| 2023 | 12 | 16 | TS-P2 | SEAT NUMBER |
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# IIT INSPIRE
## ACADEMY OF SCIENCE
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XI & XII Science (CBSE/state)
IIT- JEE (Mains + Advance)
NEET, MH-CET , NDA
Mo. No. 9595445177/9021445177
Branches : Chhatrapati Sq., Mangalmurti Sq.

1

(4 Pages)

calculator is not allowed.
5. All symbols having their usual meanings unless otherwise stated.
6. For each MCQ, correct answer must be written along with its alphabet.
7. Evaluation of each MCQ would be done for the first attempt only.

**Physical Constants:**
(1) $\pi=3.142$ (2) $g=10\,m/s^2$ (3) $h=6.63\times10^{-34}\,J.s$ (4) $c=3\times10^8\,m/s$
(5) $e=1.6\times10^{-19}C$ (6) $\varepsilon_0=8.85\times10^{-12}C^2/N.m^2$ (7) $\mu_0=4\pi\times10^7\,T.m/A$,
(8) $\sigma=5.7\times10^{-8}W/m^2K^4$

## SECTION-A

**Q.1 Select and write the correct answers to the following questions:** **[10]**

1) Which of the following is an example of the first law of thermodynamics? _________
   **d) After falling down the hill, a ball's kinetic energy plus heat energy equals the initial potential energy** **(1)**
2) One beat means that intensity of sound should be _____________
   **c) Once maximum and once minimum** **(1)**
3) The ___________ is equal to the ratio of the speed of light in vacuum (c) to the speed of light in the medium (v).
   **c) Refractive index** **(1)**
4) If the charge on condenser of $10\,\mu F$ is doubled then the energy stored in it becomes ____
   **d) Four times the initial energy** **(1)**
5) A charge q moves in a region where electric field $\vec{E}$ and magnetic field $\vec{B}$ both exist then the force on it is
   **b)** $q\vec{E}+q(\vec{v}\times\vec{B})$ **(1)**
6) A conductor rod of length (l) is moving with velocity (v) in a direction normal to a uniform magnetic field (B). What will be the magnitude of induced emf produced between the ends of moving conductor?
   **a)** $B/v$ **(1)**

7) Solar cell operates on the principle of:
    **c) Photo voltaic action** **(1)**

8) The ratio of emissive power of perfectly black body at $1327^0 C$ and $527^0 C$ is
    **b)** $16:1$ **(1)**

9) If the relative permeability of iron is 2000, its absolute permeability in SI unit is
    **a)** $8\pi \times 10^{-4} Tm/A$ **(1)**

10) The Boolean equation $A+\overline{A}$ is equal to
    **d) 1** **(1)**

**Q.2 Answer the following questions in one sentence:** **[8]**

(1) **State the analogous of torque in rotational motion with force in the linear motion.**
**Ans:**
The relation $\tau = I \propto$ is analogous to $f = ma$ for the translational motion if the moment of inertia I replaces mass, which is its physical significance.

(2) **What is meant by angle of contact?** **(1)**
**Ans:**
The angle of contact $(\theta)$ between a liquid and a solid surface is defined as the angle between the tangents drawn to the free surface of the liquid and surface of the solid at the point of contact, measured within the liquid.

(3) **What is the coefficient of absorption?** **(1)**
**Ans:**
The ratio of amount of heat absorbed to total quantity of heat incident is called the coefficient of absorption.

(4) **State zeroth law of thermodynamics.** **(1)**
**Ans:**
If two systems are each in thermal equilibrium with a third system, they are also in thermal equilibrium with each other.

(5) **Define magnetization.** **(1)**
**Ans:**
The ratio of magnetic moment to the volume of the material is called magnetization.

(6) **Define Threshold frequency.** **(1)**
**Ans:**
The minimum frequency of incident radiation required to start a photoemission in any photosensitive material is known as threshold frequency.

(7) **A potential drop per unit length along a wire is** $5 \times 10^{-3} V/m$. **If the emf of the cell balances against the length 216 cm of this potentiometer wire, find the emf of the cell.** **(1)**
**Ans:**
Given: Potential drop per unit length (K)
¿ $5 \times 10^{-3} \frac{V}{m}, l = 216\, cm = 2.16\, m$
To find: Emf of cell (E)
Formula: $E = Kl$

Calculation: From formula,

$$E = 5\times 10^{-3}\times 2.16 = 10.80\times 10^{-3}$$

¿ $0.0108\,V$

The emf of the cell is 0.0108 V.

(8) **In a common-base connection, the emitter current is 6.28 mA and collector current is 6.20 mA. Determine the common base DC current gain.** **(1)**

**Ans:**

Given: $I_E = 6.28\,mA,\ I_C = 6.20\,mA$

To find: DC current gain $(\alpha_{DC})$

Formula: $\alpha_{DC} = \dfrac{I_C}{I_E}$

Calculation: From formula,

$$\alpha_{DC} = \frac{6.20}{6.28} = 0.987$$

The value of DC current gain is 0.987.

## SECTION-B

**Attempt any eight of the following questions:** **[16]**

**Q.3 State and explain Newton's law of viscosity.** **(2)**

**Ans:**

According to Newton's law of viscosity, for a streamline flow, viscous force (f) acting on any layer is directly proportional to the area (A) of the layer and the velocity gradient (dv/dx) i.e.,

$$f \propto A\left(\frac{dv}{dx}\right)$$

$$\therefore f = \eta A\left(\frac{dv}{dx}\right) \quad \ldots\ldots\ldots (1)$$

Where, η is a constant, called coefficient of viscosity of the liquid. From Eq. 1 we can write,

$$\eta = \frac{f}{A\left(\frac{dv}{dx}\right)}$$

**Q.4 Distinguish between reversible and irreversible processes.** **(2)**

**Ans:**

| No. | Reversible process | Irreversible process |
|---|---|---|
| 1. | A reversible process is a change that can be retraced in reverse (opposite) direction. | An irreversible process is a change that cannot be retracted in reverse (opposite) direction. |
| 2. | The path of a reversible process is the same in the forward and the reverse direction. | The path of an irreversible process is not the same in the forward and the reverse direction. |

| | | |
|---|---|---|
| 3. | Reversible changes are very slow and there is no loss of any energy in the process. | There is a permanent loss of energy from the system due to friction or other dissipative forces in an irreversible process, |
| 4. | The system comes back to its initial state after it is taken along the reverse path. | The change of state depends upon the path taken to change the state during an irreversible process. |
| 5. | Reversible processes are ideal processes. | Irreversible processes are real processes. |

**Q.5 Show that for pipe open at both ends, the end correction is** $e=\frac{n_2 l_2-n_1 l_1}{2(n_1-n_2)}$

**(2)**

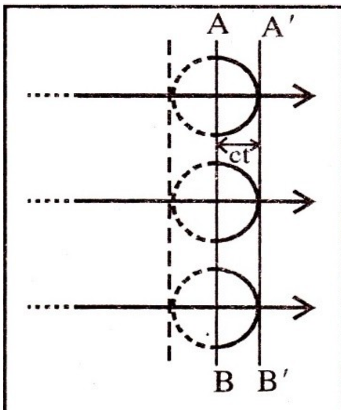
**Ans:**

1. Let,
   $l_1$ and $l_2=¿$ Vibrating length of pipe
   $n_1$ and $n_2=¿$ Resonating frequency
   $v=¿$ Velocity of sound
   $e=¿$ End correction
2. For the first resonance,
   $v=2n_1 L_1$
   $\therefore v=2n_1(l_1+2e)$ .............. (1)
3. For the second resonance,
   $v=2n_2 L_2$
   $\therefore v=2n_2(l_2+2e)$ .............. (2)
4. From equations (1) and (2) we get,
   $n_1(l_1+2e)=n_2(l_2+2e)$
   $\therefore n_1 l_1+2en_1=n_2 l_2=2en_2$
   $\therefore 2en_1-2en_2=n_2 l_2-n_1 l_1$
   $\therefore 2e(n_1-n_2)=n_2 l_2-n_1 l_1$
   $\therefore e=\frac{n_2 l_2-n_1 l_1}{2¿¿¿}$

**Q.6 Explain the Huygen's construction of plane wavefront.** **(2)**

**Ans:**

**Construction of plane wave front:**



**Huygens' construction for plane wavefront**

1. For the construction of plane wave front, let AB is the plane wave front. Every point on the wave front act as secondary source of light. These wave front reaches at distance 'ct' in time t.
2. Draw the sphere from different points on AB of radius ct. c is velocity of light.
3. The envelope of all these spheres acts as secondary wave front at distance ct.
4. Let us suppose the wave front AB is moving from left to right direction. A', B' is the new wave front in the same direction.

