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TS-P3

SEAT NUMBER



IIT INSPIRE
ACADEMY OF SCIENCE
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XI & XII Science (CBSE/state)
 IIT- JEE (Mains + Advance)
 NEET, MH-CET, NDA

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Branches : Chhatrapati Sq., Mangalmurti Sq.

(4 Pages)

calculator is not allowed.

5. All symbols having their usual meanings unless otherwise stated.
6. For each MCQ, correct answer must be written along with its alphabet.
7. Evaluation of each MCQ would be done for the first attempt only.

Physical Constants:

- (1) $\pi=3.142$ (2) $g=10\text{ m/s}^2$ (3) $h=6.63 \times 10^{-34}\text{ J.s}$ (4) $c=3 \times 10^8\text{ m/s}$
 (5) $e=1.6 \times 10^{-19}\text{ C}$ (6) $\epsilon_0=8.85 \times 10^{-12}\text{ C}^2/\text{N.m}^2$ (7) $\mu_0=4\pi \times 10^7\text{ T.m/A}$,
 (8) $\sigma=5.7 \times 10^{-8}\text{ W/m}^2\text{ K}^4$

SECTION-A

Q.1 Select and write the correct answers to the following questions: [10]

- 1) The potential energy of a molecule on the surface of liquid compared to one inside the liquid is _____.
d) greater (1)
- 2) For an isothermal process, which of the following quantities are non-zero?
b) Q and W (1)
- 3) A parallel plane capacitor is charged. If the plates are pulled apart,
a) the potential difference is increases. (1)
- 4) Magnetic field of an infinitely long wire is
a) proportional to the current flowing but inversely proportional to distance from the wire (1)
- 5) S.I. unit of magnetic dipole moment is _____.
c) Am^2 (1)
- 6) A logic gate is an electronic circuit which:
c) works using binary algebra (1)
- 7) The series limit wavelength of the Lyman series for the hydrogen atom is given by
a) $1/R$ (1)
- 8) A thin ring has mass 0.25 kg and radius 0.5 m. Its M.I. about an axis passing through its centre and perpendicular to its plane is _____.

a) 0.0625 kg m^2 (1)

9) The K.E of the photoelectrons is 2.1 eV and the work function is 1.2 eV. The wavelength of incident light is

d) 3767 \AA (1)

10) In the purely resistive A.C circuit _____.

c) **current and e.m.f are in phase** (1)

Q.2 Answer the following questions in one sentence: [8]

(1) **What is centrifugal force?** (1)

Ans: The force acting on a particle performing UCM which is along the radius and directed away from centre of circle is called centrifugal force.

(2) **What is sphere of influence?** (1)

Ans:

An imaginary sphere with a molecule at its centre and radius equal to the molecular range is called the sphere of influence of the molecule.

The inter-molecular force is effective only within the sphere of influence.

(3) **State Wien's displacement law?** (1)

Ans:

The wavelength, for which emissive power of a blackbody is maximum is inversely proportional to the absolute temperature of the blackbody.

This is Wien's displacement law, i.e.,

$$\lambda_{max} \propto \frac{1}{T} \text{ Or } \lambda_{max} \propto \frac{b}{T}$$

$$\therefore \lambda_{max} T = b$$

Where, b is called the Wien's constant and its value is $2.897 \times 10^{-3} \text{ m K}$.

(4) **State zeroth law of thermodynamics.** (1)

Ans:

If two systems are each in thermal equilibrium with a third system, they are also in thermal equilibrium with each other.

(5) **For a stationary wave set up in the string having both ends fixed, what is the ratio of the fundamental frequency to the second harmonic?** (1)

Ans:

Fundamental frequency of vibration, $n = \frac{1}{2l} \sqrt{\frac{T}{m}}$

Frequency of second harmonic, $n_1 = \frac{1}{l} \sqrt{\frac{T}{m}}$

$$\therefore \frac{n}{n_1} = \frac{\frac{1}{2l} \sqrt{\frac{T}{m}}}{\frac{1}{l} \sqrt{\frac{T}{m}}} = \frac{1}{2}$$

$$\therefore \frac{n}{n_1} = \frac{1}{2}$$

Alternate method:

For a string of length l fixed at both ends, the wavelengths of the first and second harmonics are given as $l = \lambda/2$ and $l = \lambda_1$ respectively.

Hence the ratio of their frequencies is

$$\frac{n}{n_1} = \frac{v/\lambda}{v/\lambda_1} = \frac{\lambda_1}{\lambda} = \frac{l}{2l} = \frac{1}{2}$$

(6) **In a Wheatstone's metrebridge experiment, the null point obtains in the middle one third portion of wire. Why it is recommended?** (1)

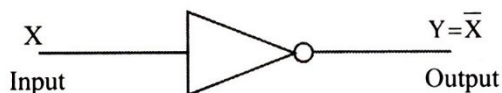
Ans:

In a Wheatstone's metrebridge experiment, the null point is obtained in middle one third portion of the wire (between 34 cm and 66 cm) so that percentage error in the measurement of l_X and l_R are minimum and nearly the same.

(7) **Draw the schematic symbol for NOT gate.** (1)

Ans:

Schematic symbol:



NOT gate symbol

(8) **Calculate the ratio of mean square speeds of molecules of a gas at 30K and 120K.** (1)

Ans:

Given: $T_1 = 30 \text{ K}$, $T_2 = 120 \text{ K}$

To find: Ratio of mean square speed $\left(\frac{\overline{v_1^2}}{\overline{v_2^2}}\right)$

Formula: $\overline{v^2} = \frac{3RT}{M_0}$

Calculation: Using formula,

$$\overline{v^2} \propto T$$

$$\therefore \frac{\overline{v_1^2}}{\overline{v_2^2}} = \frac{T_1}{T_2} = \frac{30}{120} = \frac{1}{4}$$

$$\therefore \overline{v_1^2} : \overline{v_2^2} = 1 : 4$$

Ans: The ratio of mean square speed is 1:4.

SECTION-B

Attempt any eight of the following questions: [16]

Q.3 Explain effect of presence of impurities on the surface tension of liquid. (2)

Ans:

Effect of impurities on surface tension:

1. When highly soluble impurity like common salt (or sodium chloride) is dissolved in water, it is found that surface tension of water increases.
2. When a sparingly soluble substance like phenol or alcohol is dissolved in water, surface tension of water decreases.
3. If insoluble impurity added into water, surface tension of water decreases.

Q.4 How is the kinetic energy of the gas molecule is related with the temperature? Derive the relation using kinetic theory of gases. (2)

Ans:

1. From kinetic theory of gas pressure of the gas is given as,

$$P = \frac{1}{3} \frac{N}{V} m \overline{v^2}$$

$$\therefore \frac{2}{3} N \left(\frac{1}{2} m \overline{v^2} \right) \dots \dots \dots (1)$$

2. The quantity $\frac{1}{2} m \overline{v^2}$ is the average translational kinetic energy of a molecule. In an ideal gas, the molecules are no interacting. And hence there is no potential energy term. Thus, the internal energy of an ideal gas is purely kinetic.

3. The average total energy E, therefore, is

$$E = N \cdot \frac{1}{2} m \overline{v^2} \dots\dots\dots (2)$$

From Eq. (1),

$$PV = \frac{2}{3} E \dots\dots\dots (3)$$

Using ideal gas equation,

$$PV = Nk_B T = \frac{2}{3} E \dots\dots (4)$$

$$\therefore E = \frac{3}{2} Nk_B T \dots\dots\dots (5)$$

$$\text{Or } \frac{E}{N} = \frac{3}{2} k_B T \dots\dots\dots (6)$$

This means that the average energy per molecule is proportional to the absolute temperature T of the gas. This equation relates the macroscopic parameter of the gas, T, to the kinetic energy of a molecule.

Q.5 Draw a P-V diagram and explain the concept of positive and negative work. Give one example each. (2)

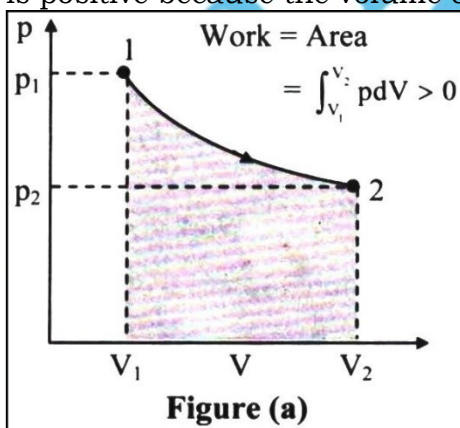
Ans:

1. A gas confined to a cylinder with a movable, frictionless, and massless piston can be, expanded with varying pressure or it can be compressed with varying pressure or it can expand at constant pressure.

