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TS-M1

SEAT NUMBER

IIT INSPIRE
ACADEMY OF SCIENCE
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XI & XII Science (CBSE/state)
IIT- JEE (Mains + Advance)
NEET, MH-CET , NDA
Mo. No. 9595445177/9021445177

Branches : Chhatrapati Sq., Mangalmurti Sq.

3

(4 Pages)

5. Use of only logarithmic table is allowed. Use of calculator is not allowed.
6. All symbols having their usual meanings unless otherwise stated.
7. For each MCQ, correct answer must be written along with its alphabet.
8. Evaluation of each MCQ would be done for the first attempt only.

SECTION-A

Q.1 Select and write the correct answers to the following questions: **[16]**

1) The Cartesian co-ordinates of the point whose polar co-ordinates are $\left(3, 150^0\right)$ are

a) $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ **(2)**

2) Negation of $p \rightarrow (p \vee \sim q)$ is

b) $p(\sim pq)$ **(2)**

3) The combined equation of lines through the points (3, 4) and parallel to the coordinate axes is

b) $xy - 4x - 3y + 12 = 0$ **(2)**

4) The area of the region bounded by the curve $xy = a^2$, X-axis and the lines $x = a$, $x = 2a$ is

c) $a^2 \log 2\, sq.units$ **(2)**

5) $\int_{-1}^{1} \frac{1-x^2}{1+x^2} dx = ¿$

a) $\pi - 2$ **(2)**

6) $y = \tan^{-1} x + c$ is a solution of

d) $\left(1+x^2\right)\frac{d^2 y}{dx^2} + 2x\frac{dy}{dx} = 0$ **(2)**

7) y ¿ $e^{\log_e 5x} + e^{\log_e 7x}$, then $\frac{dy}{dx} = ¿$

a) 12 **(2)**

8) If $A(1,2,3)$, $B(3,4,5)$ & $C(p,q,7)$ are collinear, then the values of p & q are

c) $p=5, q=6$ **(2)**

Q.2 Answer in short: **[04]**

(1) **Write the equation of plane in parametric form passing through one point & parallel to two non-zero vectors.** **(1)**

Ans:

Equation of plane in parametric form passing through one point and parallel to two non-zero vectors is

$\bar{r}=\bar{a}\lambda\bar{b}+M\bar{c}$, where $\lambda \wedge M$ are real parameters.

(2) **Write the principal value branches of range & domain of** $\sin^{-1}x \wedge \cos^{-1}x$ **(1)**

Ans:

Function	Domain	Range
$\sin^{-1}x$	$[-1,1]$	$\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1,1]$	$[0,\pi]$

(3) **State projection rule.** **(1)**

Ans:

In ΔABC, with usual notations,

$a=b\cos C+c\cos B$

$b=a\cos C-c\cos A$

$c=a\cos B+b\cos A$

(4) **Evaluate:** $\int_{-\pi/2}^{\pi/2}\sin^3 x\,dx$ **(1)**

Ans:

I¿ $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\sin^3 x\,dx$

$f(x)=\sin^3 x \quad \Rightarrow \quad f(-x)=\sin^3(-x)$

¿ $-\sin^3 x$

$\therefore f(-x)=-f(x)$

$\therefore f(x)$ is an odd function

$\therefore \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\sin^3 x\,dx=0 \quad \left(\because \int_{-a}^{a} f(x)\,dx=0, \text{if } f(x) \text{ is an odd function}\right)$

SECTION-B

Attempt any eight of the following questions: **[16]**

Q.3 What If $\bar{a}=7\hat{i}-\hat{j}+2\hat{k}, \bar{b}=\hat{i}+3\hat{j}-\hat{k}, \bar{c}=4\hat{i}+5\hat{k}$ **then find** $\bar{a}.(\bar{b}\times\bar{c})$ **.** **(2)**

Ans:

$\bar{a}=7\hat{i}-\hat{j}+2\hat{k}, \hat{b}=\hat{i}+3\hat{j}-\hat{k}, \hat{c}=4\hat{i}+5\hat{k}$

$\therefore \hat{a}.(\hat{b}\times\hat{c})=\begin{vmatrix}7 & -1 & 2\\ 1 & 3 & -1\\ 4 & 0 & 5\end{vmatrix}$

¿ $7(0)-(-1)(5-(-4))+2(0-12)$

¿ $105+9-24$

¿ $105-15$
¿ 90

Q.4 Derive Using truth table, prove the equivalence: $p\,q \equiv \sim(p \rightarrow \sim q)$ **(2)**

Ans:

(1)	(2)	(3)	(4)	(5)	(6)
p	q	$\sim q$	$(p \wedge q)$	$(p \rightarrow \sim q)$	$\sim(p \rightarrow \sim q)$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	F	T	F

Since all the entries in column (4) and column (6) are absolutely identical

$\therefore p \wedge q \equiv \sim(p \rightarrow \sim q)$ Hence, proved.

Q.5 Find the area bounded by the curve $y^2 = 16x$ **, the X-axis and lines** $x=0$**,** $x=4$ **(2)**

Ans:

$y^2 = 16x \qquad \therefore y = 4\sqrt{x} \qquad \therefore y = 4x^{1/2}$

$\therefore Area = \int_0^4 4x^{1/2} dx$

¿ $4\left[\frac{x^{3/2}}{3/2}\right]_0^4 = \frac{8}{3}(4)^{3/2}$

¿ $\frac{8}{3}.(2^2)^{3/2} = \frac{8}{3} \times 2^3 = \frac{64}{3} sq\,units.$

Q.6 Distinguish If $|\bar{u}| = 3$ **and the vector** $\bar{u}$ **is equally inclined to the unit vectors** $\hat{i}, \hat{j}, \hat{k}$ **, then find** $\bar{u}$. **(2)**

