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TS-M5

SEAT NUMBER

IIT INSPIRE
ACADEMY OF SCIENCE
A Place for Excellence & Perfection

XI & XII Science (CBSE/state)
IIT- JEE (Mains + Advance)
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Branches : Chhatrapati Sq., Mangalmurti Sq.

(4 Pages)

5. Use of only logarithmic table is allowed. Use of calculator is not allowed.
6. All symbols having their usual meanings unless otherwise stated.
7. For each MCQ, correct answer must be written along with its alphabet.
8. Evaluation of each MCQ would be done for the first attempt only.

SECTION-A

Q.1 Select and write the correct answers to the following questions: **[16]**

1) $\int \frac{1}{1+\cos x} dx =$ ¿

a) $\tan\frac{x}{2}+c$ **(2)**

2) Which of the following represent direction cosines of line?

c) $0, \frac{\sqrt{3}}{2}, \frac{1}{2}$

(2)

3) $\tan^{-1}\sqrt{3}-\cot^{-1}\sqrt{3}=$ ¿

c) $\frac{\pi}{6}$

(2)

4) The area of the region bounded by the curve $xy=a^2$, X-axis and the lines $x=a$, $x=2a$ is

c) $a^2 \log 2\, sq.units$ **(2)**

5) The vector form of the equation of the line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ is

c) $\bar{r}=$ ¿ **(2)**

6) The differential equation of $y=c^2+\frac{c}{x}$ is

a) $x^4\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx}=y$ **(2)**

7) If $sec\left(\frac{x+y}{x-y}\right)=a^2$, then $\frac{d^2y}{dx^2}=¿$

c) $\frac{y}{x}$

(2)

8) If the lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{\frac{z-3}{3}\wedge x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer $k=¿$

b) **-5**

(2)

Q.2 Answer in short: **[04]**

(1) **Find** $\frac{dy}{dx}$ **if** $y=\cos(logx)$ **(1)**

Ans:

$$y=\cos(\log x)$$
$$\therefore \frac{dy}{dx}=-\sin(\log x).\frac{1}{x}=\frac{-\sin(\log x)}{x}$$

(2) **Write the principle solution of** $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ **(1)**

Ans:

Principle solution of $\sin^{-1}\left(\frac{-1}{2}\right) is \frac{-\pi}{4}$

(3) **Write Sine Rule.** **(1)**

Ans:

Since rule: In ΔABC ,with usual notations

$$\frac{a}{sinA}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$$

Where R = circumradius of ΔABC

(4) **Write the formula for inverse of matrix by adjoint method.** **(1)**

Ans:

If A is a non-singular matrix i.e., $|A|\neq 0$

Then $A^{-1}$by adjoint method is given by-

$$A^{-1}=\frac{1}{|A|}(adj.A)$$

SECTION-B

Attempt any eight of the following questions: **[16]**

Q.3 Find the cartesian equation of the line which passes through the point $(-2,4,-5)$ and parallel to the line given by $\frac{x+2}{3}=\frac{y-3}{5}=\frac{z+5}{6}$

(2)

Ans:

Here $x_1=-2, y_1=4, z_1=-5, a=3, b=5, c=6$

$\therefore$ requiered cartesian equation is $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

$\therefore \frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$

Q.4 Find the approximate value of ¿. **(2)**

Ans:

Let one part be x

$\therefore$ *other part* $=30-x$

$\therefore f(x)=x(30-x)$

$\therefore f(x)=30x-x^2$

$\therefore f'(x)=30-2x$

$\therefore f''(x)=-2$

For $\therefore f(x)=0$, *we get*

$30-2x=0 \rightarrow x=15$

$\therefore f''(15)=-2<0$

$\therefore f(x)$ is maximum at $x=15$

$\therefore$ one part = 15 & other part = 30 – 15 = 15

$\therefore$ 30 should be divided equally into two parts 15, 15 such that their product is maximum.

Q.5 If A, B, C, D are four non-collinear points in a plane such that $\overline{AB}+\overline{BD}+\overline{CD}=\overline{0}$, then prove that the point D is the centroid if ΔABC.

(2)

Ans:

first term on L.H.S. should be $\overline{AD}$

L.H.S. ¿ $\overline{AD}+\overline{BD}+\overline{CD}$

Consider $\overline{AD}+\overline{BD}+\overline{CD}=0$

$\therefore \overline{d}-\overline{a}+\overline{d}-\overline{b}+\overline{d}-\overline{c}=0$

$\therefore 3\overline{d}=\overline{a}+\overline{b}+\overline{c}$

$\therefore \overline{d}\frac{\overline{a}+\overline{b}+\overline{c}}{3}=\overline{g}$

$\therefore D$is the centroid of ΔABC.