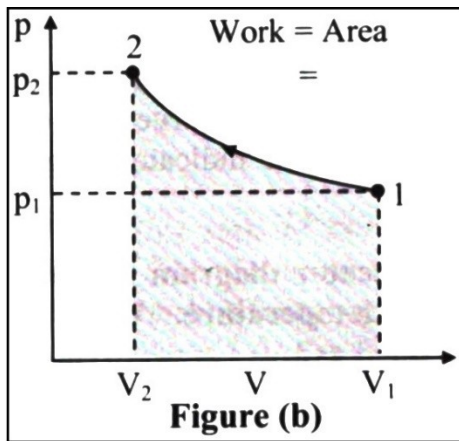
2. The area under the curve in p-V diagram, is the graphical representation of the

$$W = \int_{V_i}^{V_f} p \cdot dV$$

3. Figure shows expansion of the gas. Its volume changes due to outward displacement of the piston and the pressure of the gas decreases. The work done by the gas in this case is positive because the volume of the gas has increased.



4. Similarly, figure shows compression due to inward displacement of the piston. The pressure of the gas is increased and the work done by the gas is now negative.



Example:

1. When the milk is boiled, it expands and does work on the surrounding. This is the positive work done by the milk on the surrounding.
2. In the compression of a spring, considering spring to be system, the work is done on the system. Thus, its volume decreases and the work done are negative.

Q.6 Derive an expression for equation of stationary wave on a stretched string. (2)

Ans:

1. The equation of a stationary wave is given as,

$$y = A \sin \omega t$$

Where, $A = 2a \cos \frac{2\pi x}{\lambda}$

2. The terms in position x and time t appear separately and not as a combination $2\pi(nt \pm x/\lambda)$. The wave is not a progressive wave. x is present only in the expression for the amplitude.
3. The amplitude of the resultant wave is given as $A = 2a \cos \frac{2\pi x}{\lambda}$. It is a periodic function of x i.e., the amplitude is varying periodically in space.
4. The amplitudes are different for different particles but each point on the string oscillates with the same frequency ω (same as that of the individual progressive wave). All the particles of the string pass through their mean positions simultaneously twice during each vibration.
5. The string as a whole is vibrating with frequency ω with different amplitudes at different points. The wave is not moving either to the left or to the right. We therefore call such a wave a stationary wave or a standing wave.
6. Particles move so fast that the visual effect is formation of loops. It is therefore customary to represent stationary waves as loops.
7. In case of a string tied at both the ends, loops are seen when a stationary wave is formed because each progressive wave on a string is a traverse wave.

Q.7 Explain what is optical path length. How it is different from actual path length? (2)

Ans:

1. When a wave travels a distance Δx through a medium having refractive index of n , its phase changes by the same amount as it would if the wave had travelled a distance $n\Delta x$ in vacuum.
2. Thus, a path length of Δx in a medium of refractive index n is equivalent to a path length of $n\Delta x$ in vacuum.
3. $n\Delta x$ is called the optical path travelled by a wave.
4. Thus, optical path through a medium is the effective path travelled by light in vacuum

to generate the same phase difference. In vacuum, the optical path is equal to the actual path travelled as $n=1$.

5. Optical path in a medium can also be defined as the corresponding path in vacuum that the light travels in the same time as it takes in the given medium.

Now,

$$\text{time} = \frac{\text{distance}}{\text{speed}} \therefore t = \frac{d_{\text{medium}}}{v_{\text{medium}}} = \frac{d_{\text{vacuum}}}{v_{\text{vacuum}}}$$

$$\therefore \text{Optical path} = d_{\text{vacuum}} = \frac{v_{\text{vacuum}}}{v_{\text{medium}}} \times d_{\text{medium}}$$

$$= n \times d_{\text{medium}}$$

6. Thus, a distance d travelled in a medium of refractive index n introduces a path difference $nd - d = d(n-1)$ over a ray travelling equal distance through vacuum.

Q.8 Derive expression for energy stored in capacitor.

(2)

Ans:

1. A capacitor is a device used to store energy.
2. Charging a capacitor means transferring electron from one plate of the capacitor to the other. Hence work will have to be done by the battery in order to remove the electrons against the opposing forces.
3. These opposing forces arise since the electrons are being pushed to the negative plate which repels them and electrons are removed from the positive plate which tends to attract them.
4. In both the cases, the forces oppose the transfer from one plate to another. As the charges on the plate increases, opposition also increases.
5. This work done is stored in the form of electrostatic energy in the electric field between the plates, which can later be recovered by discharging the capacitor.

Q.9 State the postulates of Bohr's atomic model.

(2)

Ans:

Postulate-I:

"An electron revolves only in circular orbit and the centripetal force required to circulate the electron is provided by electrostatic force of attraction between nucleus and electron".

Postulate-II:

"An electron can revolve, without radiating energy, only in those orbits for which, angular momentum of electron is equal to integral multiple of $\frac{h}{2\pi}$ "

Postulate-III:

"An electron radiates energy only when it jumps from an orbit of higher energy to an orbit of lower energy".

Q.10 A needle of a sewing machine moves along a path of amplitude 4 cm with frequency 5 Hz. Find its acceleration 1/30 s after it has crossed the mean position.

(2)

Ans:

Given: $A = 4 \text{ cm} = 0.04 \text{ m}$, $n = 5 \text{ Hz}$

To find: Acceleration after $\left(\frac{1}{30}\right)\text{s}$ from mean position (a)

Formulae: i. $\omega = 2\pi n$
ii. $|a| = \omega^2 A \sin \omega t$

Calculation:

From formula (i),

$$\omega = 2\pi \times 5 = 10\pi \text{ rad/s}$$

From formula (ii),

$$|a| = (10\pi)^2 \times 0.04 \times \sin\left(10\pi \times \frac{1}{30}\right)$$

$$= 100 \times 9.872 \times 0.04 \times \sin \frac{\pi}{3}$$

$$= 4 \times 9.872 \times \frac{\sqrt{3}}{2}$$

$$= 2 \times 9.872 \times 1.732$$

$$= \text{antilog} \{ \log(2) + \log(9.872) + \log(1.732) \}$$

$$= \text{antilog} \{ 0.3010 + 0.9944 + 0.2385 \}$$

$$= \text{antilog} \{ 1.5339 \}$$

$$= 34.19 \text{ m/s}^2$$

Ans: The magnitude of acceleration at the required instant is 34.19 m/s^2 .

Q.11 A wave of frequency 500 Hz is travelling with a speed of 350 m/s.

- 1) What is the phase difference between two displacements at a certain point at time 1.0 ms apart? (1)**
- 2) What will be the smallest distance between two point which are 45° out of phase at an instant of time? (1)**

Ans:

Given: $n = 500 \text{ Hz}$, $v = 350 \text{ m/s}$,

$$\Delta t = 1.0 \text{ ms} = 10^{-3} \text{ s},$$

$$\Delta\phi = 45^\circ = \left(\frac{\pi}{4}\right)^c$$

To find: i. Phase difference at an instant ($\Delta\phi$)
ii. Smallest distance between two points at an instant (Δx)



Formulae: i. $\lambda = \frac{v}{n}$ ii. $\Delta\phi = \frac{2\pi}{T}\Delta t$

iii. $\Delta x = \frac{\lambda}{2\pi} \times \Delta\phi$

Calculation: From formula (i),

$$\lambda = \frac{350}{500} = \frac{7}{10} \text{ m}$$

From formula (ii),

$$\Delta\phi = \frac{2\pi}{T}\Delta t$$

$$= 2\pi n\Delta t \quad \dots \left(\because n = \frac{1}{T} \right)$$

$$= 2 \times \pi \times 500 \times 10^{-3}$$

$$= \pi \text{ rad}$$

From formula (iii),

$$\Delta x = \frac{\lambda}{2\pi} \times \frac{\pi}{4}$$

$$= \frac{\lambda}{8}$$

$$= \frac{7}{10 \times 8}$$

$$= 0.0875 \text{ m}$$

$$= \mathbf{8.75 \text{ cm}}$$

Ans: i. Phase difference between two displacements at an instant is π rad.

ii. Smallest distance between the two points is **8.75 cm.**

Q.12 A voltmeter has a resistance 30Ω . What will be its reading, when it is connected across a cell of emf 2 V having internal resistance 10Ω ? (2)

Ans:

Given: $R = 100 \Omega$, $E = 2 \text{ V}$, $r = 20 \Omega$

To find: Potential difference across voltmeter
i.e., terminal potential (V).

