depends on its temperature. Therefore, during an isothermal process, the internal energy of an ideal gas remains constant $(\Delta U = 0)$ because its temperature is constant $(\Delta T = 0)$.
7. The first law of thermodynamics when applied to an isothermal process,
   $Q = W$ .... (3)

   $$\therefore Q = WnRTln\frac{V_f}{V_i} \text{ .... (4)}$$
8. Thus, the heat transferred to the gas is completely converted into the work done, i.e., for expansion of the gas.

Q.18 What are the condition for obtaining good interference pattern? Give reasons. (3)

Ans:

The following conditions have to be satisfied for the interference pattern to be steady and clearly visible.

1. **The two sources of light should be coherent:** This is the essential condition for getting sustained interference pattern. As we have seen, the waves emitted by two coherent sources are always in phase or have a constant phase difference between them at all times. If the phases and phase difference vary with time, the positions of maxima and minima will also change with time and the interference pattern will not be steady. For this reason, it is preferred that the two secondary sources used in the interference experiment are derived from a single original source.
2. **The two sources of light must be monochromatic:** As can be seen from the condition for bright and dark fringes, the position of these fringes as well as the width of the fringes depends on the wavelength of light and the fringes of different colours are not coincident. The resultant pattern contains coloured, overlapping bands. (In fact, original Young's experiment was with pin holes (not slits) and for sunlight, producing coloured interference pattern with central point as white).
3. **The two interfering waves must have the same amplitude:** Only if the amplitudes are equal, the intensity of dark fringes (destructive interference) is zero and the contrast between bright and dark fringes will be maximum.
4. **The separation between the two slits must be small in comparison to the distance between the plane containing the slits and the observing screen:** This is necessary as only in this case, the width of the fringes will be sufficiently large to be measurable and the fringes are well separated and can be clearly seen.
5. **The two slits should be narrow:** If the slits are broad, the distances from different points along the slit to a given point on the screen are significantly different and therefore, the waves coming through the same slit will interfere among themselves, causing blurring of the interference pattern.
6. **The two waves should be in the same state of polarization:** This is necessary only if polarized light is used for the experiment.

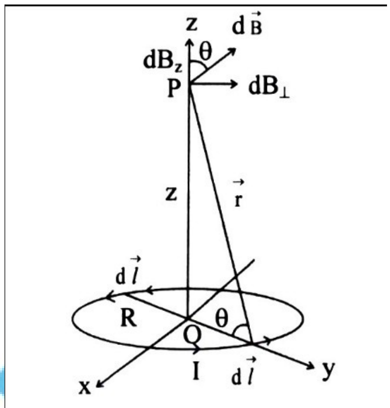
Q.19 Derive an expression for axial magnetic field produced by current in a circular loop. Then, using the derived expression, obtain equation of field at centre of loop as a special case. (3)

Ans:

1. Consider loop of radius R carrying I placed in x-y plane with its centre at origin O as shown in figure below.
2. Let point P can be on z-axis at distance $\vec{r}$ from line element $d\vec{l}$ of the loop.

3. Using Biot-Savart law, the magnitude of the magnetic field dB is given by

   $$dB = \frac{\mu_0}{4\pi} I \frac{|d\vec{l} \times \vec{r}|}{r^3} \quad \text{.......... (1)}$$



Magnetic field on the axis of a circular current loop of radius R

4. Any element $d\vec{l}$ will always be perpendicular to the vector $\vec{r}$ from the element to the point P. The element $d\vec{l}$ is in the x-y plane, while the vector $\vec{r}$ is in the y-z plane. Hence $d\vec{l} \times \vec{r} = dl.r$

   $$\therefore dB = \frac{\mu_0}{4\pi} I \frac{dl}{r^2} \quad \text{............. (2)}$$

   ¿ $\frac{\mu_0}{4\pi} I \frac{dl}{(z^2+R^2)}$ .... (3)

5. Now, the direction of $d\vec{B}$ is perpendicular to the plane formed by $d\vec{l}$ and $\vec{r}$. Its z component is dB_z and the component perpendicular to the z-axis is $dB_\perp$. The component $dB_\perp$ when summed over, yield zero s they cancel out due to symmetry. Hence, only z component remains.

6. ∴ The net contribution along the z axis is obtained by integrating $dB_z = dB\cos\theta$ over the entire loop.

   From figure,

   $$\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{z^2+R^2}}$$

   $$\therefore B_z = \int dB_z = \frac{\mu_0}{4\pi} I \int \frac{dl}{(z^2+R^2)} . \cos\theta$$

   ¿ $\frac{\mu_0}{4\pi} I \int \frac{R\,dl}{(z^2+R^2)^{\frac{3}{2}}}$

   ¿ $\frac{\mu_0}{4\pi} \frac{IR}{(z^2+R^2)^{\frac{3}{2}}} . 2\pi R$

   $$B_z = \frac{\mu_0}{2} \frac{IR^2}{(z^2+R^2)^{\frac{3}{2}}} \quad \text{........... (4)}$$

   This is the magnitude of the magnetic field due to current I in the loop of radius R, on a point at P on the z axis of the loop.