5. The semisphere in the diagram is shown by doted lines. It indicates that light cannot travel in backward direction.

**Q.7 Derive an expression for potential energy of a system of two charges in an external field. (2)**

**Ans:**

1. Let a system of two charges $q_1$ and $q_2$ be located at $r_1$ and $r_2$ respectively in an external field.
2. To bring the charge $q_2$ to $r_2$, the work is done not only against the external field E but also against the field due to $q_1$.

   ∴ Work done on $q_2$ against the external field $¿ q_2 V(\vec{r}_2)$ and Work done on $q_2$ against the field due to $q_1 = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$,

   Where, $r_{12} = ¿$ distance between $q_1$ and $q_2$.
3. By the principle of superposition for field, we add up the work done on $q_2$ against the two fields.

   ∴ Work done in bringing $q_2$ to $r_2$.

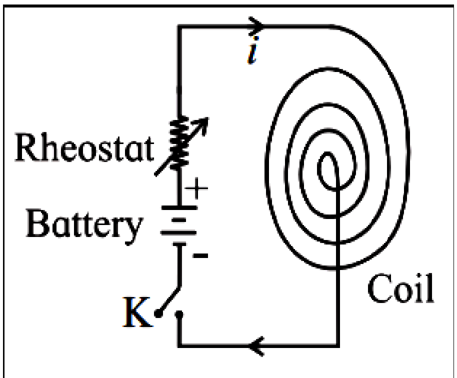
   $¿ q_2 V(\vec{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$ ........ (2)

   Thus from (1) and (2) potential energy of the system
   = Total work done in assembling the configuration

   $¿ q_1 V(\vec{r}_1) + q_2 V(\vec{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

**Q.8 Explain what is self-inductance? (2)**

**Ans:**



**Changing current in a coil**

1. Consider a circuit (coil) in which the current is changing.
2. The changing current will vary the magnitude of magnetic flux linked with the coil (circuit) itself and consequently an emf will be induced in the circuit.
3. The production of induced emf, in the circuit (coil) itself, on account of a change in the current in it, is termed as the phenomenon of self-inductance
4. Let at any instant, the value of magnetic flux linked with the circuit itself be $\phi$ corresponding to current I in it figure. It is obvious that $\phi$ will be proportional to current I.

   $i.e., \phi \propto I$

   or $\phi = LI \vee L = \phi / I$ ............. (1)

   Where, L is a constant of proportionality and is termed as the self-inductance (or coefficient of self-induction) of the coil.
5. For a closely wound coil of N turns, the same magnetic flux will be linked with all the turns.
6. When the flux through the coil changes each turn of the coil contributes towards the induced emf. Therefore, a term flux linkage is used for a closely wound coil.
7. The flux linkage for a coil with N turns corresponding to current I will be written as

   $N\phi_B \propto I$

   $N\phi_B = LI$

   $L = N\phi_B / I$ ............... (2)

   The inductance (L) depends only on the geometry and material properties of the coil.

8. According to Faraday's law, induced emf e is given by

$$e=\frac{d\phi}{dt}$$

Using Eq. (1)

$$e=\frac{-d}{dt}(LI)=-L\frac{dI}{dt} \;\; ..... (3)$$

Unit of $L=\frac{|e|}{¿dI/dt\vee¿}=\left[\frac{volt}{A/s}\right]=Henry¿$

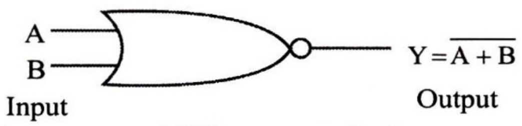
**Q.9 How does NOR gate work? Draw the schematic symbol for NOR gate. Write its Boolean expression and truth table. (2)**

**Ans:**

i. The NOR gate is formed by connecting the output of a NOT gate to the input of an OR gate.

ii. The output of a NOR gate is exactly opposite to that of an OR gate. The output Y of a NOR gate is high or 1 only when both the inputs are low or 0.

iii. **Schematic symbol:**




**NOR gate symbol**

iv. **Boolean expression:**

$Y = \overline{A+B}$

v. **Truth table:**

| Input A | Input B | Output Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

**Q.10 A small-blackened solid copper sphere of radius 2.5 cm is placed in an evacuated chamber. The temperature of the chamber maintained at $100^0C$. At what rate energy must be supplied to the copper sphere to maintain its temperature at $110^0C$? (Take Stefan's constant $\sigma$ to be $5.67\times10^{-8}Js^{-1}m^{-2}K^{-4}$ and treat the sphere as blackbody.) (2)**

**Ans:**

*Given:* r = 2.5 cm,

$T_0 = 100\ °C = 100 + 273 = 373\ K$

$T = 110\ °C = 110 + 273 = 383\ K$

$\sigma = 5.67 \times 10^{-8}\ J\ s^{-1}\ m^{-2}\ K^{-4}$

*To find:* Rate of energy supplied $\left(\frac{dQ}{dt}\right)$

*Formulae:* i. $A = 4\pi r^2$

ii. $\frac{dQ}{dt} = \sigma A\,(T^4 - T_0^4)$

*Calculation:*

From formula (i),

$A = 4 \times 3.142 \times (2.5)^2 \times 10^{-4}$

$= 4 \times 3.142 \times 6.25 \times 10^{-4}$

$= 3.142 \times 25 \times 10^{-4} = 7.855 \times 10^{-3}\ m^2$

From formula (ii),

$$\frac{dQ}{dt} = 5.67 \times 10^{-8} \times 7.855 \times 10^{-3} \times (383^4 - 373^4)$$

$$= 5.67 \times 7.855 \times 10^{-11} \times [\text{antilog}\ \{4 \times \log(383)\} - \text{antilog}\ \{4 \times \log(373)\}]$$

$$= 44.54 \times 10^{-11} \times [\text{antilog}\ \{10.3328\} - \text{antilog}\ \{10.2868\}]$$

$$= 44.54 \times (2.152 \times 10^{10} - 1.936 \times 10^{10}) \times 10^{-11}$$

$$= 44.54 \times 0.216 \times 10^{-1} = \mathbf{0.962\ W}$$

**Ans:** Rate of energy supplied is **0.962 W.**

**Q.11 At what distance from the mean position is the speed of particle performing S.H.M. half its maximum speed. Given: path length of S.H.M. = 10 cm.** **(2)**

**Ans:**

*Given:* Path length of SHM = 10 cm, $v = \frac{v_{max}}{2}$

*To find:* Distance (x)

*Formulae:* i. $v_{max} = \omega A$

ii. $v = \omega\sqrt{A^2 - x^2}$

*Calculation:*

$$\text{Amplitude} = \frac{\text{Path length}}{2} = 5\ \text{cm}$$

From formula (ii),

$$v = \omega\sqrt{A^2 - x^2}$$

But $v = \frac{v_{max}}{2}$

$$\omega\sqrt{A^2 - x^2} = \frac{v_{max}}{2}$$

$$\omega\sqrt{A^2 - x^2} = \frac{\omega A}{2} \quad \text{.... [From formula (i)]}$$

$$\therefore \quad A^2 - x^2 = \frac{A^2}{4}$$

$$\therefore \quad x = \frac{\sqrt{3}}{2} \times A$$

$$= \frac{\sqrt{3}}{2} \times 5$$

$$= 0.866 \times 5$$

$$= \mathbf{4.33\ cm}$$

**Ans:** The distance at which speed of particle is half of its maximum value of **4.33 cm**.

**Q.12 A galvanometer has a resistance of 40 Ω and a current of 4 mA is needed for a full-scale deflection. What is the resistance and how it is to be connected to convert the galvanometer?**

**1) into an ammeter of 0.4 A range and** **(1)**

**2) into a voltmeter of 0.5 V range?** **(1)**

**Ans:**

*Given:* $G = 40\ \Omega$, $I_g = 4\ mA = 4 \times 10^{-3}\ A$,
$I = 0.4\ A$, $V = 0.5\ V$

*To find:*
i. Resistance to convert into ammeter(S)
ii. Resistance to convert into voltmeter(X)

*Formulae:* i. $S = \frac{I_g}{I - I_g} \times G$ ii. $V = I_g(G + X)$

*Calculation:* From formula (i),

$$S = \left(\frac{0.004}{0.4 - 0.004}\right) \times 40$$

$$= \frac{0.004 \times 40}{0.396}$$

$$= \frac{0.16}{0.396} = \mathbf{0.4040\ \Omega}$$

From formula (ii),

$0.5 = 0.004(40 + X)$

$$\therefore \quad X = \frac{0.5}{0.004} - 40 = 125 - 40 = \mathbf{85\ \Omega}$$

**Ans:** i. Resistance of **0.404 Ω** is connected in parallel to convert it into ammeter.
ii. Resistance of **85 Ω** is connected in series to convert it into voltmeter.