Formulae: i. $V = I \times R$ ii. $I = \frac{E}{R + r}$

Calculation: From formula (i) and (ii),

$$V = \frac{E}{R + r} \times R$$

$$= \frac{2}{100 + 20} \times 100$$

$$= \frac{2}{120} \times 100$$

$$= \frac{5}{3} = \mathbf{1.67 \text{ V}}$$

Ans: The reading of the voltmeter is **1.67 V.**

Q.13 A toroidal ring, made from a bar of length (l) 1 m and diameter (d) 1 cm , is bent into a circle. It is wound tightly with 100 turns per cm. If the permeability of bar is equal to that of free space (μ_0), calculate the magnetic field inside the bar (B) when the current (i) circulating through the turns is 100 A . Also determine the self-inductance (L) of the coil. (2)

Ans:

Given: $l = 1 \text{ m}$, $d = 1 \text{ cm} = 10^{-2} \text{ m}$,
 $n = 100 \text{ turns/cm} = 10000 \text{ turns/m}$
 $i = 100 \text{ A}$

To find: i. Magnetic field inside the bar (B)
ii. Self inductance (L)

Formulae: i. $B = \mu_0 nI$ ii. $L = \frac{N\phi}{I}$

Calculation: From formula (i)

$$B = 4 \times 3.142 \times 10^{-7} \times 10000 \times 100 \\ = \mathbf{1.256 \text{ T}}$$

$$\text{Area of cross section} = \pi \times \frac{d^2}{4} \\ = 3.142 \times \frac{(10^{-2})^2}{4} \\ = 0.7855 \times 10^{-4} \text{ m}^2$$

From formula (ii),

$$L = \frac{n^2 BA}{l} \quad \dots (\because N = nl) \\ = \frac{10000 \times 1 \times 1.256 \times 0.7855 \times 10^{-4}}{100} \\ = 0.98 \times 10^{-2} \\ \approx \mathbf{10 \text{ mH}}$$

- Ans:** i. Magnetic field is **1.256 T**.
ii. Self inductance is **10 mH**.

Q.14 When two cells of emfs. E_1 and E_2 are connected in series so as to assist each other, their balancing length on a potentiometer is found to be 2.7 m. When the cells are connected in series so as to oppose each other, the balancing length is found to be 0.3 m. Compare the emfs of the two cells. (2)

Ans:

Given: $l_1 = 2.7 \text{ m}, l_2 = 0.3 \text{ m}$

To find: Ratio of emfs of two cells $\left(\frac{E_1}{E_2} \right)$

Formula: $\frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2}$

Calculation: From formula,

$$\frac{E_1}{E_2} = \frac{2.7 + 0.3}{2.7 - 0.3} = \frac{3.0}{2.4} = \frac{5}{4} = \mathbf{1.25}$$

Ans: The ratio of emfs of two cells is **1.25**.

SECTION-C

Attempt any eight of the following questions:

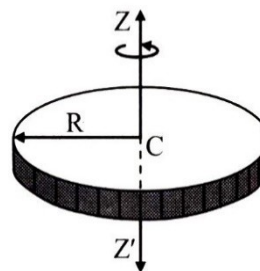
[24]

Q.15 State an expression for moment of inertia of a thin uniform disc about an axis passing through its centre and perpendicular to its plane. (3)

Ans:

Expression of M.I of uniform disc about an axis passing through its centre and perpendicular to its plane:

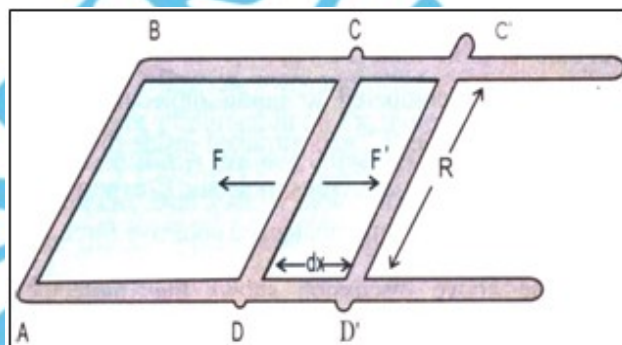
- a. Let, M = mass of disc,
 R = radius of disc
 ZZ' = axis passing through the centre of disc and perpendicular to the plane.
- b. The M. I of a thin uniform disc about an axis passing through its centre and perpendicular to its plane is given by, $I_C = \frac{1}{2} MR^2$



Q.16 Derive the relation between surface tension and surface energy per unit area. (3)

Ans:

Potential energy per unit area of liquid surface under isothermal conditions is the surface energy per unit area. S.I. unit of surface energy is J/m^2 and dimensions are $[M^1 L^0 T^{-2}]$. Consider a rectangular frame of wire (ABCD) on which wire CD can slide without friction. Suppose this frame is dipped in soap solution, taken outside and held horizontally. Then a film of soap solution will be formed in the frame. If l is the length of wire CD and T is surface tension of soap solution.



The wire CD experiences an inward force $[F=2Tl]$ due to each side of film. Due to this force, film will move towards side AB, so as to decrease its area.

To increase the surface area of liquid, imagine that external force F' equal and opposite to F is applied to side CD under isothermal condition, so that CD moves to $C'D'$ through distance dx .

Work done \propto Force \times Displacement

$$\therefore dw = F' \cdot dx \dots\dots (1)$$

By definition of surface tension,

$$T = \frac{F'}{l}$$

$$\therefore F' = Tl$$

But the total force due to surface tension on the wire is $2Tl$ because the film has two surfaces of length l .

$$\therefore F' = 2Tl \dots\dots (2)$$

From equations (1) and (2) we get,

$$dw = 2Tl dx = T(2l dx)$$

But increase in area of the two surfaces of the film,

$$dA = 2l dx$$

$$\therefore dw = T dA$$

This work done is in the form of potential energy which is called as surface energy.

$$\therefore \text{Surface energy} \propto T dA$$

Surface energy \propto surface Tension \times total increase in area

Q.17 State and prove Kirchhoff's law of heat radiation. (3)

Ans:

Statement:

At a given temperature, the ratio of emission power to coefficient of absorption of a body is equal to the emission power of a perfect blackbody at the same temperature for all wavelengths.

OR

For a body emitting and absorbing thermal radiation in thermal equilibrium, the emissivity is equal to its absorptivity.

Symbolically, $a=e$ or $a(\lambda)=e(\lambda)$

Theoretical proof:

1. Consider an ordinary body A and a perfect blackbody B of identical geometric shapes placed in an enclosure. In thermal equilibrium, both bodies will be at same temperature as that of the enclosure.
2. Let R_A Emissive power of body A,
 R_B Emissive power of blackbody B.
 a Coefficient of absorption of body A
 Q Quantity of radiant heat incident on each body in unit time and
 Q_a Quantity of radiant heat absorbed by the body A.
 Then $Q_a = aQ$
3. As the temperature of the body A and blackbody B remain the same, both must emit the same amount as they absorb in unit time.
 \therefore Quantity of radiant heat absorbed by body A = Quantity of heat emitted by body A
 $\therefore aQ = R_A$ (1)
4. For the perfect blackbody B,
 $Q = R_B$ (2)
5. Dividing equation (1) by equation (2), we get,

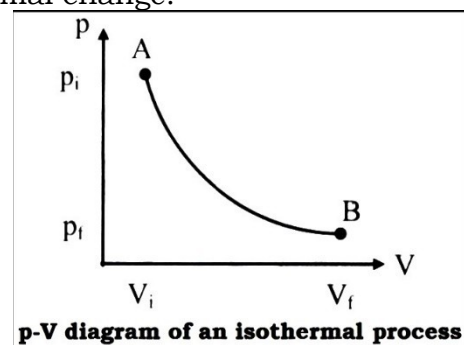
$$a = \frac{R_A}{R_B}$$
6. But by definition of emissivity, $e = \frac{R_A}{R_B}$
 $\therefore a = e$
 Hence, Kirchhoff's law is theoretically proved.