Hence, proved.

Q.6 Using the rules of logic, prove that: $(pq)(\ pq)=\ p$ **(2)**

Ans:

L.H.S = $(p\vee q)(\ p\wedge q)$

$\equiv (\ p\wedge\ q)\vee(\ p\wedge q)$ (De – margaric law)

$\equiv\ p\wedge(\ q\vee q)$ (Distri butler law)

$\equiv\ p\wedge T$ (Complement law)

$\equiv \; p$ (Identity law)

$\equiv R.H.S.$ Hence, proved.

Q.7 In ΔABC, if $\angle A=45^0$, $\angle B=60^0$ and $\angle C=75^0$, find the ratio of its sides. (2)

Ans:

By sine rule,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k$$

$\therefore a=ksinA$

$\therefore b=ksinB$

$\therefore c=ksinC$

$\therefore \frac{a}{b}=\frac{\sin A}{\sin B}$ and $\frac{b}{c}=\frac{\sin B}{\sin C}$

$\therefore a:b:c=sinA:sinB:sinC$

$\therefore a:b:c=\sin 45^0:\sin 60^0:\sin 75^0$

Now $\sin 45^0=\frac{1}{\sqrt{2}}$

$\sin 60^0=\frac{\sqrt{3}}{2}$

and $\sin 75^0=\sin\left(30^0+45^0\right)$

¿ $\sin 30^0\cos 45^0+\cos 30^0\sin 45^0$

¿ $\frac{1}{2}\times\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}$

$\sin 75^0=\frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore$ the required ratio of the sides of ΔABC i.e.

$\therefore a:b:c=\frac{1}{\sqrt{2}}:\frac{\sqrt{3}}{2}:\frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore a:b:c=2:\sqrt{6}:\sqrt{3}+1$

Q.8 Find $\frac{dy}{dx}$, if $y=x^{e^x}$ **(2)**

Ans:

$y=x^{e^x}$

Taking log on both sides

$\therefore \log y=\log x^{e^x}$

$\therefore \log y=e^x\log x.$

Diff. w. r. to x.

$\therefore \frac{1}{y}\frac{dy}{dx}=e^x.\frac{1}{x}+\log x.\,e^x$

$\therefore \frac{dy}{dx}=y\,e^x\left(\frac{1}{x}+\log x\right)$

$\therefore \frac{dy}{dx}=x^{e^x}.e^x\left(\frac{1}{x}+\log x\right)$

Q.9 Evaluate: $\int \frac{1}{\sin x \cos x} dx$

(2)

Ans:

$I = \int \frac{1}{\sin x \cos x} dx$

¿ $\int \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right) dx$

¿ $\int \left(\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x} \right) dx$

¿ $\int (\tan x + \cot x) dx$

¿ $\log|sec\, x| + \log|\sin x| + c$

¿ $\log|sec\, x . \sin x| + c$

¿ $\log\left|\frac{\sin x}{\cos x}\right| + c$

$\therefore I = \log|\tan x| + c$

Q.10 If $|\bar{u}| = 3$ and the vector $\bar{u}$ is equally inclined to the unit vectors $\hat{i}, \hat{j}, \hat{k}$, then find $\bar{u}$(2)

Ans:

$|\overline{u}| = 3, \bar{u}$ is equally inclined to the unit vectors $\hat{i},\ \hat{j}, \hat{k}$

$\therefore l = m = n = \pm \frac{1}{\sqrt{3}}$

$\therefore \bar{u} = |\overline{u}|\ (l\,\hat{i}, m\,\hat{j}, n\,\hat{k})$

¿ $3\left(\pm \frac{1}{\sqrt{3}} \hat{i} \pm \frac{1}{\sqrt{3}} \hat{j} \pm \frac{1}{\sqrt{3}} \hat{k} \right)$

¿ $\pm \frac{3}{\sqrt{3}} (\hat{i}, \hat{j}, \hat{k})$

$|\overline{u}| = \pm \sqrt{3} (\hat{i} + \hat{j} + \hat{k})$

Q.11 Find the matrix of co-factors for the matrix $\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$ **(2)**

Ans:

$A = \begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$ $\therefore A_{11} = -1, A_{12} = -4$

$A_{21} = -3, A_{22} = 1$

$\therefore$ matrix of cofactors = $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ -3 & 1 \end{bmatrix}$