Q.20 Obtain an expression for the radius is proportional to square of the principal quantum number. (3)

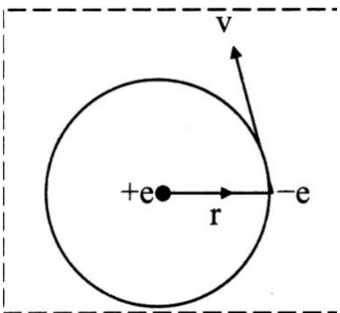
Ans:

1. To find out the radius of Bohr's orbit let us consider an electron of mass m and change –e revolves in a circular orbit of radius r as shown in the diagram.

   According to Bohr's first postulate,

   $$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2}$$

   $$\therefore v^2 = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{mr} \quad \text{......... (1)}$$



2. According to Bohr's second postulate

   Angular momentum ¿ $\frac{nh}{2\pi}$

   $$\therefore mvr = \frac{nh}{2\pi}$$

Squaring both the sides

$m^2 v^2 r^2 = \frac{n^2 h^2}{4\pi^2}$

$\therefore v^2 = \frac{n^2 h^2}{4\pi^{2¿¿}¿}$ .... (2)

3. On equating (1) and (2)

   $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr} = \frac{n^2 h^2}{4\pi^{2¿¿}¿}$

   $\therefore r = \left(\frac{\varepsilon_0 h^2}{\pi m e^2}\right) n^2$ .... (3)

4. All the quantities from bracket are constant.

   $\therefore r \propto n^2$

   Radius of Bohr orbit is directly proportional to square of principle quantum number. We get different values of radius, by keeping the values of n ¿ 1, 2, 3, ... in above equation.

Q.21 Draw a neat diagram of full wave rectifier and explain its working. **(3)**

Ans:

1. Full wave rectifier using diode is as shown in the diagram. In this circuit two diodes and a central taped transformer is used. P terminal of both the diodes are connected to the secondary of transformer and N terminals of both the diodes are connected to a common point. It is then connected to central tap of transformer through load resistance R_L. Output voltage is taken across load resistance.
2. When AC voltage is applied to primary of the transformer, voltage across secondary is developed. Terminals S_1 and S_2 become +ve and –ve alternately with respect to central tap C.
3. When S_1 is at +ve potential with respect to C, diode D_1 will be in forward bias and current flows through load resistance R_L from A to B. The output voltage is obtained in +ve half cycle. At this time diode D_2 is in reverse bias.
4. When S_2 is at +ve potential with respect to C. diode D_2 is in forward bias. In this case also current flows through load resistance R_L, in the same direction i.e., from A to B.
5. In both the half cycle we get the output voltage across load resistance. So, it is called as full wave rectifier.

   The variation output DC voltage with respect to AC input voltage is as shown in the diagram.

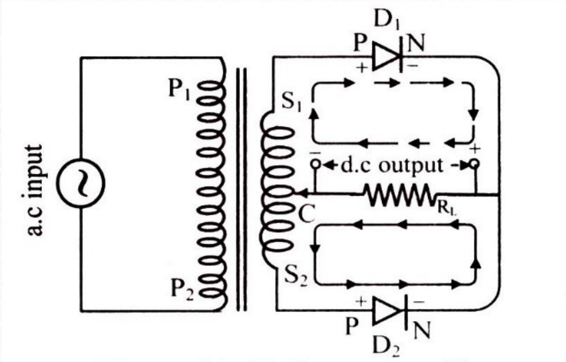


Figure (b): Full wave rectifier

P_1P_2 : Primary coil of step down transformer

S_1S_2 : Secondary coil of step down transformer

D_1 and D_2 : P-N junction diodes

C : Centre tap transformer

Q.22 A rod of magnetic material of cross section $0.25\,cm^2$ is located in $4000\,Am^{-1}$ magnetising field. Magnetic flux passing through the rod is $25\times10^{-6}\,Wb$. Find out

1) relative permeability

2) magnetic susceptibility and

3) magnetization of the rod **(3)**

Ans:

Given: $A = 0.25\ cm^2$,
$= 0.25 \times 10^{-4}\ m^2$,
$= 25 \times 10^{-6}\ m^2$,
$\phi_B = 25 \times 10^{-6}$ Wb,
$H = 4000\ Am^{-1}$,
$\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$

To find:
i. Relative permeability (μ_r)
ii. Magnetic susceptibility (χ)
iii. Magnetisation of the rod (M)

Formulae:
i. $\phi = BA$ ii. $B = \mu H$
iii. $\mu_r = \dfrac{\mu}{\mu_0}$ iv. $\mu_r = \chi + 1$
v. $M = \chi H$

Calculation: From formula(i),
$25 \times 10^{-6} = B \times 25 \times 10^{-6}$
$\therefore\ B = \dfrac{25 \times 10^{-6}}{25 \times 10^{-6}} = 1\ T$