Q.18 What is an isothermal process? Explain in brief.

(3)

Ans:

1. A process in which change in pressure and volume takes place at a constant temperature is called an isothermal process or isothermal change.
2. For such a system $\Delta T = 0$.
3. Isothermal process is a constant temperature process.
4. This is possible when a system is in good thermal contact with its environment, and the transfer of heat from, or to the system, is extremely slow so that thermal equilibrium is maintained throughout the change.
5. For example, melting of ice, which takes place at constant temperature, is an isothermal process.
6. For an isothermal process, none of the quantities Q and W is zero.
7. For an isothermal change, total amount of heat of the system does not remain constant.
8. The temperature of a system remains constant in an isothermal change and Boyle's law can be applied.
9. Therefore, the equation of state for an isothermal change is given by,
 $pV = \text{constant}$
 If p_i, V_i and p_f, V_f are the variable of a system in its initial and the final states respectively, then for an isothermal change,
 $p_f V_f = p_i V_i = \text{constant}$.



10. In an isothermal process, internal energy of system does not change, i.e., $\Delta U = 0$.
 11. Energy exchanged is used to perform work, i.e., $Q = W$.

Q.19 What is meant by coherent sources? What are the two methods of obtaining coherent sources in laboratory?

(3)

Ans:

- Two sources, which emit waves of the same frequency having a constant phase difference, independent of time, are called coherent sources.
- Different sources emit waves of different frequencies and even if they emit waves of the same frequency, they are not in phase. Thus, the interference pattern changes every instant of time and no pattern is sustained over a significant length of time.
- For interference to be seen over sustained periods we need two sources of light which emit waves of the same frequency and the waves emitted by them are in phase or have a constant phase difference between them, i.e., we need coherent sources of light. This criterion cannot be satisfied by two independent primary sources as they emit waves independently and there need not be a constant phase relation between them. Thus, to obtain sustained interference pattern one usually obtains two secondary sources from the same primary source.

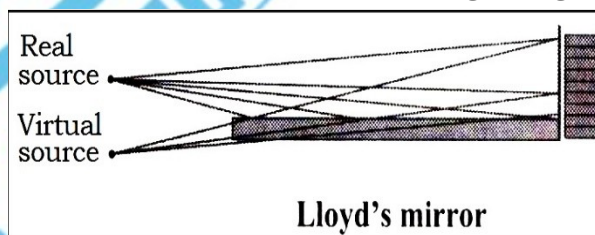
Example: Monochromatic light, laser light, etc

Method:

1. Lloyd's Mirror:

This is an extensively used device. The light from a source is made to fall at a grazing angle on a plane mirror as shown in figure.

Some of the light falls directly on the screen as shown by the blue lines in the figure and some light falls after reflection, as shown by red lines. The reflected light appears to come from a virtual source and so we get two sources. They are derived from a single source and hence are coherent. They



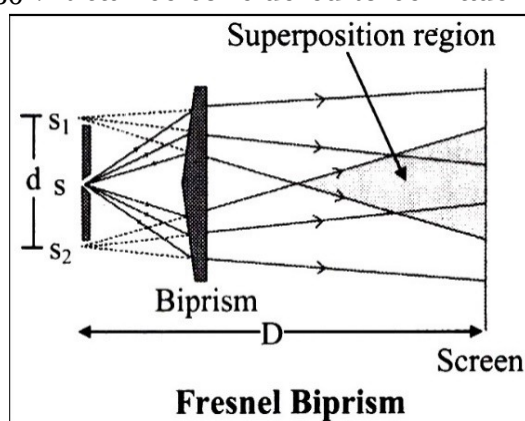
interfere and an interference pattern is obtained as shown in the figure. Note that even though we have shown the direct and reflected rays by blue and red lines, the light is monochromatic having a single wavelength.

2. Fresnel Biprism:

A biprism is a prism with vertex angle of nearly 180° . It can be considered to be made up of two prisms with very small refracting angle ranging from $30'$ to 1° , joined at their bases. In experimental arrangement, the refracting edge of the biprism is kept parallel to the length of the slit.

Monochromatic light from a source is made to pass through a narrow-slit S as shown in figure and fall on the biprism. The two halves of the biprism form virtual images S_1 and S_2 . These are coherent sources having obtained from a single secondary source S .

The two waves coming from S_1 and S_2 interfere and form interference fringes like that in Young's double slit experiment in the shaded region shown in the figure.



Q.20 Derive an expression for the effective capacitance of three parallel plate capacitors connected in series. (3)

Ans:

Condensers in Series:

1. The arrangement of condensers in series combination is as shown in diagram.
2. Let C_1 , C_2 and C_3 are the capacities of condensers and C_s is the equivalent capacity of series combination.
3. $+Q$ is the charge supplied to the plate of first condensers and same charges are induced on the plates of all the condensers.
4. V is the potential of the battery and V_1 , V_2 and V_3 are the potentials on respective condensers C_1 , C_2 and C_3 as shown in the diagram.

From the diagram,

$$V = V_1 + V_2 + V_3 \text{ ---- (1)}$$

Using the relation $V = \frac{Q}{C_s}$, we can write

$$V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2} \text{ and } V_3 = \frac{Q}{C_3}$$

Substitute these values in equation (1)

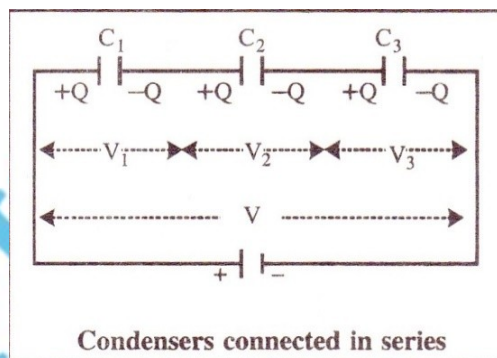
$$\therefore \frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \text{ ---- (2)}$$

5. From its relation it is clear that series combination of condenser is used to decrease the value of condensers because $\frac{1}{C_s} > \frac{1}{C_1}$ i.e. $C_s < C_1$, which means the equivalent value of series combination is always less than the minimum value of capacitor connected in series combination.

6. For n number of capacitors connected in series, then equivalent capacitance of series combination is

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$



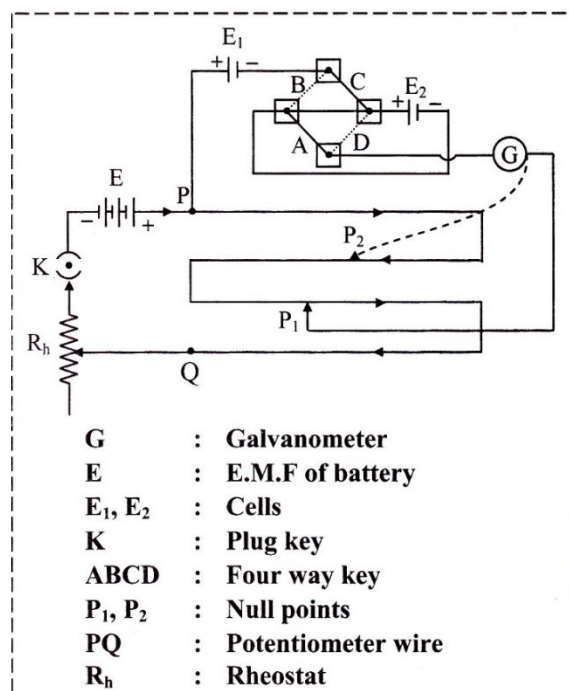
Q.21 Describe how a potentiometer is used to compare the emfs of the two cells by combination method. (3)

(3)

Ans:

Experimental arrangement:

1. Uniform wire PQ of the potentiometer is connected in series with a battery of constant e.m.f E , a plug key K and a rheostat R_h as shown in figure.
2. Let E_1 and E_2 be the e.m.f of the two given cells such that $E_1 > E_2$.
Also $E > (E_1 + E_2)$
3. Now the negative terminal of the cell E_1 is connected to the positive terminal of the cell E_2 . The cells are in assisting mode or in sum stable. The combination behaves as a new cell of e.m.f $E_1 + E_2$.



4. When the negative terminal of E_1 is connected to negative terminal of E_2 then combination behaves as a new cell of e.m.f ($E_1 - E_2$).

