2023	12	20

TS-M2

SEAT NUMBER

(4 Pages)

5. Use of only logarithmic table is allowed. Use of calculator is not allowed.
6. All symbols having their usual meanings unless otherwise stated.
7. For each MCQ, correct answer must be written along with its alphabet.
8. Evaluation of each MCQ would be done for the first attempt only.

SECTION-A

Q.1 Select and write the correct answers to the following questions: **[16]**

1) Inverse of the statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is **(2)**
   Ans: d) $(\ p \wedge\ q) \rightarrow (\ p \vee\ q)$
2) If the sum of the slopes of the lines represented by $x^2+kxy-3y^2=0$ is twice their product, then the value of 'k' is: **(2)**
   Ans: d) -2
3) The direction ratios of the line which is perpendicular to the lines with direction ratios –1, 2, 2 and 0, 2, 1 are: **(2)**
   Ans: d) –2, 1, –2
4) $\int e^x\left(\frac{x-1}{x^2}\right)dx$ is **(2)**
   Ans: a) $\frac{e^x}{x}+c$
5) The mean and variance of a binomial distribution are 18 and 12 respectively, then $n=¿$
   Ans: b) 54 **(2)**
6) In ΔABC, if $a=2$, $b=3$ and $\sin A=\frac{2}{3}$, then $\angle B=¿$

Ans: **b)** $\frac{\pi}{2}$

(2)

7) The lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{6}$ are

(2)

Ans: b) intersecting

8) The solution of $\frac{1}{x}\cdot\frac{dy}{dx}=\tan^{-1}x$ is

(2)

Ans: d) $y=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\left(x-\tan^{-1}x\right)+c$

Q.2 Answer in short: **[04]**

(1) Test whether the matrix $\begin{bmatrix} 7 & -6 \\ 13 & -5 \end{bmatrix}$ **is invertible or not.** **(1)**

Ans:

$A=\begin{bmatrix} 7 & -6 \\ 13 & -5 \end{bmatrix}$

$\therefore |A|=-35-(-78)=-35+78=43\neq 0$

$\therefore$ A is inversible.

(2) Write the principle solution of $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ **(1)**

Ans:

Principle solution of $\sin^{-1}\left(\frac{-1}{2}\right) is \frac{-\pi}{4}$

(3) If $y=\tan^{-1}(\cot 5x)$**, find** $\frac{dy}{dx}$ **(1)**

Ans:

$y=\tan^{-1}(\cot 5x)$

$y=\tan^{-1}\left[\tan\left(\frac{\pi}{2}-5x\right)\right]$

$\therefore y=\frac{\pi}{2}-5x$

$\therefore \frac{dy}{dx}=-5$

(4) State the "Fundamental Theorem of Integral Calculus". **(1)**

Ans:

Fundamental theorem of integral calculus:

If $\int f(x)dx=g(x)+c, then$

$\int_a^b f(x)dx=[g(x)]_a^b=g(b)-g(a)$

SECTION-B

Attempt any eight of the following questions: **[16]**

Q.3 If $\bar{a}=\hat{i}+\hat{j}+\hat{k}, \bar{b}=2\hat{i}+q\hat{j}+\hat{k}, \bar{c}=\hat{i}-\hat{j}+4\hat{k} \wedge ¿\ \bar{a}.(\bar{b}\times\bar{c})=1$**, find the value of q** **(2)**

Ans:

$\bar{a}.(\bar{b}\times\bar{c})=1$, $\bar{a}=\hat{i}+\hat{j}+\hat{k}, \bar{b}=\widehat{2i}+9\hat{j}+\hat{k}$, $\bar{c}=\hat{i}-\hat{j}+4\hat{k}$

$\therefore 1(49+1)-1(8-1)+1(-2-9)=1$

$\therefore 49+1-7-2-9=1$

$\therefore 39-8=1 \Rightarrow 39=9$

$\therefore 9=3$

Q.4 If $A=\{1,2,3,4,5,6,7,8,9\}$**, determine the truth value of each of the following statements:**

i) $\exists x \in A$ **such that** $x+8=15$ **ii)** $\forall x \in A$ **,** $x+5<12$ **(2)**

Ans:

$A=\{1,2,3,4,5,6,7,8,9\}$

i) $x \in A$ such that $x+8=15$

since $x+8=15$ is true only for $x=7$

$\therefore$This is a true statement.

ii) $\forall x \in A, x+5<12$

Since every element in A does not satisfy the relation $x+5<12$, $x \in A$

$\therefore$ This is a false statement.