**Q.13 Current of equal magnitude flows through two long parallel wires having separation of 1.35 cm. If the force per unit length on each of the wires in $4.76 \times 10^{-2} N$, what must be I? (2)**

**Ans:**

*Given:* $I_1 = I_2 = I$, $\frac{F}{L} = 4.76 \times 10^{-2}\ N$

$d = 1.35\ cm = 1.35 \times 10^{-2}\ m$

*To find:* Electric current

*Formula:* $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

*Calculation:* From formula,

$$4.76 \times 10^{-2} = \frac{4\pi \times 10^{-7} \times I \times I}{2 \times \pi \times 1.35 \times 10^{-2}}$$

$$\therefore \quad I^2 = \frac{4.76 \times 10^{-2} \times 1.35 \times 10^{-2}}{2 \times 10^{-7}}$$

$$= 1.35 \times 2.38 \times 10^{-2-2+7}$$

$$I = \sqrt{1.35 \times 2.38 \times 10^3}$$

$$= \sqrt{13.5 \times 2.38} \times 10$$

$$= \left\{\text{anti}\log\left(\frac{1}{2}(\log 13.5 + \log 2.38)\right)\right\} \times 10$$

$$= \left\{\text{anti}\log\left(\frac{1}{2}(1.1303 + 0.3766)\right)\right\} \times 10$$

$= \{\text{antilog}\ (0.7535)\} \times 10$

$= 5.669 \times 10$

$= \mathbf{56.7\ A}$

**Ans:** The electric current is **56.7 A**.

**Q.14 If the effective current in a 50 cycle AC circuit of 5 A, what the peak value of current? What is current 1/600 sec. after if was zero? (2)**

**Ans:**

*Given:* $f = 50$ Hz, $i_{rms} = 5$ A, $t = \frac{1}{600}$ s

*To find:* i. Peak value of current ($i_0$)
ii. Instantaneous current (i)

*Formula:* i. $i_0 = \sqrt{2}\, i_{rms}$  ii. $i = i_0 \sin(\omega t)$

*Calculation:*

From formula (i),

$$i_0 = \sqrt{2} \times 5$$
$$= 1.414 \times 5$$
$$= \mathbf{7.07\ A}$$

From formula (ii),

$$i = 7.07 \sin\left(2\pi \times 50 \times \frac{1}{600}\right) \quad ....(\because \omega = 2\pi f)$$
$$= 7.07 \sin\left(\frac{\pi}{6}\right)$$
$$= 7.07 \times 0.5$$
$$= \mathbf{3.535\ A}$$

**Ans:** i. Peak value of current is **7.07 A.**
ii. Instantaneous value of current is **3.535 A.**

## SECTION-C

**Attempt any eight of the following questions: [24]**

**Q.15 Derive an expression that relates angular momentum with the angular velocity of a rigid body. (3)**

**Ans:**

*The sum of the moments of linear momentum of all the particles of body is called as angular momentum of body.*

It is denoted by = L

It is vector quantity.

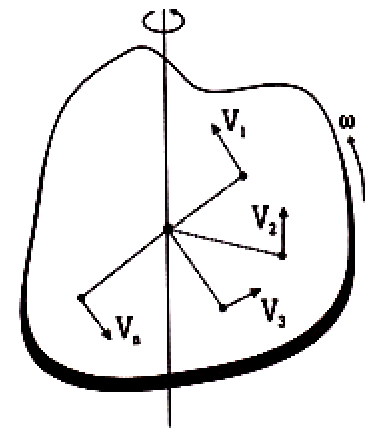
Mathematically it is equal to product of MI and angular velocity of body.

$\therefore L = I\omega$

**SI unit**: $Kg\, m^2/s$

**CGS unit**: $gr\, cm^2/s$

**Dimension**: $[L^2 M^1 T^{-1}]$




Moment of linear momentum of particle is called as angular momentum of that particle.

Consider a rigid body of any shape is rotating about an axis passing through of perpendicular to its plane.

Suppose body is made up of 'n' particle having masses $m_1$, $m_2$, $m_3$ .... $m_n$ situated at distance $r_1$, $r_2$, $r_3$ ... $r_n$.

When body rotates each and every particle of body perform circular motion with same velocity ω but different linear velocity such as $v_1$, $v_2$, $v_3$ ... $v_n$.

Linear momentum of particle of mass m, situated distance $r_1$ from axis of rotation

$p_1 = m_1 v_1$

But, $v_1 = r_1 \omega$

$p_1 = m_1 r_1 \omega$
Moment of linear momentum of particle having mass $m_1 = ¿$ linear momentum × momentum
$L_1 = p_1 \times r_1$
$L_1 = m_1 r_1^2 \omega$
Moment of linear moment of particle having mass $m_2$. Situated at distance $r_2$ from axis of rotation
$L_2 = m_2 r_2^2 \omega \ldots . L_n = m_n r_n^2 \omega$
By definition,
Angular momentum of body,
$L = L_1 + L_2 + \ldots \ldots . . + L_n$
$L = m_1 r_1^2 \omega + m_2 r_2^2 \omega \ldots . m_n r_n^2 \omega$
$L = \omega \left( \sum_{i=1}^{n} m_i r_i \right)$
$L = I \omega$
Angular momentum is a vector quantity, whose direction is same as that of $\vec{\omega}$ i.e. along the axis of rotation.
$\therefore \vec{L} = I . \vec{\omega}$

**Q.16 Derive the Mayer's relation for a molar as well as principal specific heats. (3)**

**Ans:**

1. Consider one mole of an ideal gas that is enclosed in a cylinder by light, frictionless air tight piston.
2. Let P, V and T be the pressure, volume and temperature respectively of the gas.
3. If the gas is heated so that its temperature rises by dT, but the volume remains constant, then the amount of heat supplied to the gas, $dQ_1$, is used to increase the internal energy of the gas(dE). Since, volume of the gas is constant, no work is done in moving the piston.
   $\therefore d Q_1 = dE = C_4 dT$ ……….. (1)
4. On the other hand, if the gas is heated to the same temperature, at constant pressure, volume of the gas increases by an amount say dV. The amount of heat supplied to the gas is used to increase the internal energy of the gas as well as to move the piston backwards to allow expansion of gas (the work done to move the piston $dW = PdV$)
   $d Q_2 = dE + dW = C_P dt$ ……. (2)
   Where, $C_P$ is the molar specific heat of the gas at constant pressure.
5. But $dE = C_v dT$ from Eq. (1) as the internal energy of an ideal gas depends only on its temperature.
   $\therefore C_P dT = C_V dT + dW$
   Or, $\left( C_P - C_V \right) dT = P dV$ ……….. (3)
6. For one mole of gas,
   $PV = RT$
   $\therefore P dV = R dT$, since pressure is constant.
   Substituting in Eq. (3), we get
   $\left( C_P - C_v \right) dT = R dT$
   $\therefore C_P - C_v = R$
   This is known as Mayer's relation between $C_P$ and $C_V$.
7. The above relation has been derived assuming that the heat energy and mechanical work are measured in the same units. Generally, heat supplied is measured in calories and work done is measured in joules. the above relation then is modified to $C_P - C_V = R / J$ where J is mechanical equivalent of heat.
8. Also $C_P = M_0 S_P$ and $C_V = M_0 S_V$, where $M_0$ is the molar mass of the gas and $S_P$ and $S_V$ are

respective principal specific heats. (In many books, $c_P$ and $c_v$ are used to denote the principal specific heats). Thus,