Working:

1. Pass the current in the wire PQ by using the plug key K. Check the deflections of the G by contacting the jockey at the ends of the wire. If necessary, adjust the rheostat to obtain the deflections on both the sides of the zero of the galvanometer. This means the P.D. applied across the wire is greater than the e.m.f ($E_1 + E_2$) of the cell.
2. Now contact the jockey at different points on the wire till a point P_1 is obtained for which (G) shows no deflection. Such a point is known as the balance point.
3. Let the balancing length PP_1 be l_1 . Since no current flows through the cells, therefore, its e.m.f ($E_1 + E_2$) is balanced by the fall of potential from P to P_1 .

$$\therefore E_1 + E_2 = V_{PP_1}$$

4. According to the principle of potentiometer, $V_{PP_1} \propto l_1$

$$\therefore V_{PP_1} = K l_1$$

$$\therefore E_1 + E_2 = K l_1 \dots\dots\dots (1)$$

5. Now experiment is repeated for opposing cell of e.m.f ($E_1 - E_2$) and balance point P_2 is obtained.

Let, balancing length $PP_2 = l_2$

By the same consideration:

$$E_1 - E_2 = K l_2 \dots\dots\dots (2)$$

6. Dividing equation (1) by (2), we get,

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{K l_1}{K l_2}$$

$$\therefore \frac{E_1 + E_2}{E_1 - E_2} = \frac{l_1}{l_2} \dots\dots\dots (3)$$

7. Applying component and dividend in equation (3), we get,

$$\frac{E_1 + E_2 + E_1 - E_2}{E_1 + E_2 - E_1 + E_2} = \frac{l_1 + l_2}{l_1 - l_2}$$

$$\therefore \frac{2 E_1}{2 E_2} = \frac{l_1 + l_2}{l_1 - l_2}$$

$$\therefore \frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2} \dots\dots\dots (4)$$

Thus, knowing the balancing lengths l_1 and l_2 the e.m.f of two cells can be compared. If the e.m.f of one cell is known, e.m.f of other cell can be calculated.

Q.22 A circular loop of radius 9.7 cm carries a current 2.3 A. Obtain the magnitude of the magnetic field

1) at the centre of the loop and

2) at a distance of 9.7 cm from the centre of the loop but on the axis. (3)

Ans:

Given: $R = 9.7 \text{ cm} = 9.7 \times 10^{-2} \text{ m}$,
 $I = 2.3 \text{ A}$,
 $z = 9.7 \text{ cm} = 9.7 \times 10^{-2} \text{ m}$

To find: Magnetic field
 i. at the centre of the loop
 ii. on the axis

Formulae: i. $B_c = \frac{\mu_0 I}{2R}$
 ii. $B_a = \frac{\mu_0 IR^2}{2(z^2 + R^2)^{3/2}}$

Calculation:

From formula (i),

$$\begin{aligned} B_c &= \frac{4\pi \times 10^{-7} \times 2.3}{2 \times 9.7 \times 10^{-2}} \\ &= \frac{2 \times 3.142 \times 2.3}{9.7} \times 10^{-7+2} \\ &= \{\text{antilog} [\log 2 + \log 3.142 + \log 2.3 \\ &\quad - \log 9.7]\} \times 10^{-5} \\ &= \{\text{antilog} (0.3010 + 0.4972 + 0.3617 \\ &\quad - 0.9868)\} \times 10^{-5} \\ &= \{\text{antilog} (0.1731)\} \times 10^{-5} \\ &= 1.489 \times 10^{-5} \\ &\approx 1.49 \times 10^{-5} \text{ T} \end{aligned}$$

From formula (ii),

$$\begin{aligned} B_a &= \frac{4\pi \times 10^{-7} \times 2.3 \times (9.7 \times 10^{-2})^2}{2 \left[(9.7 \times 10^{-2})^2 + (9.7 \times 10^{-2})^2 \right]^{3/2}} \\ &= \frac{4\pi \times 10^{-7} \times 2.3 \times (9.7 \times 10^{-2})^2}{2 \times 2^{3/2} \times (9.7 \times 10^{-2})^3} \\ &= \frac{\pi \times 10^{-7} \times 2.3}{2^{1/2} \times (9.7 \times 10^{-2})} \\ &= 5.268 \times 10^{-6} \text{ T} \end{aligned}$$

Ans: i. Magnetic field at the centre of $1.49 \times 10^{-5} \text{ T}$.
 ii. Magnetic field on the axis is $5.268 \times 10^{-6} \text{ T}$.

Q.23 A $25 \mu\text{F}$ capacitor, a 0.10 H inductor and a 25Ω resistor are connected in series with an AC source whose emf is given by $e = i 310 \sin 314t$ (volt). What is the frequency, reactance, impedance, current and phase angle of the circuit? (3)

Ans:

Given: $C = 25 \mu\text{F} = 25 \times 10^{-6} \text{ F}$, $L = 0.10 \text{ H}$,
 $R = 25 \Omega$, $e = 310 \sin 314 t$,

To find:

- i. Frequency (f)
- ii. Reactance of circuit $|X_L - X_C|$
- iii. Impedance (Z)
- iv. Current (i)
- v. Phase Angle (ϕ)

Formula:

- i. $e = e_0 \sin \omega t$
- ii. $X_C = \frac{1}{\omega C}$
- iii. $X_L = \omega L$
- iv. $Z = \sqrt{R^2 + (X_C - X_L)^2}$
- v. $i_{\text{rms}} = \frac{e_{\text{rms}}}{Z}$
- vi. $\phi = \tan^{-1} \frac{(X_C - X_L)}{R}$

Calculation:

On comparing $e = 310 \sin 314t$ with formula (i)

We get,

$e_0 = 310 \text{ V}$ and $\omega = 314 \text{ rad/s}$

$$\therefore f = \frac{\omega}{2\pi} = \frac{314}{2 \times 3.14} = \mathbf{50 \text{ Hz}}$$

From formula (ii),

$$\begin{aligned} X_C &= \frac{1}{3.14 \times 25 \times 10^{-6}} \\ &= \frac{100}{314 \times 25} \times 10^4 \\ &= \frac{4}{314} \times 10^4 \\ &= \mathbf{127.4 \Omega} \end{aligned}$$



From formula (iii),

$$X_L = 314 \times 0.10 \\ = \mathbf{31.4 \Omega}$$

Reactance of the circuit due to inductor and capacitor,

$$|X_L - X_C| = |31.4 - 127.4| = \mathbf{96 \Omega}$$

From formula (iv),

$$Z = \sqrt{(25)^2 + (96)^2} \\ = \sqrt{625 + 9216} \\ = \sqrt{9841} \\ = \mathbf{99.2 \Omega}$$

From formula (v),

$$i_{\text{rms}} = \frac{310 \times 0.707}{99.2} \quad \dots \left(\because e_{\text{rms}} = \frac{e_0}{\sqrt{2}} = 0.707 e_0 \right) \\ = \text{antilog} \{ \log(310) + \log(0.707) \\ \quad \quad \quad - \log(99.2) \} \\ = \text{antilog} (2.4914 + \bar{1}.8494 - 1.9965) \\ = \text{antilog} (0.3443) \\ = 2.210 = \mathbf{2.21 A}$$

From formula (vi),

$$\tan \phi = \frac{96}{25} \\ = \frac{96 \times 4}{25 \times 4} = 3.84 \\ \phi = \tan^{-1}(3.84) = 75.4^\circ = \frac{75.4 \times 3.142}{180} \\ = \mathbf{1.316 \text{ rad}}$$

Ans: For the given LCR series circuit,

- i. The frequency of the circuit is **50 Hz**
- ii. The reactance of the circuit is **96 Ω**
- iii. The impedance of the circuit is **99.2 Ω**
- iv. The rms current of the circuit is **2.21 A**
- v. The phase angle of the circuit is **1.316 rad**.