Q.12 Given: $X \quad B(n, p)$. **If** $n = 10$ **and** $p = 0.4$, **then find Var (X)** **(2)**

Ans:

$n = 10, p = 0.4 \Rightarrow q = 1 - p = 1 - 0.4 = 0.6$

$\therefore Var(x) = npq = 10 \times 0.4 \times 0.6 = 2.4$

Q.13 Evaluate: $\int_{1}^{2} \frac{\sqrt{x}\,dx}{\sqrt{3-x}+\sqrt{x}}$

(2)

Ans:

$$I=\int_{1}^{2} \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}\,dx \to (1)$$

¿ $\int_{1}^{2} \frac{\sqrt{3-x}}{\sqrt{\sqrt{3-3+x}+\sqrt{2+1-x}}}\,dx$ $\left(\therefore \int_{a}^{b} f(x)\,dx=\int_{a}^{b} f(a+b-x)\,dx\right)$

¿ $\int_{1}^{2} \frac{\sqrt{2+1-x}}{\sqrt{\sqrt{3-(2+1-x)}+\sqrt{3-x}}}\,dx$

$$\therefore I=\int_{1}^{2} \frac{\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}}\,dx \to (2)$$

Adding (1) & (2), we get

$$2I=\int_{1}^{2} \left(\frac{\sqrt{x}+\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}}\right)dx$$

$$\therefore 2I=\int_{1}^{2} 1.dx$$

$$\therefore 2I=[x]_{1}^{2}$$

$$\therefore 2I=2-1$$

$$\therefore I=\frac{1}{2}$$

Q.14 Find the value of: $cosec^{-1}(-\sqrt{2})+\cot^{-1}(\sqrt{3})$ **(2)**

Ans:

$cosec^{-1}(-\sqrt{2})+\cot^{-1}(\sqrt{3})$

¿ $cosec^{-1}\left[cosec\left(\frac{-\pi}{4}\right)+\cot^{-1}\left(\cot\frac{\pi}{6}\right)\right]$

¿ $\frac{-\pi}{4}+\frac{\pi}{6}=\frac{-6\pi+4\pi}{24}=\frac{-2\pi}{24}$

¿ $\frac{-\pi}{12}$

SECTION-C

Attempt any eight of the following questions:
[24]

Q.15 The probability that certain kind of component will survive a check test is 0.6. Find the probability that exactly 2 of the next 4 tested components survive. (3)

Ans:

$p=0.6 \Rightarrow q=1-p=1-0.6=0.4, n=4$

$\therefore p(x=2)=$ ¿ $^{4}C_{2}(0.6)^{2}(0.4)^{4-2}$

¿ $\dfrac{4\cdot!}{(4-2)!,2!}\left(\dfrac{3}{5}\right)^{2}\left(\dfrac{2}{5}\right)^{2}$

¿ $\dfrac{4\times3\times2!}{2!2\times1}\,\dfrac{9\times4}{5^{4}}$

$\therefore p(x=2)=\dfrac{216}{625}$

Q.16 Find the co-ordinates of the point which divides the line segment joining the points A(4, -2, 5) and B(-2, 3, 7) externally in the ratio 8:5. (3)

Ans:

$m=8, n=5, \bar{a}=4\hat{i}-2\hat{j}+5\hat{k}\ ,\ \bar{b}=-2\hat{i}+3\hat{j}+7\hat{k}$

$\therefore \bar{r}=\dfrac{m\bar{b}-n\bar{a}}{m-n}=\dfrac{8(-2\hat{i}+3\hat{j}+7\hat{k})-5(4\hat{i}-2\hat{j}+5\hat{k})}{8-5}$

¿ $\dfrac{(-60-20)\hat{i}+(24+10)\hat{j}+(56-25)\hat{k}}{3}$

¿ $\dfrac{-36}{3}\hat{i}+\dfrac{34}{3}\hat{j}+\dfrac{31}{3}\hat{k}$

$\therefore \bar{r}=-12\hat{i}+\dfrac{34}{3}\hat{j}+\dfrac{31}{3}\hat{k}$

$\therefore R\equiv\left(-12,\dfrac{34}{3},\dfrac{31}{3}\right)$

Q.17 Find the vector and Cartesian equations of the plane that passes through the point (0, 1, 2) and normal to the plane $(\hat{i}+\hat{j}+\hat{k})$. **(3)**

Ans:

Here $\bar{a}=0\hat{i}+\hat{j}+2\hat{k}, \bar{n}=\hat{i}+\hat{j}+\hat{k}$

$\therefore$ required plane is

$\bar{r}.\bar{n}=\bar{a}.\bar{n}$

$\therefore \bar{r}.(\hat{i}+\hat{j}+\hat{k})=(0\hat{i}+\hat{j}+2\hat{k}).(\hat{i}+\hat{j}+\hat{k})$

¿ $0+1+2$

$\therefore \bar{r}.(\hat{i}+\hat{j}+\hat{k})=3$

Put $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore (x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}+\hat{j}+\hat{k})=3$

$\therefore x+y+z=3$

$x+y+z-3=0$

$\therefore$ vector equation of plane is $\bar{r}.(\hat{i}+\hat{j}+\hat{k})=3$

& contain aquation of plane is $x+y+z-3=0$

Q.18 Determine 'k' such that the following function is a p.m.f.

p (x) = k $\begin{pmatrix}4\\x\end{pmatrix}$, x = 0,1,2,3,4 ; k > 0

= 0 , otherwise. **(3)**

Ans:

$p(x)=k\begin{pmatrix}4\\x\end{pmatrix}=k\,{}^4C_x$

For $x=0,$ $p(0)=k\,{}^4C_0=k$

For $x=1,$ $p(1)=k\,{}^4C_1=4k$

For $x=2,$ $p(2)=k\,{}^4C_2=6k$

For $x=3,$ $p(3)=k\,{}^4C_3=4k$

For $x=4,$ $p(4)=k\,{}^4C_4=k$

Since p(x) is a p.m.f

$\therefore \sum p(x)=1$

$\therefore k+4k+6k+4k+k=1$

$\therefore 16k=1$

$\therefore k=\frac{1}{16}$

Q.19 Find the general solution of $\sin x=\tan x$. **(3)**

Ans:

L.H.S.¿ a ¿

¿ $ab\cos C-ac\cos B$

¿ $ab\frac{(a^2+b^2-c^2)}{2ab}-ac\frac{(a^2+c^2-b^2)}{2ac}$

¿ $\frac{1}{2}(a^2+b^2-c^2-a^2-c^2+b^2)$

¿ $\frac{1}{2}(2b^2-2c^2)$

¿ b^2-c^2

¿ $R.H.S$

Hence, proved.

Q.20 If Rolle's theorem holds for the function $f(x)=(x-2)\log x,\ x\epsilon[1,2]$**, show that the equation** $x\log x=2-x$ **is satisfied by at least one value o x in (1, 2).** **(3)**

Ans:

The Rolle's theorem holds for the function $f(x)=(x-2)\log x,\ x\epsilon[1,2]$.

$\therefore$ there exists at least one real number $c\,\epsilon(1,2)$ such that $f'(c)=0$

Now, $f(x)=(x-2)\log x$

$\therefore f'(x)=\frac{d}{dx}[(x-2)\log x]$

¿ $(x-2).\frac{d}{dx}(\log x)+\log x.\frac{d}{dx}(x-2)$

¿ $(x-2)\times\frac{1}{x}+(\log x)(1-0)$

¿ $1-\frac{2}{x}+\log x$

$\therefore f'(c)=1-\frac{2}{c}+\log c$

$\therefore f'(c)=0$ *gives* $1-\frac{2}{c}+\log c=0$

$\therefore c-2+c\log c=0$

$\therefore c\log c=2-c$, where $c\epsilon(1,2)$

$\therefore$ c satisfies the equation $x\log x=2-x$, $c\epsilon(1,2)$

Hence, the equation $x\log x=2-x$ is satisfied by at least one value of x in (1, 2).

Q.21 Find the inverse of $A=\begin{bmatrix}1&2&3\\1&1&5\\2&4&7\end{bmatrix}$ **by adjoint method.** **(3)**

Ans:

A¿ $\begin{bmatrix}1&2&3\\1&1&5\\2&4&7\end{bmatrix}$

$\therefore |A|=\begin{vmatrix}1&2&3\\1&1&5\\2&4&7\end{vmatrix}=-13+6+6=-1\neq 0$

$\therefore A^{-1}\exists$

$A_{11}=-13, A_{12}=-6, A_{13}=6$

$A_{21}=-2, A_{22}=1, A_{23}=0$

$A_{31}=7, A_{32}=-2, A_{33}=1$

$\therefore adjA=\begin{bmatrix}A_{11}&A_{21}&A_{31}\\A_{12}&A_{22}&A_{32}\\A_{13}&A_{23}&A_{33}\end{bmatrix}=\begin{bmatrix}-13&-2&7\\-6&1&-2\\6&0&-1\end{bmatrix}$