Q.5 Find the principle solution of $cos\theta=\frac{1}{2}$ **(2)**

Ans:

$\cos\theta=\frac{1}{2} \Rightarrow \cos\theta=\cos\frac{\pi}{3}$

Also, $\cos(2\pi-x)=\cos x$

$\therefore \cos\left(2\pi-\frac{\pi}{3}\right)=\cos\frac{\pi}{3}$

$\therefore \cos\frac{5\pi}{3}=\cos\frac{\pi}{3}$

$\therefore \cos\theta=¿\cos\frac{\pi}{3}=\cos\frac{5\pi}{3}¿$

$0\le\frac{\pi}{3}\le 2\pi, 0\le\frac{5\pi}{3}\le 2\pi$

$\therefore \theta=\frac{\pi}{3}, \theta=\frac{5\pi}{3}$ is the required principle solution

Q.6 A particle moves along the curve $6y=x^3+2$ **. Find the points on the curve at which y co-ordinate is changing 8 times as the x co-ordinate.** **(2)**

Ans:

$6y=x^3+2$

Differentiating w.r.to t

$\therefore 6\frac{dy}{dt}=3x^2\frac{dx}{dt} \rightarrow (1)$

Given: - $\frac{dy}{dt}=8\frac{dx}{dt}$

$\therefore (1)\, becomes\, 6\times 8\frac{dx}{dt}=3x^2\frac{dx}{dt}$

$\therefore 3x^2=48 \Rightarrow x^2=16 \Rightarrow x=\pm 4$

Put $x=4 \in 6y=x^3+2$

$\therefore 6y=4^3+2$

$\therefore 6y=66 \Rightarrow y=11$

Put $x=-4 \in 6y=x^3+2$

$\therefore 6y=-64+2$

$\therefore 6y=-62 \Rightarrow y=\frac{-62}{6}=\frac{-31}{3}$

$\therefore$ required points are $(4,11) \wedge \left(-4,\frac{-31}{3}\right)$

Q.7 Evaluate: $\int \frac{sec^8 x}{cosec\, x}$**dx**

(2)

Ans:

$I=\int \frac{sec^8 x}{cosec\, x}dx$

¿ $\int \frac{\sin x}{\cos^8 x}dx$

Put $\cos x=t$

$\therefore -\sin x\, dx=dt \Rightarrow \sin x\, dx=-dt$

$\therefore I=\int \frac{-dt}{t^8}$

¿ $-\int t^{-8}dt$

¿ $-\frac{t^{-8+1}}{(-8+1)}+c$

¿ $-$ ¿ $\frac{-t^{-7}}{-7}+c$

¿ $\frac{1}{7t^7}+c$

¿ $\frac{1}{7\cos^7 x}+c$

$\therefore I=\frac{sec^7 x}{7}+c$

Q.8 Find the d.e. of all the parabolas whose axis is Y-axis. **(2)**

Ans:

Given faming of parabolas is

$x^2=4a(y-k)$

Diff. w. r. to x

$\therefore 2x=4a.\frac{dy}{dx}$

$\therefore x=2a\frac{dy}{dx}$

$\therefore \frac{x}{\left(\frac{dy}{dx}\right)}=2a$

Differentiating again w. R. to x

$\therefore \frac{\frac{dy}{dx}.1-x.\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^2}=0$

$\therefore \frac{dy}{dx}-x\frac{d^2y}{dx^2}=0$

Is the required d.e.