From formula (ii),
$1 = \mu \times 4000$
$\therefore\ \mu = 2.5 \times 10^{-4}$
From formula (iii),
$\mu_r = \dfrac{2.5 \times 10^{-4}}{4 \times 3.142 \times 10^{-7}}$
$= \{\text{antilog}(\log 2.5 - \log 4 - \log 3.142)\} \times 10^{7-4}$
$= \{\text{antilog}(0.3979 - 0.6020 - 0.4972)\} \times 10^{7-4}$
$= \{\text{antilog}(\bar{1}.2987)\} \times 10^3$
$\therefore\ \mu_r = 1.99 \times 10^{-1} \times 10^3 = \mathbf{199}$
From formula (iv),
$199 = \chi + 1$
$\therefore\ \chi = \mathbf{198}$
From formula (v),
$M = 198 \times 4000$
$\therefore\ M = \mathbf{7.92 \times 10^5\ Am^{-1}}$

Ans: i. Relative permeability of the material is **199**.
ii. Magnetic susceptibility of the material is **198**.
iii. Magnetisation of the rod is $\mathbf{7.92 \times 10^5\ Am^{-1}}$.

Q.23 A long solenoid consisting of 1.5×10^3 turns/m has an area of cross section of $25\,cm^2$. A coil C, consisting of 150 turns (N_c) is wound tightly around the centre of the solenoid. Calculate for a current of 3.0 A in the solenoid
1) the magnetic flux density at the centre of the solenoid,
2) the flux linkage in the coil C,
3) the average emf induced in the coil C if the current in the solenoid is reversed in direction in time of 0.5 s. $(\mu_0 = 4\pi \times 10^{-7}\ Tm/A)$ **(3)**

Ans:

Given: $n = 1.5 \times 10^3$ turns/m
$A = 25\ cm^2 = 25 \times 10^{-4}\ m^2$,
$I = 3.0$ A, $N_c = 150$, $t = 0.5$ s,

To find:
i. Magnetic flux density (B)
ii. Flux linkage
iii. Average emf (e_{avg})

Formulae:
i. $B = \mu_0 nI$
ii. Flux linkage = $N\phi$
iii. $e_{avg} = \dfrac{\text{Change in flux}}{\text{time}}$

Calculation:

From formula (i),

$B = 4 \times 3.142 \times 10^{-7} \times 1.5 \times 10^{3} \times 3$

$= 56.56 \times 10^{-4}$

$= 5.656 \times 10^{-3} = \mathbf{5.66 \times 10^{-3}\ T}$

From formula (ii)

Flux linkage

$= N\,(BA)$ ....($\because \phi = BA$)

$= 150 \times 5.66 \times 10^{-3} \times 25 \times 10^{-4}$

$= 21225 \times 10^{-7}$

$= \mathbf{2.12 \times 10^{-3}\ Wb}$

From formula (iii),

$$E = \frac{2 \times 2.12 \times 10^{-3}}{0.5} = \mathbf{8.48 \times 10^{-3}\ V}$$

Ans: i. Magnetic field is **5.66×10^{-3} T**.

ii. Flux linkage is **2.12×10^{-3} Wb**.

iii. Induced emf is **8.48×10^{-3} V**.

Q.24 A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what will happen to the capacitive reactance and the current? (3)

Ans:

Given: $C = 15.0\ \mu F = 15 \times 10^{-6}$ F,

$e_{rms} = 220$ volt, $f_1 = 50$ Hz,

$f_2 = 2f_1 = 2 \times 50 = 100$ Hz

To find:
i. Capacitive reactance $(X_C)_1$
ii. r.m.s. current $(i_{rms})_1$
iii. Peak current $(i_0)_1$
iv. Capacitive reactance when frequency is doubled $(X_C)_2$
vi. Peak current when frequency is doubled $(i_0)_2$

Formula:
i. $X_C = \frac{1}{2\pi fC}$

ii. $i_{rms} = \frac{e_{rms}}{X_C}$

iii. $i_0 = \sqrt{2}\ i_{rms}$

Calculation: From formula (i),

$$X_C = \frac{1}{2 \times 3.142 \times 50 \times 15 \times 10^{-6}}$$

$$= \frac{10^4}{3.142 \times 15}$$

$= \text{antilog}\ \{\log(10^4) - \log(3.142) - (\log 15)\}$

$= \text{antilog}\ \{4 - 0.4972 - 1.1761\}$

$= \text{antilog}\ \{2.3267\}$

$= 2.121 \times 10^2 \approx \mathbf{212\ \Omega}$

From formula (ii),

$$i_{rms} = \frac{e_{rms}}{X_C} = \mathbf{1.04 A}$$

From formula (iii)

$i_0 = \sqrt{2} \times 1.04 = 1.414 \times 1.04 = \mathbf{1.470\ A}$

Now, $X_C = \frac{1}{2\pi fC}$

$\therefore \quad X_C \propto \frac{1}{f}$

$\therefore \quad \frac{(X_C)_1}{(X_C)_2} = \frac{2f}{f}$

$\therefore \quad (X_C)_2 = \frac{1}{2}(X_C)_1$

$\therefore$ If frequency is doubled, then capacitive reactance will be **halved**.

