IIT INSPIRE
ACADEMY OF SCIENCE
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XI & XII Science (CBSE/state)
IIT- JEE (Mains + Advance)
NEET, MH-CET , NDA
Mo. No. 9595445177/9021445177

Branches : Chhatrapati Sq., Mangalmurti Sq.

(4 Pages)

not allowed.

6. All symbols having their usual meanings unless otherwise stated.

7. For each MCQ, correct answer must be written along with its alphabet.

8. Evaluation of each MCQ would be done for the first attempt only.

SECTION-A

Q.1 Select and write the correct answers to the following questions: [16]

1) Inverse of the statement pattern $(p \lor q) \to (p \land q)$ is

   a) $(p \land q) \to (p \lor q)$ c) $(\sim p \lor \sim q) \to (\sim p \land \sim q)$

   b) $(p \lor q) \to (p \land q)$ d) $(\sim p \land \sim q) \to (\sim p \lor \sim q)$

   (2)

2) If the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product, then the value of 'k' is:

   a) 2   c) -1

   b) 1   d) -2

   (2)

3) The direction ratios of the line which is perpendicular to the lines with direction ratios −1, 2, 2 and 0, 2, 1 are:

   a) −2, −1, −2   c) 2, −1, − 2

   b) 2, 1, 2   d) −2, 1, −2   **(2)**

4) $\int e^x \left(\dfrac{x-1}{x^2} \right) dx$ is

1

a) $\dfrac{e^x}{x}+c$ c) $\left(x-\dfrac{1}{x}\right)e^x+c$

b) $\dfrac{e^x}{x^2}+c$ d) $x\,e^{-x}+c$

(2)

5) The mean and variance of a binomial distribution are 18 and 12 respectively, then $n=¿$

   a) 36   c) 18

   b) 54   d) 27

   (2)

6) In $\triangle ABC$, if $a=2$, $b=3$ and $\sin A=\dfrac{2}{3}$, then $\angle B=¿$

   a) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{3}$

   b) $\dfrac{\pi}{2}$ d) $\dfrac{\pi}{6}$

   (2)

7) The lines $\dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{3}$ and $\dfrac{x-1}{-2}=\dfrac{y-2}{-4}=\dfrac{z-3}{6}$ are

   a) perpendicular   c) skew

   b) intersecting   d) collinear   **(2)**

8) The solution of $\dfrac{1}{x}\cdot\dfrac{dy}{dx}=\tan^{-1}x$ is

   a) $\dfrac{x^2\tan^{-1}x}{2}+c=0$ c) $x-\tan^{-1}x=c$

   b) $x\tan^{-1}x+c=0$ d) $y=\dfrac{x^2\tan^{-1}x}{2}-\dfrac{1}{2}\left(x-\tan^{-1}x\right)+c$ **(2)**

Q.2 Answer in short: **[04]**

(1) Test whether the matrix $\begin{bmatrix} 7 & -6 \\ 13 & -5 \end{bmatrix}$ is invertible or not.   **(1)**

(2) Write the principle solution of $\sin^{-1}\left(\dfrac{-1}{\sqrt{2}}\right)$ **(1)**

(3) If $y=\tan^{-1}\left(\cot 5x\right)$, find $\dfrac{dy}{dx}$

   (1)

(4) State the "Fundamental Theorem of Integral Calculus".   **(1)**

SECTION-B

Attempt any eight of the following questions: **[16]**

Q.3 If $\bar{a} = \hat{i} + \hat{j} + \hat{k}, \bar{b} = 2\hat{i} + q\hat{j} + \hat{k}, \bar{c} = \hat{i} - \hat{j} + 4\hat{k} \wedge \dot\iota \quad \bar{a} \cdot (\bar{b} \times \bar{c}) = 1$, find the value of q? **(2)**

Q.4 If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, determine the truth value of each of the following statements:

   i) $\exists x \in A$ such that $x + 8 = 15$ ii) $\forall x \in A$, $x + 5 < 12$ **(2)**

Q.5 Find the principle solution of $\cos\theta = \dfrac{1}{2}$ **(2)**

Q.6 A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which y co-ordinate is changing 8 times as the x co-ordinate. **(2)**

Q.7 Evaluate: $\int \dfrac{\sec^8 x}{\csc x} dx$

(2)

Q.8 Find the d.e. of all the parabolas whose axis is Y-axis. **(2)**

Q.9 Evaluate: $\displaystyle\int_0^2 x e^x dx$ **(2)**

Q.10 Find 'k' if the slope of one of the lines given by $kx^2 + 4xy - y^2 = 0$ exceeds the slope of the other by 8. **(2)**