Q.24 An AC circuit consist of only an inductor of inductance 2 H. If the current is represented by a sine wave of amplitude 0.25 A and frequency 60 Hz, calculate the effective potential difference across the inductor ($\pi=3.142$) (3)

Ans:

Given: $L = 2 \text{ H}, i_0 = 0.25 \text{ A}, f = 60 \text{ Hz}$

To find: Effective potential difference (e_{rms})

Formulae: i. $i_{\text{rms}} = \frac{i_0}{\sqrt{2}}$ ii. $X_L = 2\pi fL$

iii. $e_{\text{rms}} = i_{\text{rms}} \times X_L$

Calculation: From formula (i),

$$i_{\text{rms}} = \frac{0.25}{\sqrt{2}} \\ = 0.25 \times 0.707 \\ = 0.1768 \text{ A}$$

From formula (ii),

$$X_L = 2 \times 3.142 \times 60 \times 2 \\ = 754.08 \Omega$$

From formula (iii)

$$\begin{aligned}
 e_{\text{rms}} &= 0.1768 \times 754.08 \\
 &= \text{antilog} \{ \log (0.1768) \\
 &\quad + \log (754.08) \} \\
 &= \text{antilog} \{ \bar{1}.2475 + 2.8775 \} \\
 &= \text{antilog} \{ 2.1250 \} \\
 &= 1.334 \times 10^2 \\
 &= \mathbf{133.4 \text{ V}}
 \end{aligned}$$

Ans: The effective potential difference across the inductor is **133.4 V**

Q.25 The half-life of a nuclear species ${}^N\text{X}$ is 3.2 days. Calculate its (i) decay constant, (ii) average life and (iii) the activity of its sample of mass 1.5 mg. (3)

Ans:

Given: $T_{1/2} = 3.2 \text{ days}$
 $= 3.2 \times 24 \times 60 \times 60 \text{ s}$
 $= 2.765 \times 10^5 \text{ s,}$

$m = 1.5 \text{ mg} = 1.5 \times 10^{-3} \text{ g}$

To find: i. decay constant (λ)
 ii. average life (τ)
 iii. activity of sample (A)

Formulae: i. $T_{1/2} = \frac{0.693}{\lambda}$

ii. $\tau = \frac{1}{\lambda}$

iii. $A(t) = \lambda N(t)$

Calculation:

From formula (i),

$$2.765 \times 10^5 = \frac{0.693}{\lambda}$$

$$\therefore \lambda = \frac{0.693}{2.765 \times 10^5}$$

$$\begin{aligned}
 \therefore \lambda &= \{ \text{antilog} (\log(0.693) - \log(2.765)) \} \times 10^{-5} \\
 &= \{ \text{antilog} (\bar{1}.8407 - 0.4417) \} \\
 &= \{ \text{antilog} (\bar{1}.3990) \} \times 10^{-5} \\
 &= \mathbf{2.506 \times 10^{-6} \text{ s}^{-1}}
 \end{aligned}$$

From formula (ii),

$$\tau = \frac{1}{2.506 \times 10^{-6}}$$

$$= 0.4 \times 10^6$$

$$= 4 \times 10^5 \text{ s}$$

$$= \frac{4 \times 10^5}{8.64 \times 10^4} \quad \dots (\because 1 \text{ day} = 8.64 \times 10^4 \text{ s})$$

$$= \mathbf{4.63 \text{ days}}$$



As number of nuclei in given sample

$$N(t) = N_A \times \text{no. of moles}$$

Assuming 'Y' as atomic mass of nuclear species 'X',

$$N(t) = 6.022 \times 10^{23} \times \frac{1.5 \times 10^{-3}}{Y}$$

$$= \frac{9.033 \times 10^{20}}{Y}$$

From formula (iii),

$$A(t) = \lambda N(t)$$

$$= 2.5 \times 10^{-6} \times \frac{9.033 \times 10^{20}}{Y}$$

$$= \frac{2.258 \times 10^{15}}{Y}$$

$$= \frac{2.258 \times 10^{15}}{Y \times 3.7 \times 10^{10}}$$

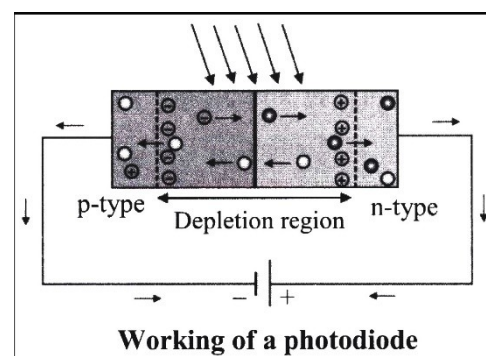
$$= \frac{0.61 \times 10^5}{Y} \text{ Ci} = \frac{6.1 \times 10^4}{Y} \text{ Ci}$$

- Ans:** i. The decay constant of reaction is $2.506 \times 10^{-6} \text{ s}^{-1}$
- ii. The mean life of the species is **4.63 days**.
- iii. The activity of its 1.5 mg sample is $\frac{6.1 \times 10^4}{Y} \text{ Ci}$.

Q.26 Explain the principle of operation of a photodiode. Represent graphically the I-V characteristics of a photodiode for different intensities of illumination. (3)

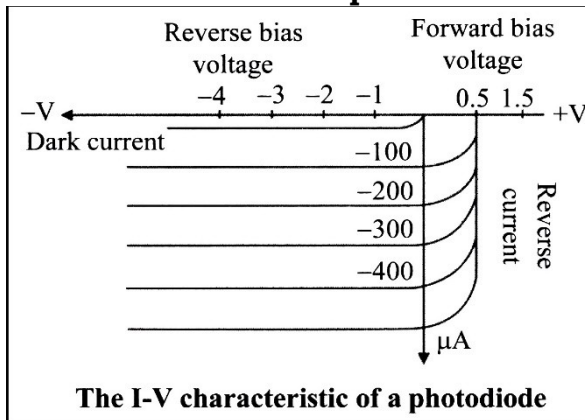
Ans:

- The p-n junction of a photodiode is placed inside a glass material so that only the junction of a photodiode is exposed to light.
- Other part of the diode is generally painted with an opaque colour or covered.
- When a p-n junction diode is reverse biased, a reverse saturation current flows through the junction.
- The magnitude of this current is constant for a certain range of reverse bias voltages.
- This current is due to the minority carriers on its either side. (Electrons are minority carriers in the p-region and the holes are minority carriers in the n-region of a diode).
- The reverse current depends only on the concentration of the minority carriers and not on the applied voltage.
- This current is called the dark current in a photodiode because it flows even when the photodiode is not illuminated. Figure schematically shows working of a photodiode.
- When a p-n junction is illuminated, electron-hole pairs are generated in the depletion region.
- The energy of the incident photons should be larger than the band gap of the semiconductor material used to fabricate the photodiode.
- The electrons and the holes are separated due to the intrinsic electric field present in the depletion region.
- The electrons are attracted towards the anode and the holes are attracted towards the cathode.
- More carriers are available for conduction and the reverse current is increased. The



reverse current of a photodiode depends on the intensity of the incident light. Thus, the reverse current can be controlled by controlling the concentration of the minority carriers in the junction.

I-V characteristic of a photodiode:



SECTION-D

Attempt any three of the following question:

[12]

Q.27 What is the need for lower and upper speed limit for a vehicle moving on a banked road? Derive expression for it.

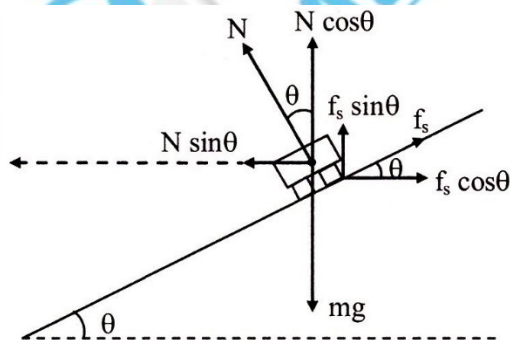
(4)

Ans:

1. If the vehical is running exactly at the speed $V_s = \sqrt{rg \tan \theta}$, the forces acting on the vehical are
 - a. weight mg acting vertically downwards
 - b. normal reaction N acting perpendicular to the road.
2. But in practice, vehicles never travel exactly with this speed.
3. Hence, for speeds other than this, the component of force of static friction between road and the tyres helps us, up to a certain limit.
4. For speeds $V_1 < \sqrt{rg \tan \theta}$

The component $N \sin \theta$ is greater than the centrifugal force $\frac{mv_1^2}{r}$.

$$\therefore \frac{mv_1^2}{r} < N \sin \theta$$



Banked road: lower speed limit

5. In this case, the direction of force of static friction (f_s) between road and the tyres is directed along the inclination of the road, upwards. Its horizontal component ($f_s \cos \theta$) is parallel and opposite to $N \sin \theta$.
6. These two forces take care of the necessary centripetal force (or balance the centrifugal force).

$$\therefore \frac{m v_1^2}{r} = N \sin \theta - f_s \cos \theta \dots (1)$$

7. The vertical components $N \cos \theta$ and $f_s \sin \theta$ is balanced by weight mg ,
 $\therefore mg = N \cos \theta + f_s \sin \theta \dots (2)$

8. For minimum possible speed, f_s is maximum and equal to $\mu_s N$.

From equation (1) and (2),

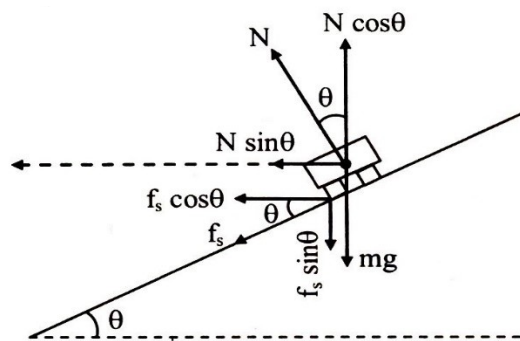
$$(v_1)_{\min} = v_{\min} = \sqrt{rg \left(\frac{\tan \theta - \mu_s}{1 + \mu_s \tan \theta} \right)}$$

For $\mu_s \geq \tan \theta$, $v_{\min} = 0$. This is true for most of the rough roads, banked at smaller angles.