For capacitive circuit,

$i = 2\pi fC \times e$

$\therefore \quad i \propto f$

$\therefore \quad \frac{i_1}{i_2} = \frac{f}{2f} \qquad \therefore \quad i_2 = 2i_1$

$\therefore$ If frequency is doubled, then current will also get **doubled**.

Ans: When the frequency is 50 Hz,

i. The capacitive reactance in the circuit is **212 Ω.**

ii. The rms current in the circuit is **1.04 A.**

iii. The peak current in the circuit is **1.47 A.**

iv. When the frequency is doubled, the capacitive reactance becomes **half** and the current becomes **double**.

Q.25 Calculate the wavelength associated with an electron, its momentum and speed
1) when it is accelerated with the potential of 54 V,
2) when it is moving with kinetic energy of 150 eV. (3)

Ans:

Given: V = 54 V, E = 150 eV

To find:
a. wavelength of electron (λ)
b. momentum (p)
c. velocity (v)

Formulae:
a. $\lambda \text{ (in nm)} = \frac{1.228}{\sqrt{V}}$
b. $\lambda = \frac{h}{p}$
c. $p = m_e v$
d. $E \text{ (in J)} = qV$

Calculation:

i. From formula (a),

$$\lambda = \frac{1.228}{\sqrt{54}} = \frac{1.228}{7.35} = \mathbf{0.167\ nm}$$

From formula (b),

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{0.167\times10^{-9}}$$
$= \text{antilog } \{\log(6.63) - \log(0.167)\} \times 10^{-25}$
$= \text{antilog } \{0.8215 - \bar{1}.2227\} \times 10^{-25}$
$= \text{antilog } \{1.5988\} \times 10^{-25}$
$= \mathbf{39.70 \times 10^{-25} kgms^{-1}}$

From formula (c),

$$v = \frac{p}{m_e} = \frac{39.70\times10^{-25}}{9.1\times10^{-31}}$$
$= \text{antilog } \{\log(39.7) - \log(9.1)\} \times 10^{6}$
$= \text{antilog } \{1.5988 - 0.9590\} \times 10^{6}$
$= \text{antilog } \{0.6398\} \times 10^{6}$
$= \mathbf{4.363 \times 10^{6}\ m/s}$

ii. From formula (d),

$$V = \frac{E(\text{in J})}{q} = \frac{q \times E(\text{in eV})}{q}$$

$\therefore \quad V = 150\ V$

From formula (a),

$$\lambda = \frac{1.228}{\sqrt{150}} = \frac{1.228}{12.25} \approx \mathbf{0.100\ nm}$$

From formula (b),

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{0.100\times10^{-9}}$$
$= \mathbf{66.3 \times 10^{-25} kgms^{-1}}$

From formula (c),

$$v = \frac{p}{m_e} = \frac{66.30\times10^{-25}}{9.1\times10^{-31}}$$
$= \text{antilog } \{\log(66.3) - \log(9.1)\} \times 10^{6}$
$= \text{antilog } \{1.8215 - 0.9590\} \times 10^{6}$
$= \text{antilog } \{0.8625\} \times 10^{6}$
$= \mathbf{7.286 \times 10^{6}\ m/s}$

Ans:

i. a. The wavelength of electron is **0.167 nm.**
b. The momentum is **39.70 × 10^{-25} kgms^{-1}.**
c. The velocity is **4.363 × 10^{6} m/s.**

ii. a. The wavelength of electron is **0.100 nm.**
b. The momentum is **66.30 × 10^{-25} kgms^{-1}.**
c. The velocity is **7.286 × 10^{6} m/s.**

Q.26 The isotopes ^{57}Co decay by electron capture to ^{57}Fe with a half-life of 272 d. The ^{57}Fe nucleus produced in an excited state, and it almost instantaneously emits gamma rays.

1) find the mean lifetime and decay constant for ^{57}Co.

2) If the activity of a radiation source ^{57}Co is 2.0 μCi now, how many ^{57}Co nuclei does the source contain? (3)

Ans:

Given: $T_{1/2}$ of nuclear reaction

$(^{57}Co + e^- \rightarrow {}^{57}Fe) = 272$ days

$= 272 \times 24 \times 60 \times 60$ sec

$= 2.35 \times 10^7$ sec

$\left(\dfrac{dN(t)}{dt}\right)_i = 2.0\ \mu Ci$

$= 2.0 \times 3.7 \times 10^{10} \times 10^{-6}$ decays/sec

$= 7.4 \times 10^4$ /s

To find:
i. Mean lifetime of ^{57}Co (τ)
ii. decay constant of ^{57}Co (λ)
iii. Number of nuclei of ^{57}Co (N)
iv. Activity after one year $\left(\dfrac{dN(t)}{dt}\right)_f$

