2023	12	29	TS-M6	SEAT NUMBER				

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XI & XII Science (CBSE/state)
IIT- JEE (Mains + Advance)
NEET, MH-CET , NDA
Mo. No. 9595445177/9021445177

Branches : Chhatrapati Sq., Mangalmurti Sq.

7

(4 Pages)

not allowed.
6. All symbols having their usual meanings unless otherwise stated.
7. For each MCQ, correct answer must be written along with its alphabet.
8. Evaluation of each MCQ would be done for the first attempt only.

SECTION-A

Q.1 Select and write the correct answers to the following questions: **[16]**

1) The negation of $p \wedge (q \rightarrow r)$ is.

a) $p \wedge (q \rightarrow r)$ c) $p \wedge (q \rightarrow r)$

b) $p \vee (q \vee r)$ d) $p \rightarrow (q \wedge r)$ **(2)**

2) In ΔABC *if* $c^2+a^2-b^2=ac$, *then* $\angle B=$¿.....

a) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$

b) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$

(2)

3) Equation of line passing through the point (0, 0, 0) and (2, 1, −¿ 3) is

a) $\frac{x}{2}=\frac{y}{1}=\frac{z}{-3}$ c) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

b) $\frac{x}{2}=\frac{y}{-1}=\frac{z}{-3}$ d) $\frac{x}{3}=\frac{y}{1}=\frac{z}{2}$

(2)

4) The value of $\hat{i}(\hat{j}\times\hat{k})+\hat{j}(k¿\times\hat{i})+\hat{k}(\hat{i}\times\hat{j})¿$ is

a) 0 c) 1

b) -1 d) 3 **(2)**

5) If $f(x)=x^5+2x-3, then (f^{-1})'(-3)=¿$.....

a) 0 c) $\frac{-1}{3}$

b) -3 d) $\frac{1}{2}$

(2)

6) The maximum value of the function $f(x)=\frac{\log x}{x}$ is

a) e c) e^2

b) $\frac{1}{e}$ d) $\frac{1}{e^2}$

(2)

7) If $\int \frac{dx}{4x^2-1}=A\log\left(\frac{2x-1}{2x+1}\right)+c, then\ A=¿¿$.....

a) 1 c) $\frac{1}{3}$

b) $\frac{1}{2}$ d) $\frac{1}{4}$

(2)

8) If the p.m.f of a r.v. X is

$P(x)=\frac{c}{x^3}, for\ x=1,2,3$

$¿0, otherwise,$

$then\ E(X)=¿$.....

a) $\frac{216}{251}$ c) $\frac{297}{294}$

b) $\frac{294}{251}$ d) $\frac{294}{297}$

(2)

Q.2 Answer in short: **[04]**

(1) Find the principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ **(1)**

(2) Write the separate equations of lines represented by the equation

$5x^2-9y^2=0$ **(1)**

(3) If $f'(x)=x^{-1}, then\ find\ f(x)$. **(1)**

(4) Write the degree of the differential equation
$(y''')^2+3(y'')+3xy'+5y=0$. **(1)**

SECTION-B

Attempt any eight of the following questions: **[16]**

Q.3 Using truth table verify that: $(p \wedge q) \vee \quad q \equiv p \vee \quad q$. **(2)**

Q.4 Find the cofactor of the elements of the matrix $\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$ **(2)**

Q.5 Find the principal solutions of $\cot\theta=0$ **(2)**

Q.6 Find the value of k, if $2x+y=0$ is one of the lines represented by $3x^2+kxy+2y^2=0$. **(2)**

Q.7 Find the cartesian equation of the plane passing through A(1, 2, 3) and the direction ratios of whose normal are 3, 2, 5. **(2)**

Q.8 Find the cartesian coordinates of the point whose polar coordinates are $\left(\frac{1}{2},\frac{\pi}{3}\right)$ **(2)**

Q.9 Find the equation of tangent to the curve $y=2x^3-x^2+2at\left(\frac{1}{2},2\right)$ **(2)**