$\therefore A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{-1}\begin{bmatrix}-13&-2&7\\-6&1&-2\\6&0&-1\end{bmatrix}$

$A^{-1}=\begin{bmatrix}13&2&-7\\6&-1&2\\-6&0&1\end{bmatrix}$

Q.22 Evaluate: $\int\sqrt{4+3x-2x^2}dx$. **(3)**

Ans:

$I=\int\sqrt{4+3x-2x^2}$ dx

¿ $\int\sqrt{2}\sqrt{\frac{4}{2}+\frac{3}{2}x-x^2}dx$

¿ $\sqrt{2}\int\sqrt{2-\left(x^2-\frac{3}{2}x\right)}$dx

¿ $\sqrt{2}\int\sqrt{2-\left(x^2-\frac{3}{2}x+\frac{9}{16}-\frac{9}{16}\right)}dx$

¿ $\sqrt{2}\int\sqrt{2+\frac{9}{16}-\left(x^2-\frac{3}{2}x+\frac{9}{16}\right)}dx$

¿ $\sqrt{2}\int\sqrt{\frac{41}{16}-\left(x-\frac{3}{4}\right)^2}dx$

$$\therefore I=\sqrt{2}\int\sqrt{\left(\frac{\sqrt{41}}{4}\right)^2-\left(x-\frac{3}{4}\right)^2}\,dx$$

$$¿\sqrt{2}\left[\frac{\left(x-\frac{3}{4}\right)}{2}\sqrt{\left(\frac{\sqrt{41}}{4}\right)^2-\left(x-\frac{3}{4}\right)^2}+\frac{\left(\frac{\sqrt{41}}{4}\right)^2}{2}\sin^{-1}\left(\frac{x-\frac{3}{4}}{\frac{\sqrt{41}}{4}}\right)\right]+c$$

$$\therefore I=\sqrt{2}\left[\frac{(4x-3)}{8}\sqrt{2+\frac{3}{2}x-x^2}+\frac{41}{32}\sin^{-1}\left(\frac{4x-3}{\sqrt{41}}\right)\right]$$

Q.23 Find k, if Sum of the slopes of the lines represented by $x^2+kxy-3y^2=0$ is twice their product. **(3)**

Ans:

$x^2+kxy-3y^2=0$

Comparing with $ax^2+2hxy+by^2=0$

$\therefore a=1, 2h=k, b=-3$

$\therefore m_1+m_2=\frac{-2h}{b}=\frac{-k}{-3}=\frac{k}{3}$

$m_1.m_2=\frac{a}{b}=\frac{1}{-3}$

Given: - $m_1+m_2=2m_1.m_2$

$\therefore \frac{k}{3}=2\times\left(\frac{-1}{3}\right)$

$\therefore k=-2$

Q.24 Using truth tables examine whether the following statement pattern is tautology, contradiction or contingency: $(\ p\wedge q)\wedge(q\rightarrow p)$ **(3)**

Ans:

p	q	p	$p\wedge q$	$q\rightarrow p$	$(\ p\wedge q)\wedge(q\rightarrow p)$
T	T	F	F	T	F
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	F	T	F

Since all entries in the cast column are F's.

$\therefore$ given statement pattern is contradiction.

Q.25 Differentiate $\log(1+x^2)\,w.r.¿\tan^{-1}x¿$ **(3)**

Ans:

Let $u=\log(1+x^2)$

$\therefore \frac{du}{dx}=\frac{1}{1+x^2}.2x=\frac{2x}{1+x^2}$

$V=\tan^{-1}x$

$\therefore \frac{dv}{dx}=\frac{1}{1+x^2}$

$\therefore \frac{du}{dx}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{\frac{2x}{1+x^2}}{\frac{1}{1+x^2}}$

$\therefore \frac{du}{dv}=2x$

Q.26 Find the area of the region bounded by the parabola $y=x^2$ & the line $y=x$ in the first quadrant. **(3)**

Ans:

$y=x^2 \rightarrow (1),\ y=x \rightarrow (2)$

Put (2) in (1)

$\therefore x=x^2 \Rightarrow x^2-x=0$

$\therefore x(x-1)=0$

$\therefore x=0, x=1$

$\therefore$ Required area = $\left|\int_0^1 x^2\,dx-\int_0^1 x\,dx\right|$

¿ $\left|\left[\frac{x^3}{3}\right]_0^1-\left[\frac{x^2}{2}\right]_0^1\right|$

¿ $\left|\frac{1}{3}-\frac{1}{2}\right|$

¿ $\left|\frac{-1}{6}\right|$

¿ $\frac{1}{6}$ sq. units

SECTION-D

Attempt any five of the following question: **[20]**

Q.27 If x and y are differential functions of t, then prove that: $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$, provided $(dx/dt)\neq 0$. If $x=at^2 \wedge y=2at$, find $\frac{dy}{dx}$. **(4)**

Ans:

Let Sx, Sy be small increments in x & y with respect to corresponding small increment St int.