Q.9 Evaluate: $\int_0^2 x e^x dx$ **(2)**

Ans:

$I=\int_0^2 x e^x dx$

¿ $\left[x.\int e^x dx\right]_0^2-\int_0^2\left\{\frac{d}{dx}(x).\int e^x dx\right\}dx$

¿ $\left[xe^x\right]_0^2-\int_0^2 1.e^x dx$

¿ $2e^2-\left[e^x\right]_0^2$

¿ $2e^2-(e^2-1)=2e^2=e^2+1$

$\therefore I=e^2+1$

Q.10 Find 'k' if the slope of one of the lines given by $kx^2+4xy-y^2=0$ **exceeds the slope of the other by 8.** **(2)**

Ans:

$kx^2+4xy-y^2=0$

Comparing with $ax^2+2hxy+by^2=0$

$\therefore a=k, 2h=4, b=-1$

$\therefore m_1+m_2=\frac{-2h}{b}=\frac{-4}{-1}=4$

¿ $m_1.m_2=\frac{a}{b}=\frac{k}{-1}=-k$

Given:- $m_1=m_2+8$

$\therefore m_2+8+m_2=4$

$\therefore 2m_2=-4$

$\therefore m_2=-2$

$Also, (m_2+8)m_2=-k$

$\therefore -k=-2(-2+8)$

$\therefore -k=-2\times 6$

$\therefore -k=-12$

$\therefore k=12$

Q.11 Find the vector equation of the plane which is at a distance of 6 units from the origin and which is normal to the vector $2\hat{i}-\hat{j}+2\hat{k}$. **(2)**

Ans:

Here, $p=6, \bar{n}=2\hat{i}-\hat{j}+2\hat{k}$

$$\therefore \hat{n}=\frac{2\hat{i}-\hat{j}+2\hat{k}}{\sqrt{2^2+(-1)^2+2^2}}$$

$$\therefore \hat{n}=\frac{2\hat{i}-\hat{j}+2\hat{k}}{3}$$

$\therefore$ required plane is

$\bar{r}\ .\ \hat{n}=p$

$$\therefore \frac{\bar{r}.(2\hat{i}-\hat{j}+2\hat{k})}{3}=6$$

$\therefore \bar{r}.(2\hat{i}-\hat{j}+2\hat{k})=18$

Q.12 Find the shortest distance between the lines $\bar{r}=(2\hat{i}-\hat{j})+(2\hat{i}+\hat{j}-3\hat{k})$ **&** $\bar{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(2\hat{i}+\hat{j}-5\hat{k})$ **(2)**

Ans:

Here $\bar{a_1}=2\hat{i}-\hat{j}, \bar{a_2}=\hat{i}-\hat{j}+2\hat{k},$

$\bar{b_1}=2\hat{i}+\hat{j}-3\hat{k}, \bar{b_2}=2\hat{i}+\hat{j}-5\hat{k}$

$\bar{a_2}-\bar{a_1}=(\hat{i}-\hat{j}+2\hat{k})-(2\hat{i}-\hat{j})=-\hat{i}+2\hat{k}$

$$\therefore \bar{b_1}\times\bar{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -3\\ 2 & 1 & -5\end{vmatrix} ¿ -2\hat{i}+4\hat{j}+0\hat{k}$$

$\therefore |\bar{b_1}\times\bar{b_2}|=\sqrt{(-2)^2+4^2}=\sqrt{20}=2\sqrt{5}$

$$\therefore d=\left|\frac{(\bar{a_2}-\bar{a_1}).(\bar{b_1}\times\bar{b_2})}{|\bar{b_1}\times\bar{b_2}|}\right|$$

$$¿\left|\frac{(-\hat{i}+2\hat{k}).(-2\hat{i}+4\hat{j}+0\hat{k})}{2\sqrt{5}}\right|$$

$$¿\left|\frac{2}{2\sqrt{5}}\right|$$

$\therefore d=\frac{1}{\sqrt{5}}$ units

Q.13 A fair coin is tossed 3 times. Let X be the number of heads obtained. Find E(X) **(2)**

Ans:

x_i	p_i	$x_i p_i$
0	1/8	0
1	3/8	3/8
2	3/8	6/8
3	1/8	$\underline{3/8}$

$$\sum x_i p_i = \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = \frac{3}{2}$$

$$\therefore E(x) = \sum x_i p_i = \frac{3}{2} = 1.5$$

Q.14 Given $X \quad B(n,p)$**. If** $E(X)=5$**,** $Var.(X)=2.5$ **; find n & p?** **(2)**

Ans:

$E(x)=5, Var.(x)=2.5$

$\therefore np=5 \rightarrow (1), npq=2.5 \rightarrow (2)$

Dividing (2) by (1)

$$\therefore \frac{npq}{np} = \frac{2.5}{5}$$

$$\therefore q = \frac{1}{2} \Rightarrow p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}$$