Ans:

$|\bar{u}| = 3$

$\bar{u}$ is equally inclined to $\hat{i}, \hat{j}, \hat{k}$

$\therefore \alpha = \beta = n$

$\Rightarrow cos\alpha = cos\beta = cosn$

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2 n = 1$

$\Rightarrow 3\cos^2\alpha = 1$

$\therefore 3\cos^2\alpha = \frac{1}{3}$

$\therefore \cos^2\alpha = \frac{1}{3}$

$\therefore cos\alpha = \pm\frac{1}{\sqrt{3}}$

$\therefore l = cos\alpha = m = cos\beta = n = cosn = \pm\frac{1}{\sqrt{3}}$

$\therefore \bar{u} = |\bar{u}|(l\hat{i} + m\hat{j} + n\hat{k})$

¿ $3\left(\pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}\right)$

¿ $\pm\frac{3}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

$\therefore \bar{u} = \pm\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$

Q.7 Explain If $y=\cot^{-1}(cosec\, x+\cot x¿)¿$ **, find** $\frac{dy}{dx}$

(2)

Ans:

$y=\cot^{-1}(cosecx+cotx)$

$¿\cot^{-1}\left(\frac{1}{sinx}+\frac{cosx}{sinx}\right)$

$¿\cot^{-1}\left(\frac{1+cosx}{sinx}\right)$

$¿\cot^{-1}\left(\frac{2\cos^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\right)$

$\therefore y=\cot^{-1}\left\{\cot\left(\frac{x}{2}\right)\right\}\Rightarrow y=\frac{x}{2}$

$\therefore \frac{dy}{dx}=\frac{1}{2}$

Q.8 Explain Find the cartesian equation of the line which passes through the point $(-2,4,-5)$ **and parallel to the line given by** $\frac{x+2}{3}=\frac{y-3}{5}=\frac{z+5}{6}$

(2)

Ans:

The given line passes through $(-2,4,-5)$

$\therefore x_1=-2, y_1=4, z_1=-5$

It is parallel to the line $\frac{x+2}{3}=\frac{y-3}{5}=\frac{z+5}{6}$

$\therefore a=3, b=5, c=6$

$\therefore$ Required line is

$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

i.e. $\frac{x-(-2)}{3}=\frac{y-4}{5}=\frac{z-(-5)}{6}$

$\therefore \frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$

Q.9 Explain A random variable X has the following probability distribution:

X = x	–2	–1	0	1	2	3
P (x)	0.1	k	0.2	2k	0.3	k

Find the value of k. **(2)**

Ans:

Since given distribution is a p.m.f.

$\therefore \sum pi=1$

$\therefore 0.1+k+0.2+2k+0.3+k=1$

$\therefore 4k+0.6=1$

$\therefore 4k=0.4$

$\therefore k=\frac{0.4}{4}$

$\therefore k=0.1 \vee k=\frac{1}{10}$

Q.10 Twenty Given that $X \quad B(n=10, p)$ **and** $E(x)=8$**, find the value of p**. **(2)**

Ans:

$n=10, E(x)=8\ p=?$

We know that $E(x)=np$.

$\therefore 8=10p$

$\therefore p=\frac{8}{10}$

$\therefore p=\frac{4}{5} \quad ¿\ p=0.8$

Q.11 Find the principle solution of $sinx=\frac{\sqrt{3}}{2}$ **(2)**

Ans:

$\sin x=\frac{\sqrt{3}}{2}$

$\therefore \sin x=\sin\left(\frac{\pi}{3}\right)$

Also, $\sin(\pi-\theta)=sin\theta$

$\therefore \sin\left(\pi-\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{3}\right)$

$\therefore \sin\left(\frac{2\pi}{3}\right)=\sin\left(\frac{\pi}{3}\right)$

$\therefore \sin x=\sin\left(\frac{\pi}{3}\right)=¿\sin\left(\frac{2\pi}{3}\right)¿$

$0\leq\frac{\pi}{3}\leq 2\pi . \wedge 0\leq\frac{2\pi}{3}\leq 2\pi .$

$\therefore$Required principle solutions are

$x=\frac{\pi}{3} \wedge x=\frac{2\pi}{3}$

Q.12 Convert $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ **into an identity matrix by suitable row transformations.** **(2)**

Ans:

$A=\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$R_2 \rightarrow R_2-2R_1$

$\therefore A \sim \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix}$

$R_2 \rightarrow \frac{1}{5}R_2$

$\therefore A \sim \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$R_1 \rightarrow R_1+R_2$

$\therefore A \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=I_2$

Q.13 Determine Evaluate: $\int \frac{1}{\sin x \cos x} \quad dx$

(2)

Ans:

$I = \int \frac{1}{sinx\, cosx} dx$

¿ $\int \left(\frac{\sin^2 x + \cos^2 x}{sinx\, cosx} \right) dx$ $\qquad \left(\therefore \sin^2 x + \cos^2 x = 1 \right)$

¿ $\int \left(\frac{\sin^2 x}{sinx\, cosx} + \frac{\cos^2 x}{sinx\, cosx} \right) dx$

¿ $\int \left(\frac{sinx}{cosx} + \frac{cosx}{sinx} \right) dx$

¿ $\int \left(\tan x + \cot x \right) dx$

$\therefore I = \log|sec\, x| + \log|\sin x| + c$

$\therefore I = \log|sec\, x . \sin x| + c$ $\qquad \left(\therefore \log m + \log n = \log(mn) \right)$

¿ $\log \left| \frac{\sin x}{\cos x} \right| + c$ $\qquad \left(\therefore sec\, x = \frac{1}{\cos x} \right)$

$\therefore I = \log|\tan x| + c$

Q.14 Let the p.m.f of a random variable X be

$p(x) = \binom{4}{x} \left(\frac{5}{9} \right)^x \left(\frac{4}{9} \right)^{4-x}$ **,** $x = 0, 1, 2, 3, 4$