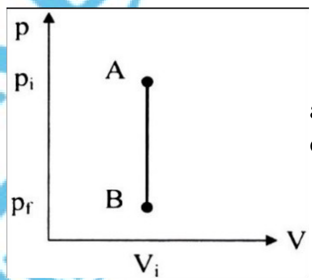
$M_0 S_P - M_0 S_V = R/J$

$\therefore S_P - S_V = R/M_0 J$ .......... (5)

**Q.17 Explain isochoric process with an expression.** **(3)**

**Ans:**

1. A thermodynamic process in which volume of the system is kept constant is called isochoric process $\Delta V = 0$.
2. A system does not work on its environment during an isochoric process $W = 0$.
3. p-V diagram of isochoric process is as shown below:




4. For an isochoric process, $\Delta V = 0$, and from the first law of thermodynamic, $\Delta U = Q$.
5. This means that for an isochoric change, all the energy added in the form of heat remains in the system itself and causes an increase in its internal energy. Also, as volume is unchanged, no work is done.
6. Temperature of the system changes, i.e., $\Delta T \neq 0$.
7. Heating a gas in a constant volume container or diffusion of a gas in a closed chamber are some examples of isochoric process.

**Expression for heat exchanged in an isochoric process:**

1. For an isochoric process, we have, $\Delta V = 0$.
2. The system does not do any work and all the energy supplied to the system is converted into its internal energy.
3. The first law of thermodynamics for isochoric process is

$Q = \Delta U$ ................ (1)

The change in internal energy is given by

$\Delta U = n C_v \Delta T$ .......... (2)

The work done is given by

$W = p \Delta V = 0 \left( beacuse \, \Delta V = 0 \right)$

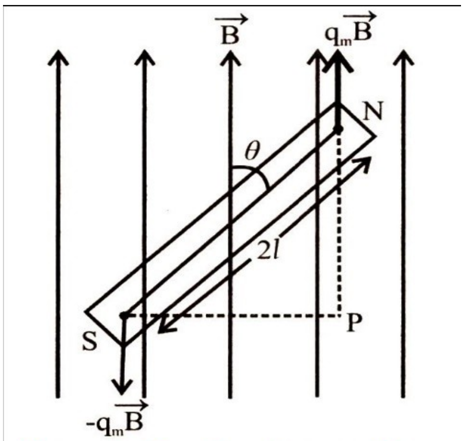
The heat exchanged is given by the first law of thermodynamics,

$Q = \Delta U + W = \Delta U = n C_v \Delta T$

**Q.18 Obtain the expression for the period of a magnet vibrating in a uniform magnetic field and performing S.H.M.** **(3)**

**Ans:**

1. If a bar magnet is freely suspended in the plane of a uniform magnetic field, it remains in equilibrium with its axis parallel to the direction of the field.




Magnet vibrating in a uniform magnetic field

2. If it is given a small angular displacement $\theta$ (about an axis passing through its centre, perpendicular to itself and to the field) and released, it performs angular oscillations.
3. Let $\mu$ be the magnetic dipole moment and B the magnetic field. In the deflected position, a restoring torque acts on the magnet that tends to bring it back to its equilibrium position.
4. The magnitude of this torque is $\tau = \mu B sin\theta$

If $\theta$ is small, $sin\theta \cong \theta^c$

$\therefore \tau = \mu B \theta$

5. For clockwise angular displacement $\theta$, the restoring torque is in the anticlockwise direction.
   $\therefore \tau = I\alpha = -\mu B\theta$
   Where, I is the moment of inertia of the bar magnet and $\alpha$ is its angular acceleration.
   $\therefore \alpha = -\left(\frac{\mu B}{I}\right)\theta$ ………… (1)
6. Since $\mu$, B and I are constant, equation (1) shows that angular acceleration is directly proportional to the angular displacement and directed opposite to the angular displacement. Hence the magnet performs angular S.H.M. The period of vibrations of the magnet is given by
   $$T = \frac{2\pi}{\sqrt{\text{angular acceleration per unit angular displacement}}}$$
   ¿ $\frac{2\pi}{\sqrt{\alpha/\theta}}$
   $\therefore T = 2\pi\sqrt{\frac{I}{\mu B}}$ ………… (2)

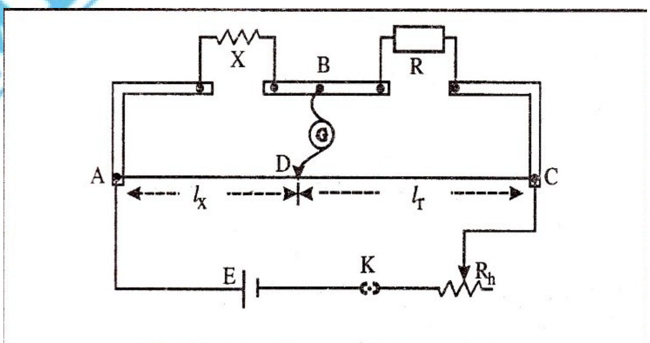
**Q.19 Explain the net circuit diagram, how you will determine the unknown resistance by using a metrebridge. (3)**

**Ans:**

This instrument is used to measure the unknown value of resistance. It is based on Wheatstone's network principle. The circuit diagram of Meter Bridge is as shown in the diagram.

**Construction:**

1. It consists of wooden platform, on which one meter uniform homogenous wire of conducting material is stretched between the points A and C.
2. The ends of wire are fixed using two L-shaped copper plates. The third copper plate is fixed on the board in such way that two gaps are formed.
3. In one gap, unknown resistance X is connected. In other gap, resistance box R is connected.
4. A source of emf 'E' plug key 'K' and variable resistance $R_h$ are connected in series with the wire. Galvanometer 'G' is connected between the junction 'B' of unknown resistance 'X' and resistance box 'R' and other terminal is connected to jockey.



Constructional diagram of Metre bridge

5. A meter scale is fixed on wooden platform to measure the distance between different points A, D and D, C on the wire.

**Working:**

1. Place the jockey at point A and at C, and see the deflection in galvanometer. The deflection must be on opposite sides otherwise adjust rheostat and the value of resistance from resistance box.
2. When current flow through circuit, by touching the jockey on different points of wire, find out the point D. The point D is called as null point and balance point.
3. This point D is such that, when we touch the jockey at that point, galvanometer shows null deflection.
4. That means potential at point B is equal to potential at point D.
5. Here, the distance $AD = l_x$ and the distance $DC = l_x$
6. In this condition, using Wheatstone's principle, we can write

$$\frac{X}{R}=\frac{resistance\,of\,l_x\,length\,of\,wire}{resistance\,of\,l_r\,length\,of\,wire}$$

If $\sigma$ is resistance per unit length of wire, then, resistance of $l_x$ length of wire is $\sigma l_x$ and length $l_x$ is $\sigma l_x$

$$\frac{X}{R}=\frac{\sigma l_x}{\sigma l_r}$$

$$X=R.\frac{l_x}{l_r} \text{ ........... (1)}$$

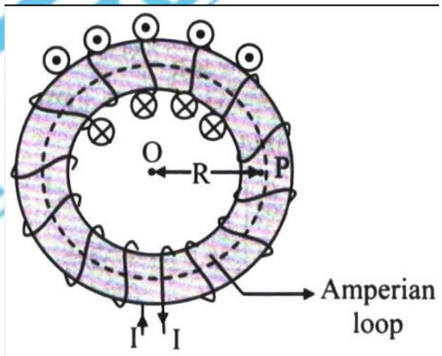
Using this equation, we can find out the value of resistance X.