Formulae:
i. $\tau = \dfrac{T_{1/2}}{0.693}$ ii. $\lambda = \dfrac{1}{\tau}$

iii. $N(t) = \dfrac{\left|\dfrac{dN(t)}{dt}\right|}{\lambda}$

iv. $A(t) = A_0 e^{(-\lambda t)}$

Calculation:

From formula (i),

$\tau = \dfrac{2.35 \times 10^7}{0.693}$ s $= \mathbf{3.391 \times 10^7}$ **s**

From formula (ii),

$\lambda = \dfrac{1}{\tau} = \dfrac{1}{3.39 \times 10^7} = \mathbf{2.945 \times 10^{-8}\ s^{-1}}$

Now for number of nuclei when activity is 2 μCi

From formula (iii),

$N(t) = \dfrac{\left(\left|\dfrac{dN(t)}{dt}\right|\right)_i}{\lambda} = \dfrac{7.4 \times 10^4}{2.95 \times 10^{-8}}$

$= \{\text{antilog}(\log 7.4 - \log 2.945)\} \times 10^{12}$

$= \{\text{antilog}(0.8692 - 0.4690)\} \times 10^{12}$

$= \{\text{antilog}(0.4002)\} \times 10^{12}$

$= \mathbf{2.513 \times 10^{12}}$

Form formula (iii),

$A(t_1) = A_0\, e^{-\lambda t_1}$

After 1 year, $t_2 = (t_1 + 1)$ year

$\therefore A(t_2) = A_0 e^{-\lambda t_2} = A_0\, e^{-\lambda(t1+1)}$

$= A_0 e^{-\lambda t_1} e^{-\lambda(1)}$

$= A_0 e^{-\lambda t_1} e^{-2.945 \times 10^{-8} \times 365 \times 24 \times 3600}$

....(Converting year into second)

$= A_0 e^{-\lambda t_1} \times 0.3951$

$= 2 \times 0.3951$ ....($\because A_0 e^{-\lambda t_1} = 2\ \mu Ci$)

$= \mathbf{0.7902\ \mu Ci}$

Ans:
i. Mean life time of ^{57}Co is $\mathbf{3.391 \times 10^7}$ **s**.
ii. Decay constant of ^{57}Co is $\mathbf{2.945 \times 10^{-8}\ s^{-1}}$.
iii. Initial nuclei number is $\mathbf{2.513 \times 10^{12}}$.
iv. Activity after one year is **0.7902 μCi**.

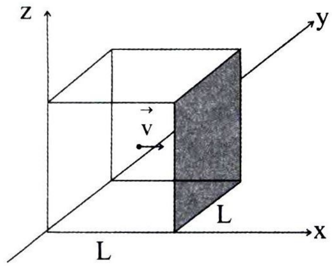
SECTION-D

Attempt any three of the following question: [12]

Q.27 Prove the relation between pressure of the gas and speed of its molecules. (4)

Ans:

1. Let there be n moles of an ideal gas enclosed in a cubical box of volume V(=L) with sides of the box parallel to the coordinate axes, as shown in figure. The walls of the box are kept at a constant Temperature T.



2. The gas molecules are in continuous random motion, colliding with each other and hitting the walls of the box and bouncing back.
3. As per one of the assumptions, we neglect intermolecular collisions and consider only elastic collisions with the walls.
4. A typical molecule moving with the velocity v, about to collide elastically with the shaded wall of the cube parallel to yz-plane.
5. During elastic collision, the component v_x of the velocity will get reversed, keeping $v_y \wedge v_z$ components unaltered.

6. Hence the change in momentum of the particle is only in the X component of the momentum, ΔP_x is given by,
ΔP_x = final momentum-initial momentum
= $(-mV_x)-(mV_x)=-2mV_x\ldots.(1)$
7. Thus, the momentum transferred to the wall during collision is + 2mV_x. The re-bounced molecule then goes to the apposite wall and collides with it.
8. After colliding with the shaded wall, the molecule travels to the opposite wall and travels back towards the shaded wall again
9. This means that the molecule travels a distance of a 2L in between two collisions.
10. As L is the length of the cubical box, the time for the molecule to travel back and forth to the shaded wall is $\Delta t=\frac{2L}{v_x}$.
11. Average force exerted on the shaded wall by molecule 1 is given as,
Average force = Average rate of change of momentum
$$\therefore F_{avg}=\frac{2mV_{x_1}}{2L/V_{x_1}}=\frac{2L}{V_x}$$
Where V_{x_1} is the X component of the velocity of molecule 1.
12. Considering other molecules 2, 3, 4 ... with the respective x components of velocities $V_{x_2}, V_{x_3}, V_{x_4}\ldots.$, the total average force on the wall is,
$$F_{AVg}=\frac{m}{L}\left(V_{x_1}^2+v_{x_2}^2+v_{x_3}^2+\ldots\right)\ldots..[¿(2)]$$
∴The average pressure
$$P=\frac{Average\ force}{Area\ of\ shaded\ wall}=\frac{m\left(V_{x_1}^2+v_{x_2}^2+\ldots.\right)}{L\times L^2}$$
13. The average of the square of the x component of the velocities is given by,
$$\overline{V_x^2}=\frac{V_{x_1}^2+v_{x_2}^2+v_{x_3}^2+\ldots+v_N^2}{N}$$
$$\therefore P=\frac{mN\overline{V_x^2}}{V}$$
Where $\overline{V_x^2}$ is the average over all possible values of V_x.
14. Now, $\overline{V^2}=\overline{V_x^2}+\overline{V_y^2}+\overline{V_z^2}$
By symmetry, $\overline{V_x^2}=\overline{V_y^2}=\overline{V_z^2}=\frac{1}{3}\overline{V^2}$ since the molecules have no preferred direction to move.
Therefore, average pressure
$$P=\frac{1}{3}\frac{N}{V}m\overline{V^2} \qquad \ldots\ldots(3)$$