Q.10 Evaluate: $\int_0^{\pi/4} sec^4 x\,dx$. **(2)**

Q.11 Solve the differential equation $y\frac{dy}{dx}+x=0$. **(2)**

Q.12 Show that function $f(x)=\tan x\, is\, increasing \in \left(2,\frac{\pi}{2}\right)$ **(2)**

Q.13 From the differential equation of all lines which makes intercept 3 on X-axis **(2)**

Q.14 If $X \quad B(n,p) \wedge E(X)=6 \wedge Var(X)=4.2, then\, find\, n \wedge p$. **(2)**

SECTION-C

Attempt any eight of the following questions:
[24]

Q.15 If 2 $\tan^{-1}$¿ **(3)**

Q.16 If angle between the lines represented by $ax^2+2hxy+by^2=0$is equal to the angle between the lines represented by $2x^2-5xy+3y^2=0$, then show that 100¿¿ **(3)**

Q.17 Find the distance he parallel lines

$$\frac{x}{2}=\frac{y}{-1}=\frac{\frac{z}{2}\wedge x-1}{2}=\frac{y-1}{-1}=\frac{z-1}{2}.$$

(3)

Q.18 If A(5, 1, p), B(1, q, p) and C(1, −¿ 2, 3) are vertices of a tringle and $G\left(r,-\frac{4}{3},\frac{1}{3}\right)$ is its centroid, then find the values of p, q, r by vector method. **(3)**

Q.19 If A $(\bar{a}) \wedge B(\bar{b})$ be any two points in the space and R($\bar{r}$¿ be a point on the line segment AB

dividing it internally in the ratio $m:n$, then prove that $\bar{r}=\frac{m\bar{b}+n\bar{a}}{m+n}$. **(3)**

Q.20 Find the vector equation of the plane passing through the point $A(-1,2,-5)$ and parallel o the vectors $4\hat{i}-\hat{j}+3\hat{k}\wedge\hat{i}+\hat{j}-\hat{k}$. **(3)**

Q.21 If $y=e^{m\tan^{-1}x}$, *then show that* $(1+x^2)\frac{d^2y}{dx^2}+(2x-m)\frac{dy}{dx}=0$. **(3)**

Q.22 Evaluate: $\int\frac{dx}{2+\cos x-\sin x}$.

(3)

Q.23 Solve $x+y\frac{dy}{dx}=sec(x^2+y^2)$. **(3)**

Q.24 A wire of length 36 metres is bent to form a rectangle. Find its dimensions if the area of the rectangle is maximum. **(3)**

Q.25 Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X. **(3)**

Q.26 If the fair coin is tossed 10 times, find the probability of getting at most six heads. **(3)**

SECTION-D

Attempt any five of the following question: **[20]**

Q.27 Without using truth table prove that
$(p\wedge q)\vee(\ p\wedge q)\vee(p\wedge\ q)\equiv p\vee q$. **(4)**

Q.28 Solve the following system of equations by the method of inversion:
$x-y+z=4, 2x+y-3z=0, x+y+z=2$. **(4)**

Q.29 Using vectors, prove that the altitudes of a triangle are concurrent. **(4)**

Q.30 Solve the LPP by graphical method;
Minimize $z=8x+10y$, *subject to* $2x+y\geqslant 7, 2x+3y\geqslant 15, y\geqslant 2, x\geqslant 0$. **(4)**

Q.31 If $x=f(t)$ and $y=g(t)$ are differentiable functions of t so that y is differentiable function of x and $\frac{dx}{dt}\neq 0$, then prove that:

$$\frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

Hence, find $\frac{dy}{dx}$ *if* $x=\sin t\wedge y=\cos t$. **(4)**

Q.32 If u and v are differentiable functions of x, then prove that:

$$\int uv\,dx=u\int v\,dx-\int\left[\frac{du}{dx}\int v\,dx\right]dx$$

Hence, evaluate $\int\log x\,dx$.

(4)

Q.33 Find the area of region between parabolas $y^2=4ax \wedge x^2=4ay$. **(4)**

Q.34 Show that: $\int_0^{\pi/4} \log$ ¿¿ **(4)**

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