**Q.20 What is toroid? Using Ampere's law, derive an expression for magnetic induction at a point along the axis of a toroid. (3)**

**Ans:**

**Expression for magnetic induction at a point along the axis of a toroid:**

1. A toroid is a solenoid of finite length bent into a hollow circular tube-like structure similar to a pressurized rubber tube inside a tyre of vehicle.
2. Schematic of a cross section of a toroid is shown in figure.
3. The magnetic field lines are concentric circles in the toroid. The direction of the field is dictated by the direction of the current i in the coil around the toroid. Again, by the Ampere's law,
   $\oint \vec{B}.d\vec{l}=\mu_0 I$
   Where, I is the net current encircled by the loop.
4. $B.2\pi R=\mu_0 i N$ ....... (1)
   Here N is the total number of turns in the toroid as the integration is over the full length of the loop, $2\pi R$.
   $\therefore B=\frac{\mu_0 i N}{2\pi R}$ ........... (2)
5. From the Eq. (2), B is inversely proportional to R. Thus, unlike the solenoid, magnetic field is not constant over the cross section of the toroid.




**Q.21 Obtain an expression for orbital magnetic moment of an electron rotating about the nucleus in an atom. (3)**

**Ans:**

An atom consists of positively charged heavy nucleus around which negatively charged electrons are revolving in circular orbit.

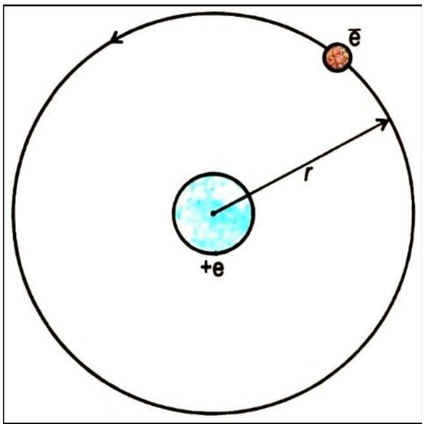
The electron of charge (-e) performs U.C.M. around a stationary nucleus with period of revolution T. If r be the radius of the orbit of revolution of the electron and v is the orbital velocity then

Period of Revolution ¿ $\frac{Circumference}{Velocity}$

$$T=\frac{2\pi r}{v} \text{ ............. (1)}$$

Circulating current $I=\frac{e}{T}$ ........ (2)

$$I=\frac{e}{\frac{2\pi r}{v}}=\frac{ev}{2\pi r} \text{ ....... (3)}$$



Magnitude of magnetic moment associated with circular current is

$$M_0 = IA = \frac{ev}{2\pi r} \times \pi r^2 \text{ .......... (4)}$$

$$\therefore\ M_0 = \frac{evr}{2} \text{ ....................... (5)}$$

The direction of this magnetic moment is into the plane of paper. Negatively charged electron is moving in anticlockwise direction, leading to a clockwise current.
Multiplying and dividing the right-hand side of equation (5) by the mass of electron $m_e$ then

$$M_0 = \frac{e}{2M_e}(m_e v r) \text{ ....... (6)}$$

$$M_0 = \frac{e}{2M_e} \times L_0 \text{ ........... (7)}$$

Here $L_0 = M_e vr =$ ¿ angular momentum of the electron revolving rounds the nucleus.

In vector $\vec{M}_0 = \frac{-e}{2M_e} \times \vec{L}_0$ ......... (8)

The negative sign indicates that the orbital angular momentum of electron is opposite in the direction to the orbital magnetic moment.

**Q.22 A pendulum consisting of a massless string of length 20 cm and a tiny bob of mass 100 g is set up as a conical pendulum. Its bob now performs 75 rpm. Calculate kinetic energy and increase in the gravitational potential energy of the bob.**
**(Use $\pi^2 = 10$, $cos\theta = 0.8$)** **(3)**
**Ans:**

*Given:* $l$ = 20 cm = 0.2 m,
m = 100 g = 0.1 kg,
$\eta$ = 75 rpm

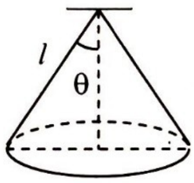
$$\omega = \frac{75\times 2\pi}{60}$$
$$= \frac{5}{2}\pi \text{ rad/s}$$

*To find:*
i. Kinetic energy of the bob
ii. Increase in gravitational potential energy of the bob w.r.t. inner edge (h)

*Formulae:*
i. $KE = \frac{1}{2}mr^2\omega^2$
ii. $\Delta P.E. = mgl\,(1 - \cos\theta)$

*Calculation:*

$$\sin\theta = \sqrt{1-(0.8)^2} = 0.6$$




$$r = l\sin\theta$$
$$= 0.2 \times 0.6$$
$$= 0.12 \text{ m}$$

From formula (i),

$$K.E. = \frac{1}{2}\times 0.1\times(0.12)^2\times\left(\frac{5}{2}\pi\right)^2$$
$$= \frac{1}{2}\times\frac{1}{10}\times 0.0144\times\frac{25}{4}\times\pi^2$$
$$= \mathbf{0.045\ J}$$

From formula (ii),

$$\Delta P.E = 0.1\times 10\times 0.2\times(1-0.8)$$
$$= \mathbf{0.04\ J}$$

**Ans:** i. Kinetic energy of the bob is **0.045 J.**
ii. Change in gravitational potential energy of the bob is **0.04 J.**

**Q.23 Find the fundamental first overtone end second overtone frequencies of a pipe, open at both the ends, of length 25 cm if the speed of sound in air is 330 m/s. (3)**

**Ans:**

Given: $L = 25\,cm = 0.25\,m, v = 330\,m/s$

*To find:*
i. Fundamental frequency ($n_0$)
ii. First overtone ($n_1$)
iii. Second overtone ($n_2$)

*Formulae:*
i. $n_0 = \frac{v}{2\,L}$
ii. $n_p = (p+1)n_0$

*Calculation:* From formula (i),

$n_0 = \frac{330}{2 \times 0.25} = \mathbf{660\ Hz}$

From formula (ii),

$n_1 = (1 + 1)n_0$

$= 2 \times 660$

$= \mathbf{1320\ Hz}$

From formula (ii),

$n_2 = (2 + 1)\ n_0$

$= 3 \times 660$

$= \mathbf{1980\ Hz}$

**Ans:** The fundamental, first overtone and second overtone frequencies are **660 Hz, 1320 Hz** and **1980 Hz** respectively.

**Q.24 In a parallel plate capacitor with the air between the plates, each plate has an area of $6 \times 10^{-3} m^2$ and the separation between the plates is 2 mm.**

**1) Calculate the capacitance of capacitor,**

**2) If this capacitor is connected to 100 v supply, what would be the charge on each plate?**

**3) How would charge on the plates be affected if a 2 mm thick mica sheet of $k=6$ is inserted between the plates while the voltage supply remains connected? (3)**

**Ans:**

*Given:* $A = 6 \times 10^{-3}\ m^2$, $r = 2\ mm = 2 \times 10^{-3}\ m$

$V = 100\ V$

*To find:*
i. Capacitance of capacitor
ii. Charge on plates of capacitor
iii. Change in charge of capacitor

*Formulae:* i. $C = \frac{\varepsilon_0 A}{d}$ ii. $Q = CV$

iii. $Q' = kQ$

*Calculation:* From formula (i)

$C = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{2 \times 10^{-3}} = \mathbf{2.655 \times 10^{-11}\ F}$

From formula (ii)

$Q = 2.655 \times 10^{-11} \times 100$

$= \mathbf{2.655 \times 10^{-9}\ C}$

From formula (iii)

$Q' = 6 \times 2.655 \times 10^{-9}$

$= \mathbf{1.593 \times 10^{-8}\ C}$

**Ans:** i. Capacitance is $\mathbf{2.655 \times 10^{-11}\ F}$
ii. Charge on plates is $\mathbf{2.655 \times 10^{-9}\ C}$
iii. Charge on plates when dielectric is inserted in it is $\mathbf{1.593 \times 10^{-8}\ C}$.