Q.28 A particle performing linear S.H.M. of period 2π seconds about the mean position O is observed to have a speed of $b\sqrt{3}m/s$, when at a distance b (metre) from O. If the particle is moving away from O at that instant, find the time required by the particle, to travel a further distance b. (4)
Ans:

Given: $T = 2\pi$ s, $v(b) = b\sqrt{3}$ m/s, $x = b$ m

To find: Time required by object to travel distance from b to 2b $(t_2 - t_1)$

Formulae: i. $\omega = \frac{2\pi}{T}$ ii. $v = \omega\sqrt{A^2 - x^2}$

iii. $v = \omega A \cos \omega t$ iv. $x = A \sin \omega t$

Calculation:

From formula (i),

$\omega = \frac{2\pi}{2\pi} = 1 \text{ rad s}^{-1}$

From formula (ii),

$b\sqrt{3} = (1) \times \sqrt{A^2 - b^2}$

Squaring both sides,

$3b^2 = A^2 - b^2$

$\therefore$ $4b^2 = A^2$

$\therefore$ $A = 2b$

From formula (iii),

$b\sqrt{3} = 1 \times 2b \times \cos(1 \times t_1)$

$\therefore$ $\frac{\sqrt{3}}{2} = \cos t_1$

$\therefore$ $t_1 = \frac{\pi}{6}$ s

From formula (iv),

$2b = 2b \sin(1 \times t_2)$

$\therefore$ $1 = \sin t_2$

$\therefore$ $t_2 = \frac{\pi}{2}$

$\therefore$ $t_2 - t_1 = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$ **s**

Ans: The time required by object is $\frac{\pi}{3}$ **s.**

Q.29 Show that only odd harmonics are present in the vibration of air column in a pipe closed at the one end. **(4)**

Ans:

If longitudinal waves are sent along the air column in a pipe closed at one end, by holding a vibrating tuning fork near the open end, they get reflected at the closed end. Thus, the air column contains the incident and reflected longitudinal waves which interfere with each other and longitudinal stationary waves are produced.

The air at the closed end is not free to vibrate and hence a node is formed at the closed end. Air at the open end is free to vibrate and so an antinode is formed at the open end. The different ways in which the air column can vibrate are called modes of vibration of the air column.

1. The simplest mode of vibration is called the first or fundamental mode of vibration.

The distance between a node and consecutive antinode is $\frac{\lambda}{4}$ where λ is the wavelength of sound.

Here, $L = \frac{\lambda}{4}$ $\therefore \lambda = 4L$

But, $v = n\lambda = n \times 4L$

$\therefore n = \frac{v}{4L}$

Where, n is the frequency of vibration at resonance.

$n = \frac{v}{4(l+e)}$ (Considering end correction)

2. In second mode of vibration, two antinodes and two nodes are formed.

$\therefore$ Length of air column ¿ $L = \frac{3\lambda_1}{4}$ $\therefore \lambda_1 = \frac{4L}{3} = \frac{4(l+e)}{3}$

λ_1 is the wavelength of in second mode of vibration of air column.

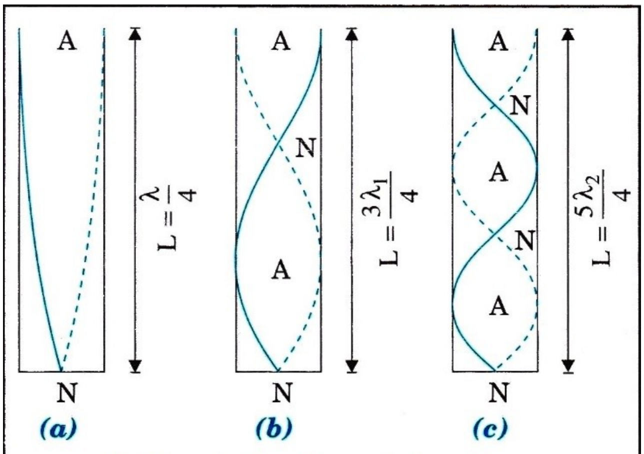
Let n_1 be the frequency of wave in second mode of vibration of air column.