$\therefore$ $as\ St \to 0 ; Sx \to 0 , Sy \to 0.$

$\therefore$ By definition of differentiation,

$\frac{dy}{dx} = \lim ¿ \delta x \to 0 \frac{\delta y}{\delta x}$

$\frac{dy}{dt} = \lim ¿ \delta t \to 0 \frac{\delta y}{\delta x} \qquad \to (1)$

$\frac{dx}{dt} = \lim ¿ \delta t \to 0 \frac{\delta y}{\delta x}$

Since $\delta x , \delta y , \delta t$ are small increments, they are real numbers.

$$\frac{\delta y}{\delta x} = \frac{\frac{\delta y}{\delta t}}{\frac{\delta x}{\delta t}}$$

Taking limit on both sides as $\delta t \to 0$

$$\therefore \lim_{\delta x \to 0} ¿ \delta t \to 0 \frac{\delta y}{\delta x} = \frac{\lim ¿ \delta t \to 0 \frac{\delta y}{\delta t}}{\lim ¿ \delta t \to 0 \frac{\delta x}{\delta t}}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$ (using (1)), provided $\frac{dx}{dt} \neq 0$

Hence, proved.

Now, $x = at^2$, $\qquad y = 2at$

$\therefore \frac{dx}{dt} = 2at \qquad \therefore \frac{dy}{dt} = 2a$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

$$\therefore \frac{dy}{dx} = \frac{1}{t}$$

Q.28 A rectangle has an area of $50\,cm^2$. Find its dimensions for least perimeter. (4)

Ans:

Let $x = lenght , y = breadth\, of\ rectangle$

$\therefore xy = 50 \Rightarrow \frac{50}{x}$

$\therefore$ perimeter of rectangle = $2(x+y)$

$\therefore f(x) = 2\left(x + \frac{50}{x}\right)$

$\therefore f'(x) = 2\left(1 - \frac{50}{x^2}\right) = 2(1 - 50x^{-2})$

$\therefore f''(x) = 2.100x^{-3} = \frac{200}{x^3}$

For $f'(x)=0$, *we get*

$2\left(1-\frac{50}{x^2}\right)=0 \Rightarrow 1-\frac{50}{x^2}=0 \Rightarrow 1=\frac{50}{x^2}$

$\therefore x^2=50 \Rightarrow x=\sqrt{50}=5\sqrt{2}$

$\therefore f''(5\sqrt{2})=\frac{200}{(5\sqrt{2})^3}>0$

$\therefore$ Perimeter of rectangle is least for $x=5\sqrt{2}$

$\therefore y=\frac{50}{5\sqrt{2}}=\frac{10}{\sqrt{2}}=5\sqrt{2}$

$\therefore$ Perimeter of rectangle is least if it becomes a square of side $5\sqrt{2}$ cm

Q.29 If p, q, r are the statements with the truth values T, F, T respectively, determine the truth values of the following:

1) $q \to (p \quad r)$ **2)** $(\quad r\, p)(\quad q)$

3) $(p \to q)\, r$ **4)** $(p q) \to (q r)$ **(4)**

Ans:

$p=T, q=F, r=T$

1) $q \to (p \vee \quad r)$

¿ $F \to (T \vee \quad T)$

¿ $F \to (T \vee F)$

¿ $F \to T$

¿ T

2) $(\quad r \wedge p) \vee (\quad q)$

¿ $(\quad T \wedge T) \vee (\quad F)$

¿ $(F \wedge T) \vee T$

¿ $F \vee T$

¿ T

3) $(p \to q) \vee r$

¿ $(T \to F) \vee T$

¿ $F \vee T$

¿ T

4) $(p \vee q) \to (q \vee r)$

¿ $(T \vee F) \to (F \vee T)$

¿ $T \to T$

¿ T

Q.30 Find the inverse of $A=\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ **by elementary row transformation.** **(4)**

Ans:

$A=\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$

$\therefore |A|=\begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{vmatrix}=0+12-9=3\neq 0$