**Q.25 A source contains two species of phosphorous nuclei, $^{32}_{15}P\left(T_{\frac{1}{2}}=14.3\,d\right)$ and $^{33}_{15}P\left(T_{\frac{1}{2}}=25.3\,d\right)$. At time $t=0$, 90% of the decay are from $^{32}_{15}P$. How much time has to elapse for only 15% of decays to be from $^{32}_{15}P$? (3)**

**Ans:**

*Given*: $^{32}_{15}P$ ($T_{1/2} = 14.3d$)

$^{33}_{15}P$ ( $T_{1/2} = 25.3$ d)

Initially, the ratio of phosphorus nuclides $^{33}_{15}P$ and $^{32}_{15}P$ is 1 : 9. This means, when the amount of $^{33}_{15}P$ is x, then amount of $^{32}_{15}P$ is 9x.

*To find*: The time at which the ratio is 85 : 15

*Formula*: $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

*Calculation*:

When 15% decays from $^{32}_{15}P$, 85% decays from $^{33}_{15}P$. This means, when the amount of $^{33}_{15}P$ is 17y, then amount of $^{32}_{15}P$ is 3y.

Number of half life , $n = \frac{t}{T_{1/2}}$

From formula,

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$\therefore \quad N = \frac{N_0}{2^{\frac{t}{T_{1/2}}}}$$

$$\therefore \quad 17\ y = \frac{x}{2^{t/25.3}} \qquad \text{....(i)}$$

$$3y = \frac{9x}{2^{t/14.3}} \qquad \text{....(ii)}$$

On dividing equation (i) by (ii)

$$\frac{17}{3} = \frac{x}{2^{t/25.3}} \times \frac{2^{t/14.3}}{9x}$$

$$51 = 2^{\frac{t}{14.3} - \frac{t}{25.3}}$$

$$\therefore \quad 51 = 2^{\frac{11t}{361.79}}.$$

$$\therefore \quad \log_{10}51 = \frac{11t}{361.79} \log_{10}2$$

$$\therefore \quad t = \frac{\log_{10}51}{\log_{10}2} \times \frac{361.79}{11} = \frac{1.7076}{0.301} \times \frac{361.79}{11}$$

$$= \mathbf{186.6\ days}$$

**Ans:** Time taken for only 15% of decays to be from $^{33}_{15}P$ is **186.6 days**.

**Q.26 What is the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of strength 10.0 mCi, its half-life being 5.3 years? (3)**

**Ans:**

*Given*: Disintegration rate,

$$\frac{dN(t)}{dt} = 10\ \text{mCi}$$

$$= 10 \times 10^{-3} \times 3.7 \times 10^{10}\ \text{decay /s}$$

$$= 3.7 \times 10^{8}\ \text{decay /s}$$

$T_{1/2}$ = 5.3 years
= $5.3 \times 3.1536 \times 10^7$ s
= $1.67 \times 10^8$ s

*To find*: Amount of ${}^{60}_{27}Co$ required

*Formulae*: i. $\lambda = \frac{0.693}{T_{1/2}}$ ii. $\left|\frac{dN(t)}{dt}\right| = \lambda N(t)$

*Calculation*:

From formula (i),

$\lambda = \frac{0.693}{1.67 \times 10^8}$

= {antilog [log (0.693) – log(1.67)]} $\times 10^{-8}$
= {antilog [$\bar{1}$.8407 – 0.2227]} $\times 10^{-8}$
= {antilog [$\bar{1}$.6180]} $\times 10^{-8}$
= $0.415 \times 10^{-8}$ s

From formula (ii),

$N(t) = \frac{\left|\frac{dN(t)}{dt}\right|}{\lambda}$

$= \frac{3.7 \times 10^8}{0.415 \times 10^{-8}}$

= antilog {log (3.7) – log(0.415)} $\times 10^{16}$
= antilog {0.5682 – ($\bar{1}$.6180)} $\times 10^{16}$
= antilog {0.9502} $\times 10^{16}$
= $8.917 \times 10^{16}$

For ${}_{27}Co^{60}$,
mass of $6.022 \times 10^{23}$ atoms = 60 g
mass of $8.915 \times 10^{16}$ atoms

$= \frac{60 \times 8.915 \times 10^{16}}{6.022 \times 10^{23}}$

= antilog {log 60 + log 8.915 – (log 6.022)} $\times 10^{-7}$ g
= antilog {1.7782 + 0.9501 – 0.7797} $\times 10^{-7}$ g
= antilog {1.9486} $\times 10^{-7}$ g
= $88.84 \times 10^{-7}$ g
= $\mathbf{8.884 \times 10^{-6}}$ **g**

**Ans:** The amount of ${}^{60}_{27}Co$ necessary to provide a radioactive source of strength 10.0 mCi is $\mathbf{8.884 \times 10^{-6}}$ **g**.

## SECTION-D

**Attempt any three of the following question: [12]**

**Q.27 Explain why the free surface of some liquid in contact with a solid is not horizontal.**

**Ans:**

1. To explain the phenomena, suppose a liquid molecule 'A' is situated in the liquid surface which is in contact with the solid as shown in fig (a).
2. Sphere of influence is drawn. Which shows that sphere of influence is partly in solid, liquid and air.
3. The molecule 'A' is acted upon by the following four forces:

a. Net adhesive force exerted by the molecules of the solid $\overrightarrow{AP}$ a $\vec{F}_A$.

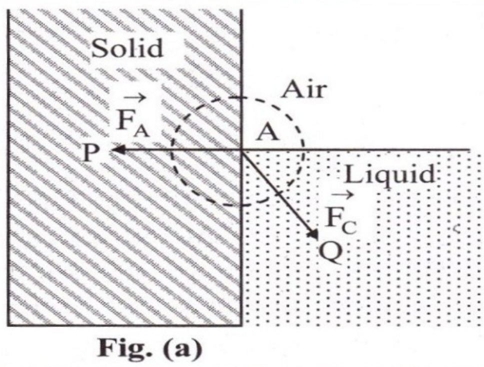


**Fig. (a)**

b. Net cohesive force exerted by the molecules of liquid $\overrightarrow{AQ}$ a $\vec{F}_C$.
c. Net adhesive force exerted by the molecules of air, as the number of air molecule in sphere of influence of molecules 'A' is very small. Hence, resultant adhesive force between air and liquid is neglected.
d. Gravitational force is equal to the weight of the molecule. It is also small and therefore neglected.

4. So, the behaviour of the molecule depends upon two forces $\vec{F}_A$ and $\vec{F}_C$.

**Case 1:**

In case of liquid which partially wets the solid (e.g. kerosene), resultant adhesive force $\vec{F}_A$ between liquid and solid acting on a molecule A is stronger than resultant cohesive force $\vec{F}_C$ between liquid molecules. Therefore, resultant force $\vec{F}_R$ lies inside the solid as shown in fig (b).
In equilibrium state, the tangent AT to the liquid surface is perpendicular to the resultant force $\vec{F}_R$. Therefore, liquid creeps upwards on the solid surface. Hence, the liquid surface in contact with solid is concave upwards and angle of contact is acute.

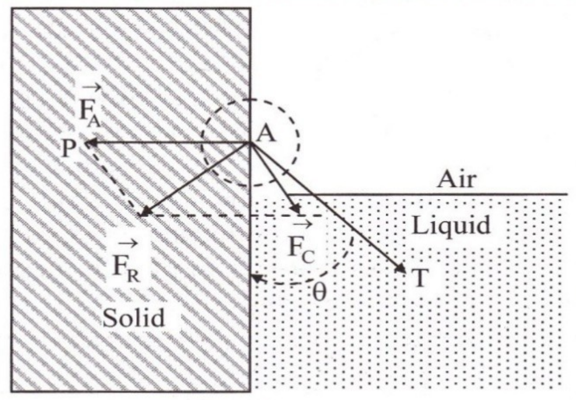


Fig. (b)

**Case 2:**

In case of the liquid which does not wet the solid (e.g., mercury), the resultant adhesive force $\vec{F}_A$ between liquid and solid on a molecule A is smaller than resultant cohesive force $\vec{F}_C$. Therefore, their resultant force $\vec{F}_R$ lies inside the liquid as shown in fig (c).
In equilibrium state, the tangent AT to the liquid surface is perpendicular to the resultant force $\vec{F}_R$. Therefore, liquid creeps downward on the solid surface. Hence, the liquid surface in contact with solid is convex upward and angle of contact is obtuse.