$\therefore p = 1/2$

$$\therefore (1)\, gives\, n \times \frac{1}{2} = 5$$

$$\Rightarrow n = 10$$

$\therefore n = 10, p = 1/2$

SECTION-C

Attempt any eight of the following questions: **[24]**

Q.15 Find the separate equations of the lines represented by the following equation:

$x^2 + 2(cosec\,\alpha)xy + y^2 = 0.$ **(3)**

Ans:

$x^2 + 2(cosec\,\alpha)xy + y^2 = 0$

Comparing with $ax^2 + 2hxy + by^2 = 0$

$\therefore a = 1, 2h = 2(cosec\,\alpha), b = 1$

$\therefore$ Auxiliary equation is $bm^2 + 2hm + a = 0$

$\therefore m^2 + 2(cosec\,\alpha)m + 1 = 0$

$$\therefore m = \frac{-2cosec\,\alpha \pm \sqrt{(2\,cosec\,\alpha)^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$¿ \frac{-2\,cosec\,\alpha \pm \sqrt{4\,cosec^2\alpha - 4}}{2}$$

$\therefore m = -cosec\,\alpha \pm \sqrt{\cot^2\alpha} = -cosec \pm \cot\alpha$

$\therefore m_1 = -cosec\,\alpha + \cot\alpha, m_2 = -cosec\,\alpha - \cot\alpha$

$\therefore$ required lines are $y = m_1 x \wedge y = m_2 x$

$y = (-cosec\,\alpha + \cot\alpha)x, y = (-cosec\,\alpha - \cot\alpha)x$

$\therefore (cosec\,\alpha - \cot\alpha)x + y = 0, (cosec\,\alpha + \cot\alpha)x + y = 0$

Q.16 Using truth tables examine whether the following statement pattern is tautology, contradiction or contingency: $[p \wedge (p \to \quad q)] \to q$ **(3)**

Ans

p	q	q	$p \to \quad q$	$p \wedge (p \to \quad q)$	$[p \wedge (p \to \quad q)] \to q$
T	T	F	F	F	T
T	F	T	T	T	F
F	T	F	T	F	T
F	F	T	T	F	T

Since the entries in the last column are mined with T, F

$\therefore$ given statement pattern is contingency.

Q.17 Find the equation of the line passing through the points $(3,1,2)$ **and perpendicular to the lines** $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ **and** $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$.

(3)

Ans:

Here $\bar{a}=3\hat{i}+\hat{j}+2\hat{k}$,

$\bar{b}_1=\hat{i}+2\hat{j}+3\hat{k};\ \bar{b}_2=-3\hat{i}+2\hat{j}+5\hat{k}$

$$\therefore \bar{b}=\bar{b}_1\times\bar{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ -3 & 2 & 5\end{vmatrix}$$

$\therefore \bar{b}=4\hat{i}-14\hat{j}+8\hat{k}$

$\therefore$ required line is

$\bar{r}=\bar{a}+\lambda\bar{b}$

$\therefore\ \bar{r}=(3\hat{i}+\hat{j}+2\hat{k})+\lambda(4\hat{i}-14\hat{j}+8\hat{k})$

Q.18 Find the inverse of the matrix $\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$ **by suitable column transformations. (3)**

Ans:

$$A=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$$

$$\therefore |A|=\begin{vmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{vmatrix}=7-3-3=1\neq 0$$

$\therefore A^{-1}$exists

Consider $\quad A^{-1}.A=I$

$$\therefore A^{-1}\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$C_2\to C_2-3C_1$

$$\therefore A^{-1}\begin{bmatrix}1 & 0 & 3\\ 1 & 1 & 3\\ 1 & 0 & 4\end{bmatrix}=\begin{bmatrix}1 & -3 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$C_3 \rightarrow C_3 - 3C_1$

$$\therefore A^{-1}\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -3 & -3\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$C_1 \rightarrow C_1 - C_3$

$$\therefore A^{-1}\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}4 & -3 & -3\\ 0 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$$

$C_1 \rightarrow C_1 - C_2$

$$\therefore A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$$

$$\therefore A^{-1}=\begin{bmatrix}7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$$

Q.19 In ΔABC, prove that: $a(b \times cosC - c \times cosB) = b^2 - c^2$ **(3)**

Ans:

L.H.S. ¿ $a(b\cos C - c\cos B)$

¿ $ab\cos C - ac\cos B$

¿ $ab\text{-}\frac{(a^2+b^2-c^2)}{2ab} - \frac{ac(a^2+c^2-b^2)}{2ac}$

¿ $\frac{1}{2}[a^2+b^2-c^2-a^2+c^2-b^2]$

¿ $\frac{1}{2}(2b^2-2c^2)$

¿ b^2-c^2

¿ $R.H.S$

Hence, proved.