$\therefore A^{-1}\exists$

Consider $AA^{-1}=I$

$\therefore \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} A^{-1}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 \to R_2+R_1$

$$\begin{bmatrix}1&2&3\\0&3&5\\1&2&4\end{bmatrix}A^{-1}=\begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix}$$

$$R_3\to\frac{1}{3}R_2$$

$$\begin{bmatrix}1&2&3\\0&1&5/3\\0&0&1\end{bmatrix}A^{-1}=\begin{bmatrix}1&0&0\\1/3&1/3&0\\-1&0&1\end{bmatrix}$$

$$R_2\to R_2-\frac{5}{3}R_3$$

$$\begin{bmatrix}1&2&3\\0&1&0\\0&0&1\end{bmatrix}A^{-1}=\begin{bmatrix}1&0&0\\2&1/3&-5/3\\-1&0&1\end{bmatrix}$$

$$R_1\to R_1-3R_3$$

$$\begin{bmatrix}1&2&0\\0&1&0\\0&0&1\end{bmatrix}A^{-1}=\begin{bmatrix}4&0&-3\\2&1/3&-5/3\\-1&0&1\end{bmatrix}$$

$$R_1\to R_1-2R_2$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}A^{-1}=\begin{bmatrix}0&-2/3&1/3\\2&1/3&-5/3\\-1&0&1\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{3}\begin{bmatrix}0&-2&1\\6&1&-5\\-3&0&3\end{bmatrix}$$

Q.31 Prove that $\int u.v\,dx=u\int v-\int\left\{\frac{du}{dx}\int v\,dx\right\}dx.$ **(4)**

Ans:

Let $\int vdx=w\to(1)$

$$\therefore v=\frac{dw}{dx}\to(2)$$

By product rule of differentiation,

$$\frac{d}{dx}(u,w)=u.\frac{dw}{dx}+w.\frac{du}{dx}$$

$\therefore$ By definition of integration

$$u.w=\int\left(u.\frac{dw}{dx}+w.\frac{du}{dx}\right)dx$$

$$\therefore u.\int vdx=\int\left[u.v+\left(\int vdx\right).\frac{du}{dx}\right]dx$$

$$\therefore u.\int vdx=\int u.v\,dx+\int\left\{\frac{du}{dx}.\int vdx\right\}dx$$

$$\therefore \int u.v\,dx=u\int vdx-\int\left\{\frac{du}{dx}.\int vdx\right\}dx$$

Hence, proved.

Q.32 Evaluate: $\int_0^{\pi} e^x \sin 2x\, dx.$ **(4)**

Ans:

$I=\int_0^{\pi} e^x \sin^2 x\, dx \rightarrow (1)$

¿ $\left[\sin 2x . \int e^x dx\right]_0^{\pi} - \int_0^{\pi} \left[\frac{d}{dx}(\sin 2x) . \int e^x dx\right] dx$

¿ $\left[\sin 2x . e^x\right]_0^{\pi} - \int_0^{\pi} 2\cos 2x . e^x dx$

¿ $(0-0)-2\left\{\left[\cos 2x . \int e^x dx\right]_0^{\pi} - \int_0^{\pi} \frac{d}{dx}(\cos 2x) . \int e^x dx\right\}$

¿ $-2\left\{\left(\cos 2x\, e^x\right)_0^{\pi} - \int_0^{\pi} -2\sin 2x . e^x dx\right\}$

¿ $-2\left(e^{\pi}-1\right)-4\int_0^{\pi} -2\sin 2x . e^x dx$

$\therefore I=2\left(1-e^{\pi}\right)-4I$ (Using (1))

$\therefore I+4I=2\left(1-e^{\pi}\right)$

$\therefore 5I=2\left(1-e^{\pi}\right)$

$\therefore I=\frac{2}{5}\left(1-e^{\pi}\right)$

Q.33 In ΔABC, if $\cos A =$ ¿ $\sin B - \cos C$ ¿, **then show that it is a right-angled triangle. (4)**

Ans:

$\cos A=\sin B-\cos C$

$\therefore \cos A+\cos C=\sin B$

$\therefore 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-C}{2}\right)=2\sin\left(\frac{B}{2}\right)\cos\left(\frac{B}{2}\right)$

$\therefore \cos\left(\frac{\pi-B}{2}\right)\cos\left(\frac{A-C}{2}\right)=$ ¿ $\sin\left(\frac{B}{2}\right)\cos\left(\frac{B}{2}\right)$ ¿