9. For speeds $v_2 > \sqrt{rg \tan \theta}$,

The component $N \sin \theta$ is less than the centrifugal force $\frac{m v_2^2}{r}$.

$$\therefore \frac{m v_2^2}{r} > N \sin \theta$$



Banked road: upper speed limit

10. In this case, the direction of force of static friction (f_s) between road and the tyres is directed along the inclination of the road, downwards.

11. The horizontal component ($f_s \cos \theta$) is parallel to $N \sin \theta$.

These two forces taken care of the necessary centripetal force (or balance the centrifugal force).

$$\therefore \frac{m v_2^2}{r} > N \sin \theta + f_s \cos \theta \dots (3)$$

12. The vertical component, $N \cos \theta$ balances the component $f_s \sin \theta$ and weight 'mg'.

$$\therefore N \cos \theta = f_s \sin \theta + mg$$

$$\therefore mg = N \cos \theta - f_s \sin \theta \dots (4)$$

13. For maximum possible speed, f_s is maximum and equal to $\mu_s N$.

From equation (3) and (4)

$$(v_2)_{\max} = v_{\max} = \sqrt{rg \left(\frac{\tan \theta + \mu_s}{1 - \mu_s \tan \theta} \right)}$$

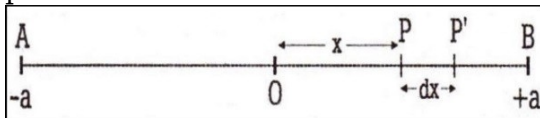
If $\mu_s = \cot \theta$, $v_{\max} = \infty$, But $(\mu_s)_{\max} = 1$. Thus, for $\theta \geq 45^\circ$, $v_{\max} = \infty$.

for heavily banked road, minimum limit may be important.

Q.28 Deduce the expression for the kinetic energy and potential energy of a particle executing S.H.M. Hence obtain the expression for total energy of a particle performing S.H.M. and show that the total energy is conserved. State the factors on which total energy depends. (4)

Ans:

Consider a particle performing S.H.M. about the equilibrium (M.P.) O, along a path length $AB=2a$. Let mass of particle is 'm' and it has a displacement 'x' from mean position at a point P.



A. P.E. of particle performing S.H.M:

1. The energy possess by the particle by virtue of its position is called potential energy at that point.
2. In this position of x from equilibrium particle is acted upon a restoring force $f = -kx$ towards the mean position.
3. If particle is further displaced through dx from p to p' against the restoring force, a small work dw is said to be done and stored in it as its potential energy.
4. $dw = -fdx = -[-kx][+dx] = +kxdx$
5. The total work done is displacing the particle from O to P is obtain by integrating

$$\therefore \int dw = \int_{x=0}^{x=x} -fdx = \int_{x=0}^{x=x} kxdx = k \int_{x=0}^{x=x} xdx$$

$$\omega = \frac{1}{2}kx^2$$

6. \therefore Total work done

$$P.E. = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 \quad [k = m\omega^2]$$

$$\therefore P.E. = \frac{1}{2}m\omega^2x^2 \dots \dots \dots (1)$$

7. Maximum/minimum P.E:

At mean position $x=0$

$$\text{At } x=0, P.E. = \frac{1}{2}m\omega^2x^2 = 0 = \text{minimum}$$

At extreme position $x=\pm a$

$$P.E. = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2a^2$$

$P.E. = \therefore$ Maximum

B. K.E. of particle performing S.H.M:

1. A body possesses K.E. by virtue of its motion.
2. A particle executing S.H.M. has linear velocity 'v' at any position 'x' from equilibrium.

$$v = \pm \omega \sqrt{a^2 - x^2}$$

3. If mass of body is 'm' then K.E. at any displacement 'x' is given by

$$4. K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m[\pm \omega \sqrt{a^2 - x^2}]^2$$

$$K.E. = \frac{1}{2}m\omega^2(a^2 - x^2) \dots \dots \dots (2)$$

$$\text{As } \frac{k}{m} = \omega^2$$

$$\therefore k = m\omega^2$$

$$5. K.E. = \frac{1}{2}k(a^2 - x^2) \dots \dots \dots (3)$$

6. Maximum and minimum K.E.:

At mean position $x=0$

$$K.E. = \frac{1}{2}mv^2(a^2 - 0)$$

$$\therefore \frac{1}{2}m\omega^2a^2 = \therefore \text{Maximum}$$

At extremity $x = \pm a$

$$K.E. = \frac{1}{2} m \omega^2 (a^2 - x^2)$$

$$\therefore \frac{1}{2} m \omega^2 (0) = 0$$

$K.E. = 0$ Minimum at extremity

C. T.E. in S.H.M.:

Total energy in S.H.M. is sum of K.E. and P.E. at that position and time.

$$\therefore T.E. = K.E. + P.E.$$

$$\therefore \frac{1}{2} m \omega^2 (a^2 - x^2) + \frac{1}{2} m \omega^2 x^2$$

$$\therefore \frac{1}{2} m \omega^2 [a^2 - x^2 + x^2]$$

$$\therefore \frac{1}{2} m \omega^2 a^2$$

$$T.E. = \frac{1}{2} m \omega^2 a^2 = \frac{1}{2} m 4 \pi^2 n^2 a^2$$

Hence, T.E. is independent of displacement and it is conserved at any position.

Conservation of energy in linear S.H.M.:

1. If particle is at mean position:

At mean position, velocity of particle in S.H.M. is maximum and displacement is minimum i.e. $x = 0$

$$\therefore K.E. = \frac{1}{2} m \omega^2 (a^2 - x^2)$$

$$\therefore \frac{1}{2} m \omega^2 (a^2 - a^2) = \frac{1}{2} m \omega^2 a^2$$

$$P.E. = \frac{1}{2} m \omega^2 (x)^2 = \frac{1}{2} m \omega^2 (0)^2 = 0$$

$$\therefore T.E. = K.E. + P.E.$$

$$\therefore \frac{1}{2} m \omega^2 a^2 + 0 = \frac{1}{2} m \omega^2 a^2 \dots\dots (2)$$

At mean position

$$T.E. = K.E.$$

2. If particle is at extreme position:

At extreme position, velocity of particle in S.H.M. is minimum and displacement is maximum i.e. $x = \pm a$

$$K.E. = \frac{1}{2} m \omega^2 (a^2 - x^2) = \frac{1}{2} m \omega^2 (a^2 - a^2) = 0$$

$$P.E. = \frac{1}{2} m \omega^2 x^2 = \frac{1}{2} m \omega^2 a^2$$

$$\therefore T.E. = K.E. + P.E.$$

$$\therefore \frac{1}{2} m \omega^2 a^2 + 0 = \frac{1}{2} m \omega^2 a^2 \dots\dots (3)$$

At extreme position

$$T.E. = P.E.$$

3. From equations (1), (2) and (3), it is observed that total energy of a particle performing linear S.H.M. at any point in its path is constant. Hence, total energy of linear S.H.M. remains conserved.