Q.20 Evaluate: $\int \frac{1}{\sin(x-a).\cos(x-b)}$**dx**

(3)

Ans:

¿ $\int \frac{1}{\sin(x-a)\cos(x-b)}dx$

¿ $\frac{1}{\cos(a-b)}\int \frac{\cos[(x-b)-(x-a)]}{\sin(x-a)\cos(x-b)}dx$

¿ $\frac{1}{\cos(a-b)}\int \left[\frac{\cos[(x-b)\cos(x-a)+\sin(x-b)\sin(x-a)]}{\sin(x-a)\cos(x-b)}\right]\text{dx}$

¿ $\frac{1}{\cos(a-b)}\int \left[\frac{\cos(x-b)\cos(x-a)}{\sin(x-a)\cos(x-b)}+\frac{\sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)}\right]dx$

¿ $\frac{1}{\cos(a-b)}\int[\cot(x-a)+\tan(x-b)]\,dx$

¿ $sec(a-b)[\log|\sin(x-a)|+\log|sec(x-b)|]+c$

¿ $sec(a-b)\log|\sin(x-a)\,sec(x-b)|+c$

$\therefore I=sec(a-b)\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+c$

Q.21 Given below is the probability distribution of X:

x	0	1	2	3	4
P(X = x)	k	2k	4k	2k	k

Find: i) the value of k, ii) $P(X\geq 2)$, **iii)** $P(X<3)$ **(3)**

Ans:

i) $\sum pi=1\Rightarrow k+2k+4k+2k+k=1\Rightarrow 10k=1$

$\therefore k=\frac{1}{10}$

ii) $p(\times\geq 2)=4k+2k+k=7k=7\times\frac{1}{10}=\frac{7}{10}$

iii) $p(\times<3)=k+2k+4k=7k=7\times\frac{1}{10}=\frac{7}{10}$

Q.22 Evaluate: $\int_{-\pi/4}^{\pi/4}\frac{dx}{1+sinx}$ **(3)**

Ans:

$I=\int_{-\pi/4}^{\pi/4}\frac{1}{1+\sin x}dx$

¿ $\int_{-\pi/4}^{\pi/4}\frac{(1-\sin x)}{(1+\sin x)(1-\sin x)}dx$

¿ $\int_{-\pi/4}^{\pi/4}\frac{(1-\sin x)}{(1-\sin^2 x)}dx$

¿ $\int_{-\pi/4}^{\pi/4}\left(\frac{1-\sin x}{\cos^2 x}\right)dx$

¿ $\int_{-\pi/4}^{\pi/4}\left(\frac{1}{\cos^2 x}-\frac{\sin x}{\cos^2 x}\right)dx$

¿ $\int_{-\pi/4}^{\pi/4}(sec^2 x-sec\,x\tan x)\,dx$

¿ $[\tan x-sec\,x]_{-\pi/4}^{\pi/4}$

¿ $\left(\tan\frac{\pi}{4}-sec\frac{\pi}{4}\right)-\left(\tan\left(\frac{-\pi}{4}\right)-sec\left(\frac{-\pi}{4}\right)\right)$

¿ $(1-\sqrt{2})-(-1-\sqrt{2})$

¿ $1-\sqrt{2}+1+\sqrt{2}$

$\therefore I=2$

Q.23 Find the area enclosed between the circle $x^2+y^2=1$ & the line $x+y=1$ in the first quadrant. **(3)**

Ans:

$x^2+y^2=1 \rightarrow (1)$ $\qquad$ $x+y=1 \rightarrow (2)$

$\Rightarrow y=\sqrt{1-x^2}$ $\qquad$ $\Rightarrow y=1-x$

Put $y=1-x \in (1)$

$\therefore x^2+(1-x)^2=1$

$\therefore x^2+1-2x+x^2=1$

$\therefore 2x^2-2x=1-1$

$\therefore 2x(x-1)=0$

$\therefore x=0, x=1$

$\therefore$ Required area is

$$A=\left|\int_0^1 \sqrt{1-x^2}\,dx-\int_0^1 (1-x)\,dx\right|$$

¿ $\left|\left[\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\log\left|x+\sqrt{1-x^2}\right|\right]_0^1-\left[x-\frac{x^2}{2}\right]_0^1\right|$

¿ $\left|0-\frac{1}{2}(\log 1-\log 1)-\left[1-\frac{1}{2}\right]\right|$

¿ $\frac{1}{2}$ sq units

Q.24 Let PQRS be a quadrilateral. If M and N are the mid points of the sides PQ and RS respectively then prove that $\overline{PS}+\overline{QR}=2\overline{MN}$ **(3)**

Ans:

Since M, N are midpoint of sides PQ & RS

$\therefore \overline{M}=\frac{\overline{p}+\overline{q}}{2} \rightarrow (1), \overline{n}=\frac{\overline{r}+\overline{s}}{2} \rightarrow (2)$

L.H.S. = $\overline{PS}+\overline{QR}$

¿ $(\overline{s}-\overline{p})+(\overline{r}-\overline{q})$

¿ $(\overline{s}+\overline{r})-(\overline{p}+\overline{q})$

¿ $2\left[\frac{(\overline{s}+\overline{r})}{2}-\frac{(\overline{p}+\overline{q})}{2}\right]$

¿ $2(\overline{n}-\overline{m})$ $\qquad$ (Using (1) & (2))

¿ $2\overline{MN}$

¿ $R.H.S$

Hence, Proved.

Q.25 Find $\frac{dy}{dx}$, if $x=3\cos t-2\cos^3 t, y=3\sin t-2\sin^3 t$ **(3)**

Ans:

$x=3\cos t-2\cos^3 t$

$$\therefore \frac{dx}{dt} = -3\sin t - 2\times 3\cos^2 t.(-\sin t)$$

$$¿ -3\sin t + 6\cos^2 t \sin t$$

$$¿ 3\sin t(2\cos^2 t - 1)$$

$$\therefore \frac{dx}{dt} = 3\,sint.\cos 2t$$

$$y = 3\,sint - 2\sin^3 t$$

$$\therefore \frac{dy}{dt} = 3\,cost - 2\times 3\sin^2 t\,cost$$

$$¿ 3\,cost(1 - 2\sin^2 t)$$

$$\therefore \frac{dy}{dt} = 3\,cost.\cos 2t$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3\cos t.\cos 2t}{3\,sint.\cos 2t}$$

$$\therefore \frac{dy}{dx} = \cot\,(t)$$

Q.26 Find the points on the curve $y = x^3 - 2x^2 - x$, where the tangents are parallel to $3x - y + 1 = 0$. **(3)**

Ans:

$y = x^3 - ¿ 2x^2 - x$, $\quad 3x - y + ¿ 0$

$\therefore \frac{dy}{dx} = 3x^2 - 4x - 1 = m \to (1)$ $\qquad \therefore m = \frac{-3}{-1}, \therefore m = 3 \to (2)$

$\therefore ¿(1)\wedge(2), 3x^2 - 4x - 1 = 3$

$\therefore 3x^2 - 4x - 4 = 0$

$\therefore 3x^2 - 6x + 2x - 4 = 0$

$\therefore 3x(x-2) + 2¿2) = 0$

$\therefore (x-2)(3x+2) = 0$

$\therefore x = 2, x = \frac{-2}{3}$

For $x = 2$

$y = 2^3 - 2\times 2^2 - 2$

$\therefore y = 2$

for $x = -2/3$

$$y = \left(\frac{-2}{3}\right)^3 - 2\left(\frac{-2}{3}\right)^2 - \left(\frac{-2}{3}\right)$$

$$\therefore y = \frac{-8}{27} - \frac{8}{9} + \frac{2}{3}$$

$$¿ \frac{-8 - 24 + 18}{27} \qquad \text{(L.C.M. 27)}$$

$$\therefore y = \frac{-14}{27}$$

$\therefore$Required points are $(2, -2) \wedge \left(\frac{-2}{3}, \frac{-14}{27}\right)$