$\therefore v = n_1\lambda_1$ $\therefore n_1 = \frac{v}{\lambda_1} = \frac{3v}{4L} = \frac{3v}{4(l+e)}$

$\therefore n_1 = 3n$ $\left(\because n = \frac{\upsilon}{4L}\right)$

This frequency is called third harmonic or first overtone.

3. In third mode of vibration of air column, three antinodes and three nodes are formed.



Modes of vibration of air column in a pipe closed at one end

$\therefore$ Length of air column¿ $L=\frac{5\lambda_2}{4}$ $\therefore \lambda_2=\frac{4L}{5}=\frac{4(l+e)}{5}$

But, $v=n_2\lambda_2$

Where, n_2, λ_2 are the frequency and wavelength of the wave in third mode of vibration of air column

$\therefore v=n_2\frac{4L}{5}$

$\therefore n_2=\frac{5v}{4L}=\frac{5v}{4(l+e)}=5\left(\frac{v}{4L}\right)=5n$ $\therefore n_2=5n$

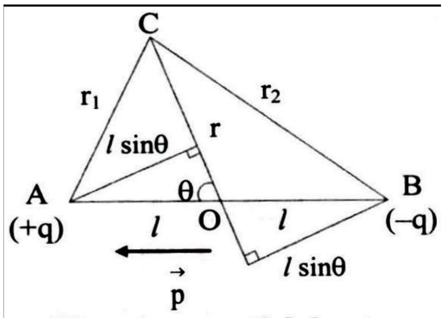
This frequency is called the frequency of fifth harmonic or second overtone.

4. Similarly, frequency of p^{th} overtone,
   $n_p=(2p+1)n$
   Where $p=0,1,2,3$ ......
   Thus, possible frequencies of the air column are n, 3n, 5n ....
   It means that only odd harmonics are present as overtone in the mode of vibration of air column closed at one end.

Q.30 Derive an expression for electric potential due to an electric dipole. Discuss the same at axial and equatorial point. **(4)**

Ans:

1. An electric dipole AB consisting of two charges +q and –q separated by a finite distance 2l. Its dipole moment is $\vec{P}$, of magnitude $p=q\times 2l$, directed from –q to +q.
2. Let C be any point near the electric dipole at a distance r from the centre O inclined at an angle θ with axis of the dipole. r_1 and r_2 are the distance of point C from charges +q and –q respectively.
3. Potential at C due to charge +q at A is,
   $V_1=\frac{+q}{4\pi\epsilon_0 r_1}$
   Potential at C due to charges –q at B is,
   $V_2=\frac{-q}{4\pi\epsilon_0 r_2}$
4. The electrostatic potential is the work done by the electric field per unit charge,
   $\left(V=\frac{W}{Q}\right)$.
   The potential at C due to the dipole is,



Electric potential due to an electric Dipole

$$V_c = V_1 + V_2 = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \quad \text{......... (1)}$$

5. By geometry,

$$r_1^2 = r^2 + l^2 - 2rl\cos\theta$$

$$r_2^2 = r^2 + l^2 + 2rl\cos\theta$$

$$r_1^2 = r^2\left[1 + \frac{l^2}{r^2} - 2\frac{l}{r}\cos\theta\right]$$

$$r_2^2 = r^2\left[1 + \frac{l^2}{r^2} + 2\frac{l}{r}\cos\theta\right]$$

For a short dipole, $2l < ¿ r$ and

If $r \gg l\ \frac{l}{r}$ is small $\therefore \frac{l^2}{r^2}$ can be neglected

$$\therefore r_1^2 = r^2\left[1 - 2\frac{l}{r}\cos\theta\right]$$

$$r_2^2 = r^2\left[1 + \frac{2l}{r}\cos\theta\right]$$

$$\therefore r_1 = r\left[1 - \frac{2l}{r}\cos\theta\right]^{\frac{1}{2}}$$

$$r_2 = r\left[1 + \frac{2l}{r}\cos\theta\right]^{\frac{1}{2}}$$

$$\therefore \frac{1}{r_1} = \frac{1}{r}\left[1 - \frac{2l}{r}\cos\theta\right]^{\frac{-1}{2}} \text{ and}$$

$$\frac{1}{r_2} = \frac{1}{r}\left[1 + \frac{2l}{r}\cos\theta\right]^{\frac{-1}{2}} \quad \text{........ (2)}$$

Using equation (1) and (2)

6. $V_c = V_1 + V_2 = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\left(1 - \frac{2l\cos\theta}{r}\right)^{\frac{-1}{2}} - \frac{1}{r}\left(1 + \frac{2l\cos\theta}{r}\right)^{\frac{-1}{2}}\right]$