Find E(X) and var (X). **(2)**

Ans:

$P(x) = \binom{4}{x} \left(\frac{5}{9} \right)^x \left(\frac{4}{9} \right)^{4-x} = {}^4C_x \left(\frac{5}{9} \right)^x \left(\frac{4}{9} \right)^{4-x}$

Comparing with $p(x) = {}^nC_x\, p^x q^{n-x}$

$\therefore n = 4, p = \frac{5}{9}, q = \frac{4}{9}$

$\therefore E(x) = np = 4 \times \frac{5}{9} = \frac{20}{9}$

$Var(x) = npq = 4 \times \frac{5}{9} \times \frac{4}{9} = \frac{80}{81}$

SECTION-C

Attempt any eight of the following questions:

[24]

Q.15 Show that the points (0, 7, 10),(-1, 6, 6) and (–4, 9, 6) form a right-angled triangle.(3)

Ans:

Let P $\equiv (0, 7, 10), Q \equiv (-1, 6, 6), R \equiv (-4, 9, 6)$

$\therefore \overline{PQ} = -\hat{i} - \hat{j} - 4\hat{k}$

$\overline{QR} = -3\hat{i} + 3\hat{j} + 0\hat{k}$

& $\overline{PR} = 0\hat{i} + 2\hat{j} - 4\hat{k}$

Clearly, $\overline{PQ} \cdot \overline{QR} = (-1) \times (-3) + (-1) \times 3 + (-4) \times C$

¿ $3 - 3 + 0$

$\therefore \overline{PQ} \cdot \overline{QR} = 0$

$\therefore \overline{PQ} \perp \overline{QR}$ $\qquad \left(\therefore \bar{a} \perp \bar{b}\, if\, \bar{a} \cdot \bar{b} = 0 \right)$

$\therefore \Delta PQR$ is right angled at Q.
Hence, proved.

Q.16 Find the co-ordinates of the point which divides the line segment joining the points A (4, -2, 5) and B (-2, 3, 7) externally in the ratio 8:5. (3)

Ans:

$A \equiv (4,-2,5) \therefore \bar{a} = 4\hat{i} - 2\hat{j} + 5\hat{k}$

$B \equiv (-2,3,7) \therefore \bar{b} = -\widehat{2i} + 3\hat{j} + 7\hat{k}$

Let R be the point which divides seg AB externally in the ratio 8:5

$\therefore$ by section formula for external division,

$$\bar{r} = \frac{m\bar{b} - n\bar{a}}{m-n}$$

We get,

$$\bar{r} = \frac{8\bar{b} - 5\bar{a}}{8-5} \qquad (\therefore m=8, n=5)$$

¿ $\dfrac{8(-\widehat{2i} + 3\hat{j} + 7\hat{k}) - 5(4\hat{i} - 2\hat{j} + 5\hat{k})}{3}$

¿ $\dfrac{(-16-20)\hat{i} + (24+10)\hat{j} + (56-25)\hat{k}}{3}$

$$\therefore \bar{r} = \frac{-36}{3}\hat{i} + \frac{34}{3}\hat{j} + \frac{31}{3}\hat{k}$$

¿ $12\hat{i} + \dfrac{34}{3}\hat{j} + \dfrac{31}{3}\hat{k}$

$$\therefore R = \left(-12, \frac{34}{3}, \frac{31}{3}\right)$$

Q.17 Find the angle between the pair of lines

$\bar{r} = (\widehat{3i} + \widehat{2j} - \widehat{4k}) + \lambda(\hat{i} + \widehat{2j} + \widehat{2k})$ **&** $\bar{r} = (\widehat{5i} - \widehat{2k}) + \mu(\widehat{3i} + \widehat{2j} + \widehat{6k})$ **(3)**

Ans:

$\bar{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) \qquad \rightarrow (1)$

$\therefore a_1 = 1, b_1 = 2, \ c_1 = 2$

$\bar{r} = (5\hat{i} - 2\hat{k}) + r(3\hat{i} + 2\hat{j} + 6\hat{k}) \qquad \rightarrow (2)$

$\therefore a_2 = 3, b_2 = 2, \ c_2 = 6$

$$\therefore \cos\theta = \left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

¿ $\left| \dfrac{1 \times 3 + 2 \times 2 + 2 \times 6}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{3^2 + 2^2 + 6^2}} \right|$

¿ $\left| \dfrac{3+4+12}{\sqrt{9}\sqrt{49}} \right|$

¿ $\left| \dfrac{19}{3 \times 7} \right|$

$$\therefore \cos\theta = \frac{19}{21}$$

$$\therefore \theta = \cos^{-1}\left(\frac{19}{21}\right)$$

Q.18 Determine 'k' such that the following function is a p.m.f.

$p(x) = k\binom{4}{x}$ **,** $x = 0,1,2,3,4$ **;** **k > 0**

¿0 , **otherwise.** **(3)**

Ans:

$p(x)=k\binom{4}{x}=k\,{}^4C_x$

For $x=0$, $p(0)=k\,{}^4C_0=k$

For $x=1$, $p(1)=k\,{}^4C_1=4k$

For $x=2$, $p(2)=k\,{}^4C_2=6k$

For $x=3$, $p(3)=k\,{}^4C_3=4k$

For $x=4$, $p(4)=k\,{}^4C_4=k$

Since p(x) is a p.m.f

$\therefore \sum p(x)=1$

$\therefore k+4k+6k+4k+k=1$

$\therefore 16k=1$

$\therefore k=\frac{1}{16}$

Q.19 Find the equation of the plane in vector form passing through the points (1, 0, 1) (1, –1, 1), (4, –3, 2). **(3)**

Ans:

$A=(1,0,1), B=(1,-1,1), C=(4,-3,2)$

$\therefore \bar{a}=\hat{i}+\hat{k}, \bar{b}=\hat{i}-\hat{j}+\hat{k}, \bar{c}=4\hat{i}+(-3)\hat{j}+2\hat{k}$

$\therefore \bar{b}-\bar{a}=-\hat{j},\ \bar{c}-\bar{a}=3\hat{i}-3\hat{j}+\hat{k}$

$\therefore (\bar{b}-\bar{a})\times(\bar{c}-\bar{a})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & -1 & 0\\ 3 & -3 & 1\end{vmatrix}$

¿ $-\hat{i}+0\hat{j}+3\hat{k}$

$\therefore (\bar{b}-\bar{a})\times(\bar{c}-\bar{a})=-\hat{i}+3\hat{k}$

$\therefore$ equation of required plane is

$(\bar{r}-\bar{a})\cdot[(\bar{b}-\bar{a})\times(\bar{c}-\bar{a})]=0$

i.e. $[\bar{r}-(\hat{i}+\hat{k})]\cdot(-\hat{i}+3\hat{k})=0$

$\therefore\ \bar{r}\cdot(-\hat{i}+3\hat{k})-(-1+3)=0$

$\therefore\ \bar{r}\cdot(-\hat{i}+3\hat{k})-2=0$

$\therefore\ \bar{r}\cdot(-\hat{i}+3\hat{k})=2$

OR

Equation of required plane is

$\bar{r}$ ¿ $\bar{a}+\lambda\bar{b}+\mu\bar{c}$

$\therefore\ \bar{r}=(\hat{i}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})+\mu(4\hat{i}-3\hat{j}+2\hat{k})$

Q.20 What Prove that: $\int \frac{1}{a^2-x^2}dx=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

(3)

Ans:

$I=\int \frac{1}{a^2-x^2}dx=\frac{1}{2a}\int \frac{2a}{a^2-x^2}dx$

¿ $\frac{1}{2a}\int \frac{2a}{(a+x)(a-x)}dx$

¿ $\frac{1}{2a}\int \left[\frac{(a+x)+(a-x)}{(a+x)(a-x)}\right]dx$

¿ $\frac{1}{2a}\left\{\int \frac{(a+x)}{(a+x)(a-x)}dx+\int \frac{(a-x)}{(a+x)(a-x)}dx\right\}$

¿ $\frac{1}{2a}\left\{\int \frac{1}{a-x}dx+\int \frac{1}{a+x}dx\right\}$

$\therefore I=\frac{1}{2a}\left[\frac{\log|a-x|}{(-1)}+\log|a+x|\right]+c$

¿ $\frac{1}{2a}\left[\log|a+x|-\log|a-x|\right]+c$

$\therefore I=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

$\therefore \int \frac{1}{a^2-x^2}dx=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

Hence, proved.

Q.21 An Find the vector and Cartesian equations of the plane that passes through the point (0, 1, 2) and normal to the plane $(\hat{i}+\hat{j}+\hat{k})$. **(3)**

Ans:

Here $A=(0,1,2)$ $\quad \therefore 0\hat{i}+\hat{j}+2\hat{k}$ & $\bar{n}=\hat{i}+\hat{j}+\hat{k}$

$\therefore$ *vector equation of required plane is*

$\bar{r}\cdot\bar{n}=\bar{a}\cdot\bar{n}$

$\therefore \bar{r}\cdot(\hat{i}+\hat{j}+\hat{k})=(0\hat{i}+\hat{j}+2\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})$

$\therefore \bar{r}\cdot(\hat{i}+\hat{j}+\hat{k})=0+1+2$

$\therefore \bar{r}\cdot(\hat{i}+\hat{j}+\hat{k})=3$

Put $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})=3$

$\therefore z+y+z=3 \vee x+y+z-3=0$

$\therefore$ *Vextore equation is* $\bar{r}\cdot(\hat{i}+\hat{j}+\hat{k})=3$

& Cartesian equation is $x+y+z-3=0$

Q.22 Solve by method of inversion: $5x-y+4z=5$**,** $2x+3y+5z=2$**,** $5x-2y+6z=-1$ **(3)**

Ans:

$5x-y+4z=5$

$2x+3y+5z=2$

$5x-2y+6z=-1$

In matrix form, above system is

$AX=B \rightarrow (1)$

i.e. $\begin{bmatrix}5 & -1 & 4\\ 2 & 3 & 5\\ 5 & -2 & 6\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}5\\ 2\\ -1\end{bmatrix}$

$\therefore A=\begin{bmatrix}5 & -1 & 4\\ 2 & 3 & 5\\ 5 & -2 & 6\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}5\\ 2\\ -1\end{bmatrix}$

$\therefore |A|=\begin{vmatrix}5 & -1 & 4\\ 2 & 3 & 5\\ 5 & -2 & 6\end{vmatrix}$

¿ $5(18+10)-(-1)(12-25)+4(-4-15)$

¿ $5\times 28+1\times(-13)+4\times(-19)$

¿ $140-13-76$

$\therefore |A| = 51 \neq 0 \Rightarrow A^{-1} \exists$

Now, $A_{11}=28, A_{12}=13, A_{13}=-19$

$A_{21}=-2, A_{22}=10, A_{23}=5$

$A_{31}=-17, A_{32}=-17, A_{33}=17$

$$\therefore adj\,A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(adj\,A) = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$

Premutiplying (1) by A^{-1}, we get $X = A^{-1}B$

$$\therefore X = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{51}\begin{bmatrix} 140-4+17 \\ 65+20+17 \\ -95+10-17 \end{bmatrix}$$

¿ $\frac{1}{51}\begin{bmatrix} 153 \\ 102 \\ -102 \end{bmatrix}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$$

$\therefore x=3, y=2, z=-2$

Q.23 Prove cosine rule using projection rule. **(3)**

Ans:

By projection rule, we have

$a = b\cos C + c\cos B$

$b = a\cos C + c\cos A$

$c = a\cos B + b\cos A$

$\therefore a^2 = ab\cos C + ac\cos B$

$b^2 = ab\cos C + bc\cos A$

$c^2 = ac\cos B + bc\cos A$

$\therefore b^2 + c^2 - a^2$

¿ $ab\cos C + bc\cos A + ac\cos B + bc\cos A - ab\cos C - ac\cos B$

$\therefore b^2 + c^2 - a^2 = 2bc\cos A$.