$\cos\left(\frac{\pi}{2}-\frac{B}{2}\right)\cos\left(\frac{A-C}{2}\right)=$ ¿ $\sin\left(\frac{B}{2}\right)\cos\left(\frac{B}{2}\right)$ ¿

$\therefore \sin\left(\frac{B}{2}\right)\cos\left(\frac{A-C}{2}\right)=$ ¿ $s\in\left(\frac{B}{2}\right)\cos\left(\frac{B}{2}\right)$ ¿

$\therefore \cos\left(\frac{A-C}{2}\right)=$ ¿ $\cos\left(\frac{B}{2}\right)$ ¿

$\therefore \frac{A-C}{2}=\frac{B}{2} \Rightarrow A-c=B$

$\therefore A=B+C \Rightarrow A+B+C=A+A=2A$

$\therefore A+B+C=\pi \Rightarrow 2A=\pi \Rightarrow A=\pi/2$

$\therefore \Delta ABC$ is right angled at A

Q.34 Minimize ¿ $6x+4y$ **, subject to** $3x+2y\geq 12$, $x+y\geq 5$, $0\leq x\leq 4$, $0\leq y\leq 4$. **(4)**

Ans:

Consider $3x+2y=12 \rightarrow (1)$

Put $x=0 \therefore y=6 \therefore A \equiv (0,6)$

Put $y=0 \therefore x=4 \therefore B \equiv (4,0)$

Consider $x+y=5 \rightarrow (2)$

Put $x=0 \therefore y=5 \therefore C \equiv (0,5)$

Put $y=0 \therefore x=5 \therefore D \equiv (5,0)$

Line $x=4 \rightarrow (3)$ passes through B $(4,0)$

Line $y=54 \rightarrow (4)$ passes through $E(0,4)$

(3) & (4) intersect at F$(4,4)$

(1) $-3 \times (2)$ gives

$3X+2y=12$

-3x $+3y=15$

$-y=-3\ y=3$

Put $y=3 \in (2) \rightarrow (¿2) \quad x=2$

$\therefore (1) \wedge (2)$ *intersect at* $G(2,3)$

Put (3) in (1) & (2) we get

For (1), 12+2y=12

$\therefore 2y=0$

$y=0$

$\therefore (1) \wedge (2)$ *intersect at* $(4,0) \equiv B$

For (2), $4+y=5$

$\therefore y=1$

$\therefore (2) \wedge (3)$ *intersect at* $H(4,1)$

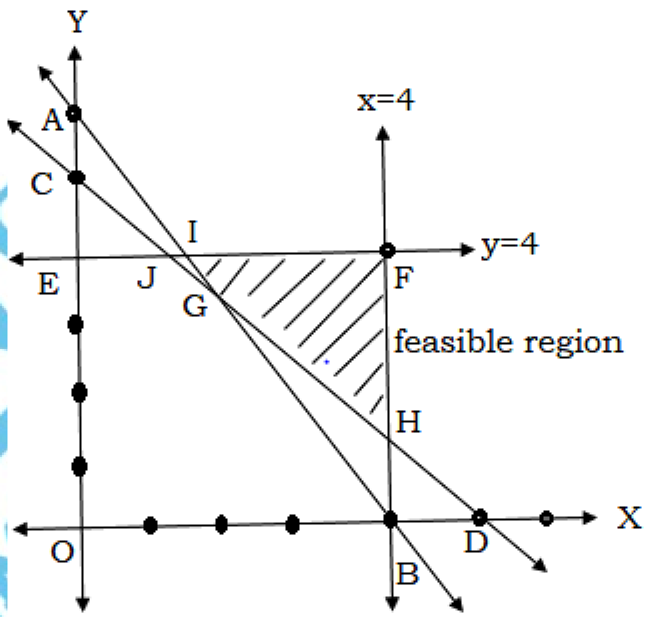
Put (4) in (1) & (2), we get

For (1), $3x+8=12$

$\therefore 3x=4$

$\therefore x=4/3$

$\therefore (1) \wedge (4)$ *intersect at* $I\left(\frac{4}{3},4\right)$



For (2), $x+4=5$

$\therefore x=1$

$\therefore (2) \wedge (4)$ *intersect at* $J(1,4)$

From graph, feasible region is IGHF points on feasible region

	$Z=6x+4y$
$I\left(\frac{4}{3},4\right)$	$Z=24$
$G(2,3)$	$Z=24$
$H(4,1)$	$Z=28$
$F(4,4)$	$Z=40$

$\therefore$minimum value of $Z=24$ which occurs at points I & G

$\therefore$ there are infinitely may solutions between points I & G