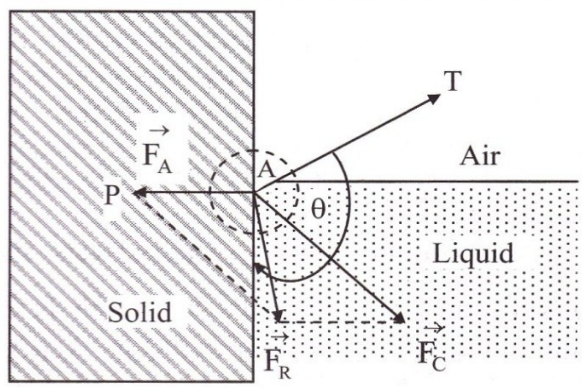


Fig. (c)

**Case 3:**

In case of the liquid which completely wets the solid (e.g. pure water) the resultant adhesive force $\vec{F}_A$ acting on molecule A is very strong and resultant cohesive force $\vec{F}_C$ can be neglected.
Therefore, their resultant force $\vec{F}_R$ lies along $\vec{F}_A$ as shown in fig (d).
In equilibrium state, the free surface of water is always perpendicular to resultant force FR acting in molecule A. the only force acting on molecule A is gravitational force acting vertically downwards and thus angle of contact is zero for pure water and clean glass.

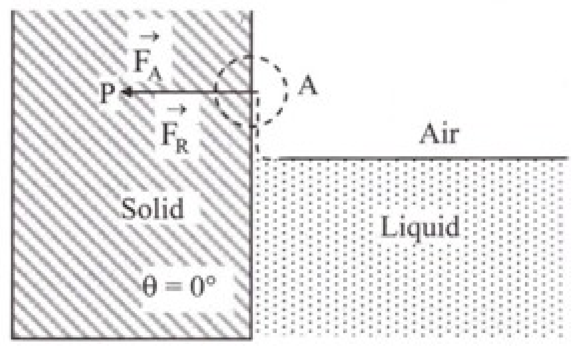
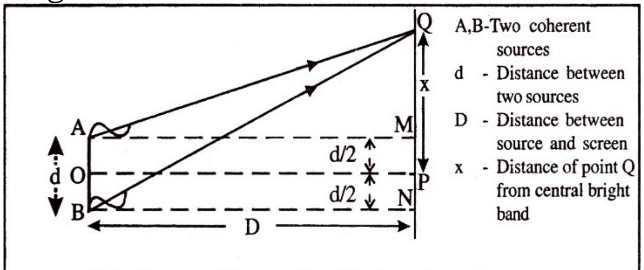


Fig. (d)

**Q.28 Derive condition for occurrence of dark and bright fringes on screen in Young's double slit interference experiment. Define fringe width and derive formula for it. (4)**

**Ans:**

1. Let us consider two coherent, monochromatic sources A and B separated by a distance d. the screen is placed at a distance D from two sources as shown in the diagram.




Interference pattern produced by two coherent sources

2. Draw a perpendicular bisector to AB to meet screen at point P. Draw perpendiculars AM and BN from point A and B respectively on the screen.
3. Let Q as any point at a distance x from point P. Join AQ and BQ.
   The path difference at point Q is $[BQ - AQ]$.
   If $[BQ - AQ] = n\lambda$, point Q will be bright.
   If $[BQ - AQ] = \left(n - \frac{1}{2}\right)\lambda$, then point Q will be dark.
4. From the geometry of the figure,
   $BQ^2 = BN^2 + NQ^2$
   ¿ $D^2 + \left(x + \frac{d}{2}\right)^2$
   $AQ^2 = AM^2 + MQ^2$
   ¿ $D^2 + \left(x - \frac{d}{2}\right)^2$
   $BQ^2 - AQ^2 = \left[D^2 + x^2 + \frac{d^2}{4} + xd - D^2 - x^2 - \frac{d^2}{4} + xd\right]$
   $BQ^2 - AQ^2 = 2xd$
   $(BQ - AQ)(BQ + AQ) = 2xd$
   $(BQ - AQ) = \frac{2xd}{BQ + AQ}$
5. In practice, the distance x and d are very small in comparison with D
   $\therefore BQ + AQ = 2D$
   $\therefore BQ - AQ = \frac{2xd}{2d}$
   $\therefore BQ - AQ = \frac{xd}{d}$
6. If the path difference $\frac{xd}{d} = n\lambda$ ......... (1)
   Then the point Q will be bright.
   If the path difference ¿ $\frac{xd}{d} = \left(n - \frac{1}{2}\right)\lambda$ ..... (2)
   The point Q will be dark.
7. For the point of symmetry P, AP = BP
   $\therefore$ Path difference = 0 that means, point P is always bright.

- **Expression for band width (Fringewidth):**

*The distance between two successive bright bands or dark bands in an interference pattern is called as band width or fringe width.*

- **Expression for band width of bright band:**

1. Let $n^{th}$ bright band is present at a distance $x_n$ and $(n+1)^{th}$ bright band is present at a distance $x_{n+1}$ from the centre of interference pattern i.e., from point P.
2. The band width, $X = x_{n+1} - x_n$
3. For $n^{th}$ bright band, path difference is
   $$\frac{x_n d}{D} = n\lambda$$
   For $(n+1)^{th}$ bright band, path difference is
   $$\frac{x_{n+1} d}{D} = (n+1)\lambda$$
4. On subtracting the above two equations,
   $$\frac{(x_{n+1} x_n) d}{D} = n\lambda + \lambda - n\lambda$$
   $$\therefore \frac{Xd}{D} = \lambda \qquad [X = x_{n+1} - x_n]$$
   $$\therefore X = \frac{\lambda D}{d} \quad \text{.........} \quad (3)$$
   This is the expression for band width of bright band.

- **Expression for band width of dark band:**

1. Let $m^{th}$ dark band is present at a distance $x_m$ and $(m+1)^{th}$ dark band is present at a distance $x_{m+1}$ from the centre of interference pattern i.e., from point P.
2. The band width, $X = x_{m+1} - x_m$
   Using the relation for path difference
3. For $m^{th}$ dark band,
   $$\frac{x_n d}{D} = \left(m - \frac{1}{2}\right)\lambda$$
   For $(m+1)^{th}$ dark band,
   $$\frac{x_{m+1} d}{D} = \left(m + 1 - \frac{1}{2}\right)\lambda$$
4. Subtracting the relations,
   $$(x_{m+1} - x_m)\frac{d}{D} = m\lambda + \lambda - \frac{\lambda}{2} - m\lambda + \frac{\lambda}{2}$$
   $$\frac{Xd}{D} = \lambda$$
   $$\therefore X = \frac{\lambda D}{d} \quad \text{........} \quad (3)$$
   This is the expression for band width of dark band.
5. From the relation (3), it is clear that width of bright and dark band is same i.e., bright and dark bands are equally spaced in interference pattern.

**Q.29 What is a transformer? Explain its working with construction. Derive an expression for e.m.f current in terms of number of turns in primary and secondary coil. (4)**

**Ans:**

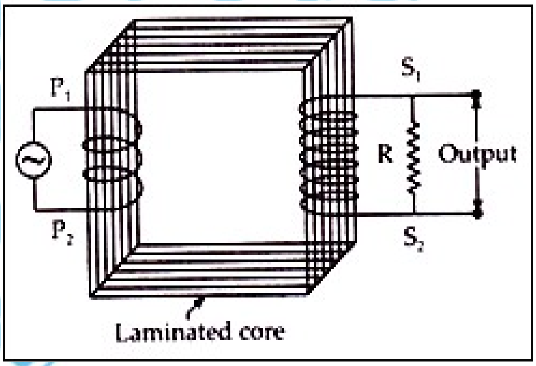
Transformer is a device used to convert low alternating voltage at lower at higher current into high alternating voltage at lower current and vice versa. In other words, a transformer is an electrical device used to increase or decrease alternating voltage.