$\therefore a^2 = b^2 + c^2 - 2bc\cos A$

Similarly, we can prove that

$b^2 = a^2 + c^2 - 2ac\cos B$

& $c^2 = a^2 + b^2 - 2ab\cos C$

Hence, proved.

Q.24 If $x = a\left(t - \frac{1}{t}\right), y = a\left(t + \frac{1}{t}\right)$ **, show that:** $\frac{dy}{dx} = \frac{x}{y}$ **(3)**

Ans:

$x = a\left(t - \frac{1}{t}\right)$

$\therefore \frac{dx}{dt} = a\left(1+\frac{1}{t^2}\right)$

$Y = a\left(t-\frac{1}{t}\right)$

$\therefore \frac{dy}{dt} = a\left(1-\frac{1}{t^2}\right)$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a\left(1-\frac{1}{t^2}\right)}{a\left(1+\frac{1}{t^2}\right)}$

¿ $\frac{a\frac{1}{t}\left(t-\frac{1}{t}\right)}{a\frac{1}{t}\left(t+\frac{1}{t}\right)}$

$\therefore \frac{dy}{dx} = \frac{a\left(t-\frac{1}{t}\right)}{a\left(t+\frac{1}{t}\right)} = \frac{x}{y}$

Hence, proved.

Q.25 Find the vector equation of the line passing through the point $(-1,-1,2)$ and parallel to the line $2x-2=3y+1=6z-2$. **(3)**

Ans:

$2x-2=3y+1=6z-2$

$\therefore 2(x-1)=3(y+1)=6(z-1/3)$

$\therefore \frac{x-1}{1/2} = \frac{y+1}{1/3} = \frac{z-1/3}{1/6}$

$\therefore \frac{x-1}{6\times 1/2} = \frac{y+1}{6\times 1/3} = \frac{z-1/3}{6\times 1/6}$

$\therefore \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1/3}{1}$

$\therefore d.r.s. \text{ of this line are } a=3, b=2, c=1$

$\therefore \bar{b} = 3\hat{i}+2\hat{j}+\hat{k}$

Given $A \equiv (-1,-1,2)$

$\therefore \bar{a} = -\hat{i}-\hat{j}+2\hat{k}$

$\therefore$ equation of required line is

$\bar{r} = \bar{a}+\lambda\bar{b}$

$\therefore \bar{r} = \left(-\hat{i}+(-\hat{j})+2\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}+\hat{k}\right)$

Q.26 Find the vector equation of the plane which is at a distance of 6 units from the origin, and which is normal to the vector $2\hat{i}-\hat{j}+2\hat{k}$. **(3)**

Ans:

Given: $\bar{n} = 2\hat{i}-\hat{j}+2\hat{k}, p=6$

$\therefore \hat{n} = \frac{2\hat{i}-\hat{j}+2\hat{k}}{\sqrt{2^2+(-1)^2+2^2}}$

¿ $\frac{2\hat{i}-\hat{j}+2\hat{k}}{\sqrt{4+1+4}} = \frac{2\hat{i}-\hat{j}+2\hat{k}}{\sqrt{9}}$

$\therefore \hat{n}=\frac{2\hat{i}-\hat{j}+2\hat{k}}{3}$

$\therefore$ *required plane is*

$\bar{r}.\hat{n}=p$

$\therefore \bar{r}.\frac{(2\hat{i}-\hat{j}+2\hat{k})}{3}=6$

$\therefore \bar{r}.(2\hat{i}-\hat{j}+2\hat{k})=18$

SECTION-D

Attempt any five of the following question: **[20]**

Q.27 Prove Evaluate: $\int \frac{1}{\cos \propto + ¿\cos x} dx ¿$

(4)

Ans:

$I=\int \frac{1}{\cos\alpha+\cos x}dx$

Put $\tan\left(\frac{x}{2}\right)=t$

$\therefore dx=\frac{2dt}{1+t^2}, \cos x=\frac{1-t^2}{1+t^2}$

$\therefore I=\int \frac{\frac{2dt}{1+t^2}}{\cos\alpha+\frac{(1-t^2)}{(1+t^2)}}$

$¿2\int \frac{1}{(1+t^2)\cos\alpha+(1-t^2)}dt$

$¿2\int \frac{1}{\cos\alpha+t^2\cos\alpha+1-t^2}dt$

$¿2\int \frac{1}{(1+\cos\alpha)+t^2(\cos\alpha-1)}dt$

$¿2\int \frac{1}{(1+\cos\alpha)-t^2(1-\cos\alpha)}dt$

$¿2\int \frac{1}{2\cos^2\left(\frac{\alpha}{2}\right)-t^2.2\sin^2\left(\frac{\alpha}{2}\right)}dt$

$¿\frac{2}{2}\int \frac{1}{\cos^2\left(\frac{\alpha}{2}\right)-t^2\sin^2\left(\frac{\alpha}{2}\right)}dt$

$¿\frac{1}{\sin^2\left(\frac{\alpha}{2}\right)}\int \frac{1}{\frac{\cos^2\left(\frac{\alpha}{2}\right)}{\sin^2\left(\frac{\alpha}{2}\right)}-t^2}dt$