7. Using binomial expansion, $(1+x)^n = 1 + nx$, $x \ll l$ and remaining terms up to the first order of $\frac{l}{r}$ only, we get

$$V_c = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left[\left(1 + \frac{l}{r}\cos\theta\right) - \left(1 - \frac{l}{r}\cos\theta\right)\right]$$

$$¿ \frac{q}{4\pi\epsilon_0 r}\left[1 + \frac{l}{r}\cos\theta - 1 + \frac{l}{r}\cos\theta\right]$$

$$¿ \frac{q}{4\pi\epsilon_0 r}\left[\frac{2l}{r}\cos\theta\right]$$

$$\therefore V_c = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2} \qquad (\because p = q \times 2l)$$

8. Electric potential at C, can also be expressed as,

$$V_c = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}.\vec{r}}{r^3}$$

$$V_c = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}.\hat{r}}{r^2}, \left[\hat{r} = \frac{\vec{r}}{r}\right]$$

Where, $\hat{r}$ is a unit vector along the position vector, $\overrightarrow{OC} = \hat{r}$

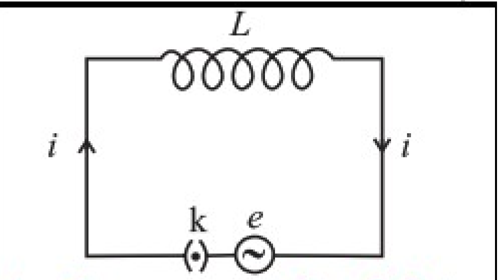
Special cases:

1. Potential at an axial point, $\theta=0^0$ (towards +q) or 180^0 (toward -q)
   $V_{axial}=\frac{\pm 1}{4\pi\epsilon_0}\frac{p}{r^2}$
   i.e., This is the maximum value of the potential.
2. Potential at an equatorial point, $\theta=90^0$ and $V=0$
   Hence, the potential at any point on the equatorial line of a dipole is zero. This is the minimum value of the magnitude of the potential of a dipole.

Q.31 Show that in an AC circuit containing a pure inductor, the voltage is ahead of current by $\pi/2$ in phase. **(4)**

Ans:

1. Consider an alternating e.m.f 'e' applied across a pure inductor of self-inductance L as shown in figure.
2. Let the alternating emf supplied is represented by
   $e=e_0\sin\omega t$ .... (1)



An AC source connected to an Inductor

3. According to Faraday's law, when the key k is closed, current i begins to grow in the inductor because magnetic flux linked with it changes and induced emf is produced which opposes the applied emf.
4. According to Lenz's law,
   $e=-L\frac{di}{dt}$ .... (2)

   Where e is the induced emf and $\frac{di}{dt}$ is the rate of change of current.
5. To maintain the flow of current in the circuit, applied emf (e) must be equal and opposite to the induced emf (e').
6. According to Kirchhoff 's voltage law as there is no resistance in the circuit,
   $e=e'$
   $\therefore e=\left(-L\frac{di}{dt}\right)=L\frac{di}{dt}$ (From Eq. (2))
   $\therefore di=\frac{e}{L}dt$
7. Integrating the above equation on both the sides, we get,
   $\int di=\int\frac{e}{L}dt$
   $i=\int\frac{e_{0\sin\omega t}}{L}dt$ $\left(\because e=e_0\sin\omega t\right)$
   $i=\frac{e_0}{L}\left[\frac{-\cos\omega t}{\omega}\right]+constant$
8. Constant of integration is time independent and has the dimensions of i. As the emf oscillates about zero, i also oscillates about zero so that there cannot be any component of current which is time independent.
   Thus, the integration constant is zero
   $\therefore i=\frac{-e_0}{\omega L}\sin\left(\frac{\pi}{2}-\omega t\right)$ $\left(\because \sin\left(\frac{\pi}{2}-\omega t\right)\right.$ ¿ $\left.\cos\omega t\right)$
   $\therefore i=\frac{e_0}{\omega L}\sin\left[\omega t-\frac{\pi}{2}\right]$
   $i=i_0\sin\left[\omega t-\frac{\pi}{2}\right]$ .... (3)

Where, $i_0 = \frac{e_0}{\omega L}$

Where, i_0 is the peak value of current

9. Eq. (3) gives the alternating current developed in a purely inductive circuit when connected to a source of alternating emf.
10. Comparing Eq. (1) and (3) we find that the alternating current i lag behind the alternating voltage emf e by a phase angle of $\pi/2$ radians (90^0) or the voltage across L leads the current by a phase angle of $\pi/2$ radians (90^0) as shown in figure.

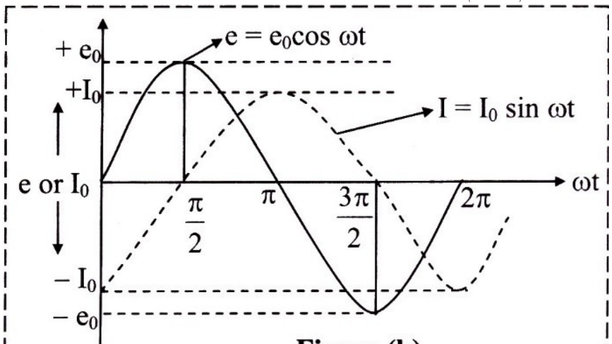


Figure (b)