Q.11 Find the vector equation of the plane which is at a distance of 6 units from the origin and which is normal to the vector $2\hat{i} - \hat{j} + 2\hat{k}$. **(2)**

Q.12 Find the shortest distance between the lines $\bar{r} = (2\hat{i} - \hat{j}) + (2\hat{i} + \hat{j} - 3\hat{k})$ &

   $\bar{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + \hat{j} - 5\hat{k})$ **(2)**

Q.13 A fair coin is tossed 3 times. Let X be the number of heads obtained. Find E(X) **(2)**

Q.14 Given $X \ B(n, p)$. If $E(X) = 5$, $Var.(X) = 2.5$; find n & p? **(2)**

SECTION-C

Attempt any eight of the following questions:
[24]

Q.15 Find the separate equations of the lines represented by the following equation:
   $x^2 + 2(\csc\alpha)xy + y^2 = 0$. **(3)**

Q.16 Using truth tables examine whether the following statement pattern is tautology, contradiction or contingency: $[p \wedge (p \to q)] \to q$ **(3)**

Q.17 Find the equation of the line passing through the points $(3, 1, 2)$ and perpendicular to the lines $\dfrac{x-1}{1} = \dfrac{y-2}{2} = \dfrac{z-3}{3}$ and $\dfrac{x}{-3} = \dfrac{y}{2} = \dfrac{z}{5}$. **(3)**

Q.18 Find the inverse of the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ by suitable column transformations. **(3)**

Q.19 In $\triangle ABC$, prove that: $a(b \times \cos C - c \times \cos B) = b^2 - c^2$ **(3)**

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Q.20 Evaluate: $\int \dfrac{1}{\sin(x-a).\cos(x-b)}dx$

(3)

Q.21 Given below is the probability distribution of X:

x	0	1	2	3	4
P(X = x)	k	2k	4k	2k	k

Find: i) the value of k,　　ii) $P(X \geq 2)$,　　iii) $P(X < 3)$ **(3)**

Q.22 Evaluate: $\displaystyle\int_{-\pi/4}^{\pi/4} \dfrac{dx}{1+sinx}$ **(3)**

Q.23 Find the area enclosed between the circle $x^2 + y^2 = 1$ & the line $x + y = 1$ in the first quadrant.
(3)

Q.24 Let PQRS be a quadrilateral. If M and N are the mid points of the sides PQ and RS respectively then prove that $\overline{PS} + \overline{QR} = 2\,\overline{MN}$ **(3)**

Q.25 Find $\dfrac{dy}{dx}$, if $x = 3\cos t - 2\cos^3 t$, $y = 3\sin t - 2\sin^3 t$ **(3)**

Q.26 Find the points on the curve $y = x^3 - 2x^2 - x$, where the tangets are parallel to $3x - y + 1 = 0$. **(3)**

SECTION-D

Attempt any five of the following question: **[20]**

Q.27 Find the inverse of the matrix $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ using elementary row transformations. **(4)**

Q.28 Find the combined equation of pair of straight lines passing through the origin each of which make an angle of 30^0 with the line $3x + 2y - 11 = 0$. **(4)**

Q.29 Maximize $z = 6x + 10y$, subject to $3x + 5y \leq 10, 5x + 3y \leq 15, x \geq 0, y \geq 0$ **(4)**

Q.30 Show that: $2\sin^{-1}\left(\dfrac{3}{5}\right) = \tan^{-1}\left(\dfrac{24}{7}\right)$ **(4)**

Q.31 Sand is pouring at the rate of $12\,cm^3/sec$. The falling sand forms a cone on the ground in such a way that the height of the cone is always $\left(\dfrac{1}{6}\right)^{th}$ of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm? **(4)**

Q.32 A body cools down according to Newton's law from 100^0C to 60^0C in 20 minutes. The temperature of the surrounding being 20^0C, how long will it take to cool down to 30^0C?
(4)

Q.33 Prove that: $\int \sqrt{x^2 - a^2}\,dx = \dfrac{x}{2}\sqrt{x^2 - a^2} - \dfrac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$ **(4)**

Q.34 Evaluate: $\displaystyle\int_{0}^{\pi/2} \dfrac{1}{5 + 4\,cosx}dx$ **(4)**

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"All the Best"

"If you are not willing to learn,

no one can help you.

If you are determined to learn,

no one can stop you."