$\therefore I=\frac{1}{\sin^2\left(\frac{\alpha}{2}\right)}\int \frac{1}{\cot^2\left(\frac{\alpha}{2}\right)-t^2}dt$

¿ $\frac{1}{\sin^2\left(\frac{\alpha}{2}\right)} \cdot \frac{1}{2\cot\left(\frac{\alpha}{2}\right)} \log\left|\frac{\cot\left(\frac{\alpha}{2}\right)+t}{\cot\left(\frac{\alpha}{2}\right)-t}\right|+c$

¿ $\frac{1}{2\sin^2\left(\frac{\alpha}{2}\right)\frac{\cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}} \log\left|\frac{\cot\left(\frac{\alpha}{2}\right)+\tan\left(\frac{x}{2}\right)}{\cot\left(\frac{\alpha}{2}\right)-\tan\left(\frac{x}{2}\right)}\right|+c$

¿ $\frac{1}{2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)} \log\left|\frac{\cot\left(\frac{\alpha}{2}\right)+\tan\left(\frac{x}{2}\right)}{\cot\left(\frac{\alpha}{2}\right)-\tan\left(\frac{x}{2}\right)}\right|+c$

¿ $\frac{1}{\sin\alpha} \log\left|\frac{\cot\left(\frac{\alpha}{2}\right)+\tan\left(\frac{x}{2}\right)}{\cot\left(\frac{\alpha}{2}\right)-\tan\left(\frac{x}{2}\right)}\right|+c$

$\therefore I = cosec\,\alpha \log\left|\frac{\cot\left(\frac{\alpha}{2}\right)+\tan\left(\frac{x}{2}\right)}{\cot\left(\frac{\alpha}{2}\right)-\tan\left(\frac{x}{2}\right)}\right|+c$

Q.28 Show that $\sin^{-1}\frac{12}{13}+$¿$\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=0$¿

(4)

Ans:

Let $\sin^{-1}\left(\frac{12}{13}\right)=\alpha \Rightarrow \sin\alpha=\frac{12}{13}$

$\therefore \cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-\left(\frac{12}{13}\right)^2}$

¿ $\sqrt{1-\frac{144}{169}}=\sqrt{\frac{25}{169}}\frac{5}{13}$

Let, $\cos^{-1}\left(\frac{4}{5}\right)=\beta \Rightarrow \cos\beta=\frac{4}{5}$

$\therefore \sin\beta=\sqrt{1-\cos^2\beta}=\sqrt{1-\left(\frac{4}{5}\right)^2}$

¿ $\sqrt{1-\frac{16}{25}}=\sqrt{\frac{9}{25}}=\frac{3}{5}$

$\therefore \tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{12/13}{5/13}=\frac{12}{5}$

$\tan\beta=\frac{\sin\beta}{\cos\beta}=\frac{3/5}{4/5}=\frac{3}{4}$

$$\therefore \tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\times\frac{3}{4}}$$

$$¿\frac{\frac{48+15}{20}}{1-\frac{36}{20}}=\frac{\frac{63}{20}}{\frac{20-36}{20}}$$

$$\therefore \tan(\alpha+\beta)=\frac{63}{-16}$$

$$\therefore \alpha+\beta=\tan^{-1}\left(\frac{63}{-16}\right)$$

$$\therefore \sin^{-1}\left(\frac{12}{13}\right)+\cos^{-1}\left(\frac{4}{5}\right)=\tan^{-1}\left(\frac{-63}{16}\right)$$

$$\therefore \sin^{-1}\left(\frac{12}{13}\right)+\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{63}{16}\right)$$

$$¿\tan^{-1}\left(\frac{-63}{16}\right)+\tan^{-1}\left(\frac{63}{16}\right)=-\tan^{-1}\left(\frac{63}{16}\right)+\tan^{-1}\left(\frac{63}{16}\right)$$

$$¿0$$

Hence, proved.

Q.29 Solve: $(x^2-y^2)dx+2xy\,dy=0$ **(4)**

Ans:

$$(x^2-y^2)dx+2xy\,dy=0$$

$$\therefore 2xy\,dy=-(x^2-y^2)dx$$

$$\therefore \frac{dy}{dx}=\frac{y^2-x^2}{2xy}\rightarrow(1)$$

Put $y=Vx\rightarrow(2)$

$$\therefore \frac{dy}{dx}=V.1+x.\frac{dV}{dx}$$

$$\therefore \frac{dy}{dx}=V+x.\frac{dV}{dx}\rightarrow(3)$$

Using (2) and (3); (1) becomes

$$v+x\frac{dV}{dx}=\frac{(Vx)^2-x^2}{2x.Vx}$$

$$\therefore v+x\frac{dV}{dx}=\frac{V^2x^2-x^2}{2x^2V}$$

$$\therefore v+x\frac{dV}{dx}=\frac{x^2(V^2-1)}{2x^2V}$$

$$x\frac{dV}{dx}=\frac{V^2-1}{2V}-V$$

$$\therefore x\frac{dV}{dx}=\frac{V^2-1-2V^2}{2V}$$

$$\therefore x\frac{dV}{dx}=\frac{-1-V^2}{2V}$$

$$\therefore x\frac{dV}{dx}=\frac{-(1+V^2)}{2V}$$

$$\therefore \left(\frac{2V}{1+V^2}\right)dv = \frac{-dx}{x}$$

Integrating both sides

$$\therefore \int \left(\frac{2v}{1+v^2}\right)dv = -\int \frac{dx}{x}$$

$$\therefore \log\left(1+v^2\right) = -\log x + \log c$$

$$\therefore \log\left(1+v^2\right) + \log x = \log x$$

$$\therefore \log x\left(1+v^2\right) = \log c$$

$$\therefore x\left(1+v^2\right) = c$$

$$\therefore x\left(1+\left(\frac{y}{x}\right)^2\right) = c \qquad \left(\therefore y = vx, v = \frac{y}{x}\right)$$

$$\therefore x + x.\frac{y^2}{x^2} = c$$

$$\therefore x + \frac{y^2}{x} = c$$

Multiplying both sides by x

$$\therefore x^2 + y^2 = cx$$

Q.30 Using vector method, prove that: $\sin\left(\alpha+\beta\right) = ¿ \sin\alpha\cos\beta + \cos\alpha\sin\beta ¿$ **(4)**