Q.29 Derive the relation between magnetic field intensity (H) and magnetization (M) for a magnetic material placed in a magnetizing field. Also find out magnetic induction in rod when a plate of magnetic material of size 10 cm × 0.5 cm × 0.2 cm (length,

breadth and thickness respectively) is located in magnetic moment of 5 Am^2 is induced in it. (4)

Ans:

1. Consider a magnetic material with some net magnetization, placed in a solenoid with n turns per unit length and carrying current I . Magnetic field inside the solenoid is given by

$$B_0 = \mu_0 n I$$

2. Let us denote the magnetic field due to the material kept inside the solenoid by B_m . Thus, the net magnetic field inside the rod can be expressed as

$$B = B_0 + B_m \dots\dots (1)$$

3. It has been observed that B_m is proportional to magnetization M of the material

$$B_m = \mu_0 M$$

Where, μ_0 is permeability of free space.

$$\therefore B = \mu_0 n I + \mu_0 M \dots\dots (2)$$

4. The net magnetic field inside the rod is,

$$B = \mu_0 H + \mu_0 M$$

$$B = \mu_0 (H + M) \dots\dots (3)$$

$$\therefore H = \frac{B}{\mu_0} - M \dots\dots\dots (4)$$

From the above expression we conclude that H and M have the same unit i.e., ampere per metre. and also have the same dimensions. Thus, the magnetic field induced in the material (B) depends on H and M .

Solution:

Given: $l = 10 \text{ cm} = 10^{-1} \text{ m}$,
 $b = 0.5 \text{ cm} = 5 \times 10^{-3} \text{ m}$,
 $h = 0.2 \text{ cm} = 2 \times 10^{-3} \text{ m}$,
 $H = 0.5 \times 10^4 \text{ Am}^{-1} = 5 \times 10^3 \text{ Am}^{-1}$,
 $\mu_0 = 4 \pi \times 10^{-7} \text{ TmA}^{-1}$
 $m_{\text{net}} = 5 \text{ Am}^2$

To find: Magnetic induction in the rod (B)

Formulae: i. $M = \frac{m_{\text{net}}}{V}$

ii. $B = \mu_0 (M + H)$

Calculation: From formula (i),

$$M = \frac{5}{10^{-1} \times 5 \times 10^{-3} \times 2 \times 10^{-3}}$$

$$= 5 \times 10^6 \text{ Am}^{-1}$$

From formula (ii),

$$B = 4 \times 3.142 \times 10^{-7} [5 \times 10^6 + 5 \times 10^3]$$

$$= 12.568 \times 10^{-7} \times 5 \times 10^3 [10^3 + 1]$$

$$= 6.284 \times 10^{-3} \times 1001$$

$$= 6.29 \times 10^3 \times 10^{-3}$$

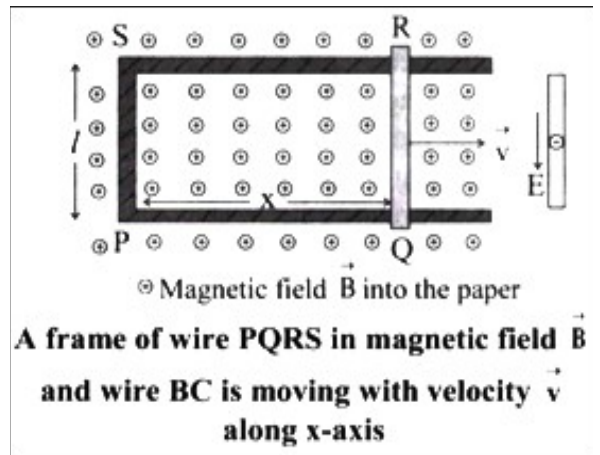
$$= \mathbf{6.29 \text{ T}}$$

Ans: Magnetic induction in the rod is **6.29 T**.

Q.30 Show that, for any circuit whose parts move in a fixed magnetic field, the induced emf is the time derivative of flux (ϕ) regardless of the shape of the circuit. (4)

Ans:

1. Consider a rectangular frame of wire ABCD of area (lx) is situated in a constant magnetic field (\vec{B}).
2. As the wire BC of length l is moved out with velocity \vec{v} to increase x the area of the loop ABCD increases. Thus, the flux of \vec{B} through the loop increases with time.
3. According to the 'Flux Rule' the induced emf will be equal to the rate at which the magnetic flux through a conducting circuit is changes.
4. The induced emf will cause a current in the loop. It is assumed that there is enough resistance in the wire so that the induced currents are very small producing negligible magnetic field.



5. As the flux ϕ through the frame ABCD is Blx , magnitude of the induced emf can be written as

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(Blx) = Bl \frac{dx}{dt} = Blv, \dots (1)$$

Where, v is the velocity of wire BC increasing the length x of wires AB and CD.

6. A charge q which is carried along by the moving wire BC, experiences Lorentz force $\vec{F} = q(\vec{v} \times \vec{B})$; which is perpendicular to both \vec{v} and \vec{B} and hence is parallel to wire BC.
7. The force \vec{F} is constant along the length l of the wire BC (as v and B are constant) and zero elsewhere ($\because v=0$ for stationary part CDAB of wire frame).
8. When a charge q moves a distance l along the wire, the work done by the Lorentz force is $W = F \cdot l = qvB \sin\theta \cdot l$, where θ is the angle between \vec{B} and \vec{v} .
9. The emf generated is work/charge i.e.,

$$e = \frac{W}{q} = vB \sin\theta \cdot l \dots (2)$$

10. For maximum induced emf, $\sin\theta = 1$

$$e_{max} = Blv \dots (3)$$

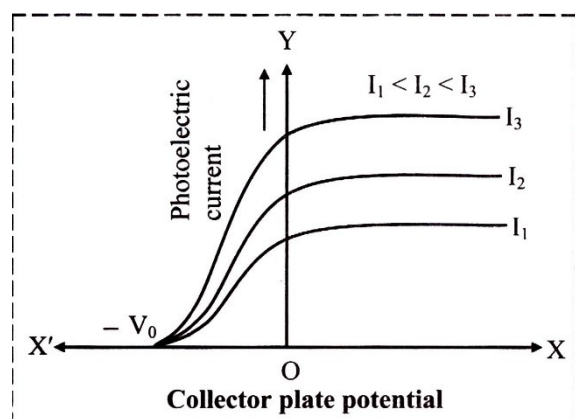
11. Thus, from Eq. (1) & (2) for any circuit whose parts move in a fixed magnetic field, the induced emf is the time derivative of flux (ϕ) regardless of the shape of the circuit.

Q.31 Explain in brief the effect of potential difference on photoelectric current at constant frequency of incident radiation. What will be the energy of each photon in monochromatic light of frequency 5×10^{14} Hz? (4)

Ans:

Effect of Potential difference on the Photoelectric Current:

1. The threshold frequency and intensity of incident radiation, both are kept constant at suitable value.
2. The positive potential of plate C is gradually increased and resulting photoelectric current is measured. It is found that photoelectric current increase with increase in positive (accelerating) potential.
3. At some stage for certain positive potential of plate C, all the emitted electrons are collected by the plate C and photoelectric current becomes maximum.
4. If we increase the positive potential of plate C further, the photoelectric current does not increase. This maximum value of photoelectric current is called saturation current.



5. After saturation state, reversing key is reversed and negative (retarding) potential is applied to the plate C with respect to plate E. Thus, negative potential of plate C gradually increases till the photoelectric current reduces to zero.
6. Photoelectric current becomes zero when the stopping potential is sufficient to repel the most energetic photoelectrons, with the maximum kinetic energy,

$$\therefore K.E_{max} = \frac{1}{2}mv_{max}^2 = eV_0$$

Where,

v_{max} = \hat{v} Maximum velocity of photoelectron

M = \hat{m} Mass of electron

E = \hat{e} Magnitude of charge on electron

V_0 = \hat{V} Stopping potential

7. If the frequency of incident radiation is kept constant and the experiment is repeated by using incident beam of different intensities, V_0 always remains the same.
8. Thus, for given frequency of incident radiation, the stopping potential and the maximum kinetic energy of photoelectrons are independent of the intensity of incident radiation.

Given: $\nu = 5 \times 10^{14}$ Hz

To find: Energy of photon (E)

Formula: $E = h\nu$

Calculation: From formula

$$\begin{aligned} E &= 6.63 \times 10^{-34} \times 5 \times 10^{14} \\ &= 3.31 \times 10^{-19} \text{ J} \\ &= \frac{3.31 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} \\ &= 2.068 \approx \mathbf{2.07 \text{ eV}} \end{aligned}$$

Ans: Energy of the photon is **2.07 eV**.