**Principle:**

*A transformer is based on the principle of mutual induction i.e., whenever the amount of magnetic flux linked with a coil change, an e.m.f is induced in the neighbouring coil.*

**Construction:**
A transformer consists of a rectangular soft iron core made of laminated sheets, well insulated from each other. Two coils $P_1 P_2$ and $S_1 S_2$ are wound on the same core, but are well insulated from each other. Note that both the coils are insulated from the core. The source of alternating emf (to be transformed) is connected to $P_1P_2$, the primary coil and the load resistance R is connected to $S_1S_2$, the secondary coil.
For an ideal transformer, we assume that the resistance of the primary and secondary windings is negligible.
**Theory and working:**
When an alternating source of emf is connected to the primary coil, an alternating current flow through it. Due to the flow of alternating current in the primary coil, an alternating magnetic field is produced and hence changing magnetic flux is linked with the coils. This changing magnetic flux induces an alternating emf in the secondary coil $(e_s)$. Let $n_p$ and $n_s$ be the number of turns in the primary and secondary coils respectively. The iron core is capable of coupling the whole of the magnetic flux $\phi$ produced by the turns of the primary coil with the secondary coil.
According to Faraday's law of electromagnetic induction, the induced emf in the primary,

$e_p = -n_p \frac{d\phi}{dt}$ .......... (1)




The induced emf in secondary coil is,

$e_s = -n_s \frac{d\phi}{dt}$ ........... (2)

Dividing equation (2) by equation (1), we have,

$\frac{e_s}{e_p} = \frac{n_s}{n_p}$

Where,

$\frac{n_s}{n_p} = K$, is the transformation ratio or turns ratio.

Thus,

$\frac{e_s}{e_p} = \frac{n_s}{n_p} = K$ ............ (3)

**For step-up transformer**:
In this transformer $K > 1$, then $n_s > n_p$ and $e_s > e_p$ i.e., output alternating voltage is greater than the input alternating voltage.
**For step-down transformer:**
In this transformer $n_s < n_p$ and $e_s < e_p$ i.e., output alternating voltage is less than the input alternating voltage.

**Q.30 Derive an expression for the impedance of an LCR circuit connected to an AC power supply.** **(4)**
**Ans:**
1. Consider an alternating e.m.f applied to a series combination of pure inductor of inductance L, capacitor of capacitance C and resistor of resistance R as shown in fig. (a).
2. Let $e_L$, $e_C$ and $e_R$ be the r.m.s voltage across inductor, capacitor and resistor. Let 'I' be the r.m.s current flowing through the circuit.
3. As '$e_R$' is in phase with current I, the vector $e_R$ is drawn in the same direction that as of I along positive direction of X axis.
4. $e_L$ leads the current by $\frac{\pi}{2}$ radian, hence vector $e_L$ is drawn in the positive direction of Y axis.

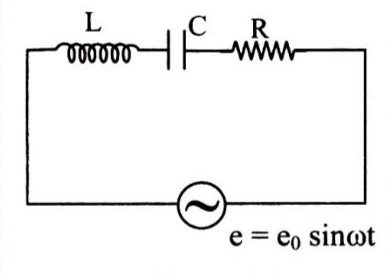


**Figure (a)**

5. $e_C$ lags the current I by $\frac{\pi}{2}$ radian, hence the vector '$e_C$' is drawn in the negative direction of the Y axis. $e_R$ and I are in phase and hence drawn in the same direction.

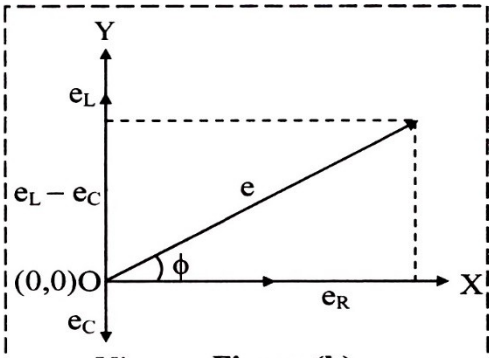



**Figure (b)**

6. The voltages through L, C and R are given by
   $e_L = I\,X_L$, $e_C = I\,X_C$ and $e_R = IR$ ...... (1)
7. Let $e_L > e_C$, the resultant e.m.f $e_L - e_C$, is in the direction of $e_L$ as shown in fig. (b). Resultant of $e_R$, $e_L$ and $e_C$ gives the r.m.s value (e) of applied e.m.f.
8. From the fig. (b), we have,
   $e^2 = e_R^2 + (e_L - e_C)^2$
   From equation (1), we have,
   $\therefore e^2 = (IR)^2 + (IX_L - IX_C)^2$
   $\therefore e^2 = I^2\left[R^2 + (X_L - X_C)^2\right]$
   $\therefore e = I\sqrt{R^2 + ¿¿}$........ (2)
   $\therefore e = IZ$
   When, $Z = \sqrt{R^2 + ¿¿}$
   ¿ impedance of circuit
9. In equation (2) substituting,
   $X_L = \omega L \wedge X_C = \frac{1}{\omega C}$
   We have,
   $e = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ ......... (3)
10. Comparing equation (3) with Ohm's relation $V = IR$, we have,
   $Z = \sqrt{R^2 + \left(\omega L + \frac{1}{\omega C}\right)^2}$ .......... (4)
   Equation (4) represents expression of impedance in LCR series circuit.
   **Phase difference in LCR series circuit:**
   From figure (b), we have,
   $\tan\phi = \frac{e_L - e_C}{e_R}$
   $¿\frac{IX_L - IX_C}{IR}$
   $¿\frac{X_L - X_C}{R}$
   $\therefore \tan\phi = \frac{X_L - X_C}{R}$
   $\therefore \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$ ............ (5)

Equation (5) represents the phase difference between $X_L$, $X_C$ and R.

**Q.31 Given the following data for the incident wavelength and topping potential obtained from an experiment of a photoelectric effect, estimate the value of Plank's constant and the work function of the cathode material. What is the threshold frequency and the corresponding wavelength? What is the most likely metal used for emitter? (4)**

| Incident wavelength (in $\dot{A}$) | 2536 | 3650 |
|---|---|---|
| **Stopping potential (in V)** | **1.95** | **0.5** |

**Ans:**

*Given:* $\lambda_1 = 2536$ Å, $(V_0)_1 = 1.95$ V,
$\lambda_2 = 3650$ Å, $(V_0)_2 = 0.5$ V

*To find:*
i. Planck's constant (h)
ii. Work function ($\phi_0$)
iii. Threshold frequency ($\nu_0$)
iv. Threshold wavelength ($\lambda_0$)

*Formulae:*
i. $\frac{hc}{e}$ = slope of graph of $V_0$ v/s $(1/\lambda)$
ii. $V_0 = \frac{hc}{e\lambda} - \frac{\phi_0}{e}$
iii. $\phi_0 = h\nu_0$
iv. $c = \nu\lambda$

*Calculation:*

i. Slope of graph $V_0$ v/s $(1/\lambda)$ would be,

$$m = \frac{V_{0_2} - V_{0_1}}{\frac{1}{\lambda_2} - \frac{1}{\lambda_1}} = \frac{0.5 - 1.95}{\frac{1}{3650 \times 10^{-10}} - \frac{1}{2536 \times 10^{-10}}}$$

$$= \frac{-1.45}{(2.74 \times 10^6) - (3.94 \times 10^6)}$$

$$= \frac{1.45}{1.2 \times 10^6} = 1.21 \times 10^{-6}$$

$\therefore \quad \frac{hc}{e} = 1.21 \times 10^{-6}$

$\therefore \quad h = \frac{1.21 \times 10^{-6} \times 1.6 \times 10^{-19}}{3 \times 10^8} = \mathbf{6.45 \times 10^{-34} Js}$

ii. From formula (ii),

$(V_0)_1 \times e = \frac{hc}{\lambda_1} - \phi_0$

$\therefore \quad 1.95\ (eV) = \frac{12500}{2536} - \phi_0$

$\therefore \quad \phi_0 = 4.93 - 1.95 = \mathbf{2.98\ eV}$

As value of work function of calcium is $\phi_0 = 2.9$, the metal used for emitter is most likely **Calcium.**

iii. Using formula (iii),

$$\nu_0 = \frac{\phi_0}{h} = \frac{2.98 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = \frac{4.768 \times 10^{-15}}{6.63}$$

$$= \mathbf{7.191 \times 10^{14}\ Hz}$$

iv. Now, from formula (iv),

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{7.191 \times 10^{14}} = 0.4172 \times 10^{-6}\ m = \mathbf{4172\ Å}$$