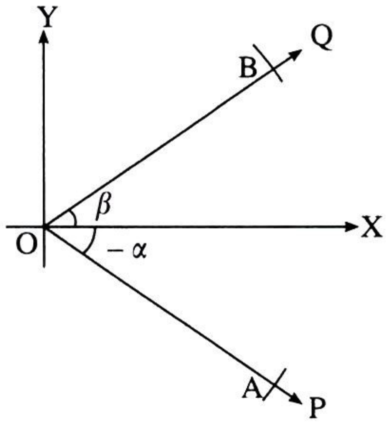
Ans:

Let $\hat{e}_1 \wedge \hat{e}_2$ be two-unit vectors in the XY- plane

Let $-\alpha \wedge \beta$ be the angles made by $\hat{e}_1 \wedge \hat{e}_2$ respectively with X-axis.

Let $\hat{e}_1 = \overline{OP}, \hat{e}_2 = \overline{OQ}$

As shown in the figure.



∴By using polar co-ordinates,

P $\equiv \left(\cos(-\alpha), \sin(-\alpha), 0\right)$

$\therefore P \equiv \left(\cos\alpha, -\sin\alpha, 0\right)$

& Q $\equiv \left(\cos\beta, \sin\beta, 0\right)$

$\hat{e}_1 = \cos\alpha\,\hat{i} - \sin\alpha\,\hat{j} + 0\,\hat{k}$

& $\hat{e}_2 = \cos\beta\,\hat{i} - \sin\beta\,\hat{j} + 0\,\hat{k}$

$$\hat{e}_1 \times \hat{e}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ cos\alpha & -sin\alpha & 0 \\ cos\beta & sin\beta & 0 \end{vmatrix}$$

$¿ 0\hat{i} + 0\hat{j} + ¿$

$\therefore \hat{e}_1 \times \hat{e}_2 = ¿ \qquad \rightarrow (1)$

Since total angle (without signs) between

$\hat{e}_1 \times \hat{e}_2 is (\alpha+\beta)$

∴ By definition of cross product,

$\hat{e}_1 \times \hat{e}_2 = |\hat{e}_1||\hat{e}_2| \sin(\alpha+\beta)\hat{k}$

$\hat{e}_1 \times \hat{e}_2 = ¿ \sin(\alpha+\beta)\hat{k} \qquad \rightarrow (2)$

$\therefore ¿ (1) \wedge (2),$

$\sin(\alpha+\beta)\hat{k} = ¿ ¿$

$\therefore \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

Hence, proved.

Q.31 A rectangular sheet of paper has its area 24 sq. meters. The margins at the top and bottom are 75 cm each and at the sides 50 cm each. What are the dimensions of the paper, if the area of the printed space is maximum? (4)

Ans:

Let $x = lenght, y = ¿$height of the rectangular sheet of paper.

∴length of printed space

$¿ x-50cm-50cm$

$¿ x-\frac{1}{2}m-\frac{1}{2}m$

$¿(x-1)m$

& height of printed space

$¿ y-75cm-75cm$

$¿ y-\frac{3}{4}m-\frac{3}{4}m$

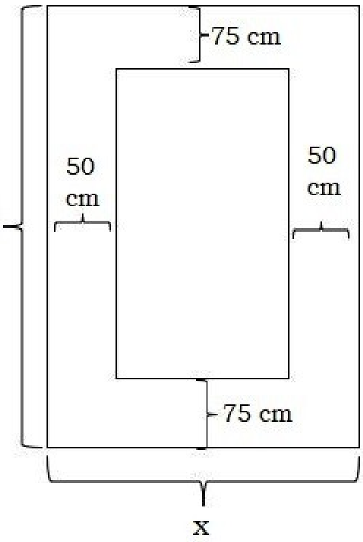
$¿\left(y-\frac{3}{2}\right)m$

∴Area of printed space = $(x-1)\left(y-\frac{3}{2}\right)$

Given: - Area of sheet = 24

$\therefore xy=24 \Rightarrow y=\frac{24}{x}$

∴Area of printed space = $(x-1)\left(\frac{24}{x}-\frac{3}{2}\right)$



$\therefore f(x)=24-\frac{3}{2}x-\frac{24}{x}+\frac{3}{2}$

$\therefore f'(x)=\frac{-3}{2}+\frac{24}{x^2}=\frac{-3}{2}+24x^{-2}$

∴ f (x) =-2×24 {x} ^ {-3} = {-48} over {{x} ^ {3}

For $f'(x)=0$, we get

$\frac{-3}{2}+\frac{24}{x^2}=0$

$\therefore \frac{24}{x^2}=\frac{3}{2}$

$\therefore \frac{24\times 2}{3}=x^2 \Rightarrow x^2-16$

$\therefore x=4$

∴ f (4) = {-48} over {43} <

$\therefore f(x) is maximum at x=4$

∴Dimension of paper are

$x=4m \wedge y=\frac{24}{4}=6m$

Q.32 Evaluate: $\int_0^1 \frac{x(\sin¿¿-1x)^2}{\sqrt{1-x^2}}¿\ dx$

(4)

Ans:

$I=\int_0^1 \frac{x(\sin^{-1}x)^2}{\sqrt{1-x^2}}dx$

Put $\sin^{-1}x=t \Rightarrow x=sint$

$\therefore \frac{1}{\sqrt{1-x^2}}dx=dt$

$$\therefore I=\int_0^{\pi/2} sint\,.\,t^2\,dt$$

$$¿\left[t^2\int \sin t\,dt\right]_0^{\pi/2}-\int_0^{\pi/2}\left\{\frac{d}{dt}(t^2).\int sint\,dt\right\}dt$$

$$¿\left[-t^2\int \cos t\right]_0^{\pi/2}-\int_0^{\pi/2}-2t\cos t\,dt$$

$$\therefore I=¿\,(0-0)+2\int_0^{\pi/2} t\cos t\,dt$$

$$¿2\left\{\left[t\int \cos t\,dt\right]_0^{\pi/2}-\int_0^{\pi/2}\left[\frac{d}{dt}(t).\int \cos t\,dt\right]dt\right\}$$

$$¿2\left\{\left[t\int \sin t\right]_0^{\pi/2}-\int_0^{\pi/2}1.\sin t\,dt\right\}$$

$$¿2\left\{\left(\frac{\pi}{2}-0\right)-\left[(-\cos t)\right]_0^{\pi/2}\right\}$$

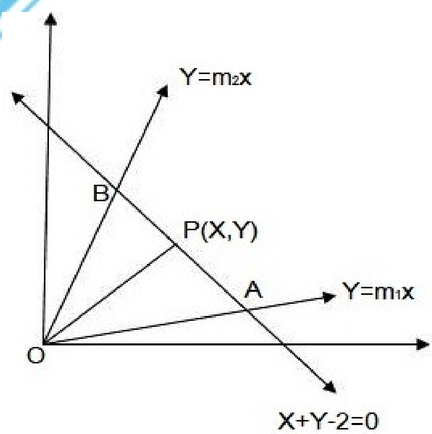
$$¿2\left\{\frac{\pi}{2}+\left(\cos\frac{\pi}{2}-\cos 0\right)\right\}$$

$$¿2\left(\frac{\pi}{2}-1\right)$$

$$\therefore I=\pi-2$$

Q.33 ΔOAB **is formed by the lines** $x^2-4xy+y^2=0$ **and the line AB. Find the equation of the median of the triangle drawn from the origin.** **(4)**

Ans:



$x^2-4xy+y^2=0\rightarrow(1)$

$x+y-2=0$

$\therefore y=2-x\rightarrow(2)$

Put (2) in (1), we get,

$x^2-4x(2-x)+(2-x)^2=0$

$\therefore x^2-8x+4x^2+4-4x+x^2=0$

$\therefore 6x^2-12x+4=0$

$\therefore 3x^2-6x+2=0$

$\therefore x_1+x_2=\frac{-(-6)}{3}$ (Since $P(x,y)=(1,1)$ is the midpoint of seg AB,

$\therefore x_1+x_2=2$ $\therefore OP$ is the median of ΔOAB)

$\therefore \frac{x_1+x_2}{2}=1=x$

Put in (2)

$\therefore y=2-1=1$

$\therefore$equation of median OP is

$\frac{y-0}{1-0}=\frac{x-0}{1-0}$

$\therefore y=x\vee x-y=0$

Q.34 Minimize $z=x+2y$**, subject to** $x+2y\geq 50$, $2x-y\leq 0$, $2x+y\leq 100$, **x,** $y\geq 0$ **(4)**

Ans:

Consider $x+2y=50 \to (1)$

Put $x=0 \therefore y=25 \therefore A\equiv(0,25)$

Put $y=0 \therefore x=50 \therefore B\equiv(50,0)$

Consider $2x-y=0 \to (2)$

Put $y=0 \therefore x=0 \therefore (2)\,passes\,through\,(0.0)$

Put $y=0 \therefore x=10 \therefore C\equiv(10.20)$

Consider $2x+y=100 \to (3)$

Put $x=0 \therefore y=100 \therefore D\equiv(0,100)$

Put $y=0 \therefore x=50 \therefore B\equiv(50,0)$

$\therefore (1)\wedge(3)$ intersect at B

Now, (2) $\Rightarrow y=2x\; Put \in (1), we\; get$

$5x=50 \Rightarrow x=10$

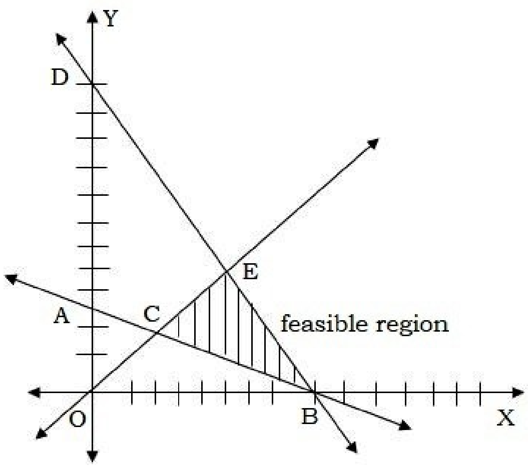
Put in (2) $\Rightarrow y=20$

$\therefore (1)\wedge(2)\, intersect\; at\; C\equiv(10,20)$

$(2)+(3) \Rightarrow 4x=100 \Rightarrow x=25$

Put in (2) $\Rightarrow y=50$

$\therefore (2)\wedge(3)\, intersect\; at\; E\equiv(25,50)$



From the graph, feasible region is BCE.

Point on feasible region	$z=x+2y$
$B(50,0)$	$z=50$
$C(10,20)$	$z=50$
$E(25,50)$	$z=25$

The minimum value of $z=50$ which occurs at points B & C

$\therefore$ The given L.P.P. has infinite number of solutions between the points B & C

"All the Best"

"Education is one thing

no one can take away

from you."