SECTION-D

Attempt any five of the following question: **[20]**

Q.27 Find the inverse of the matrix $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ **using elementary row transformations.** **(4)**

Ans:

$$A=\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\therefore |A|=\begin{vmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{vmatrix} A^{-1}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 6 \\ 2 & 2 & 5 \end{bmatrix} A^{-1}=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & 0 \\ 2 & 2 & 5 \end{bmatrix} A^{-1}=\begin{bmatrix} 0 & 1 & 0 \\ 1 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1}=\begin{bmatrix} 0 & 1 & 0 \\ 1 & -3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$R_2 \rightarrow -R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1}=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$R_1 \rightarrow R_1 - 2R_3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1}=\begin{bmatrix} 0 & 5 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$R_1 \rightarrow R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1}=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$A^{-1}=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Q.28 Find the combined equation of pair of straight lines passing through the origin each of which make an angle of 30^0 with the line $3x+2y-11=0$. **(4)**

Ans:

$3x+2y-11=0 \rightarrow (1)$

$m=\frac{-3}{2}$

Let OA & OB be the required lines

Let equation of OA be $y=m_1 x$

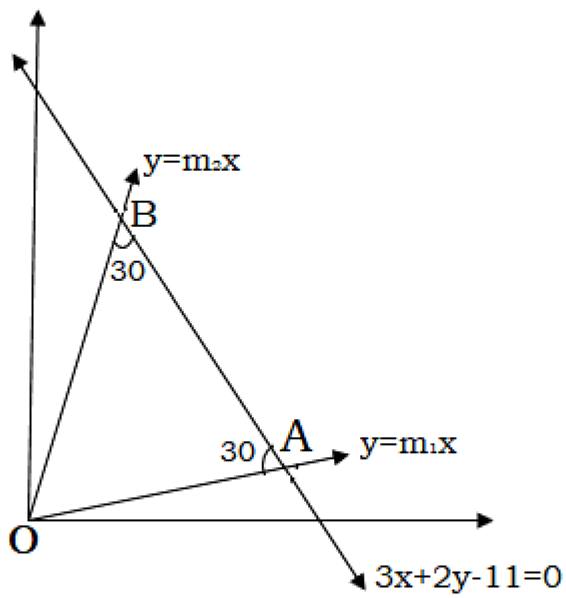
Let OA makes angle $\theta=30^\circ$ with (1)

$\therefore \tan 30^\circ=\left|\frac{m-m_1}{1+m m_1}\right|$

$\therefore \frac{1}{\sqrt{3}}=\left|\frac{\frac{-3}{2}-m_1}{1+\left(\frac{-3}{2}\right)m_1}\right|$

$\therefore \frac{1}{\sqrt{3}}=\left|\frac{\frac{-3-2m_1}{2}}{\frac{2-3m_1}{2}}\right|$

$\therefore \frac{1}{\sqrt{3}}=\left|\frac{-(3+2m_1)}{(2-3m_1)}\right|$



Squaring both sides, we get

$\frac{1}{3}=\frac{(3+2m_1)^2}{(2-3m_1)^2}$

$\therefore (2-3m_1)^2=3(3+2m_1)^2$

$\therefore 4-12m_1+9m_1^2=3(9+12m_1+4m_1^2)$

$\therefore 4-12m_1+9m_1^2=27+36m_1+12m_1^2$

$\therefore 12m_1^2-9m_1^2+36m_1+12m_1+27-4=0$

$\therefore 3m_1^2+48m_1+23=0$

Put $m_1=y/x$ $\quad (\because y=m_1 x)$

$\therefore 3\left(\frac{y}{x}\right)^2+48\left(\frac{y}{x}\right)+23=0$

$\therefore \frac{3y^2}{x^2}+\frac{48y}{x}+23=0$

Multiplying both sides by x^2, *we get*

$3y^2+48xy+23x^2=0$

OR $23x^2+48xy+3y^2=0$

Is the required combined equation

Q.29 Maximize $z=6x+10y$, subject to $3x+5y\le 10, 5x+3y\le 15, x\ge 0, y\ge 0$ **(4)**

Ans:

consider $3x+5y=10 \rightarrow (1)$

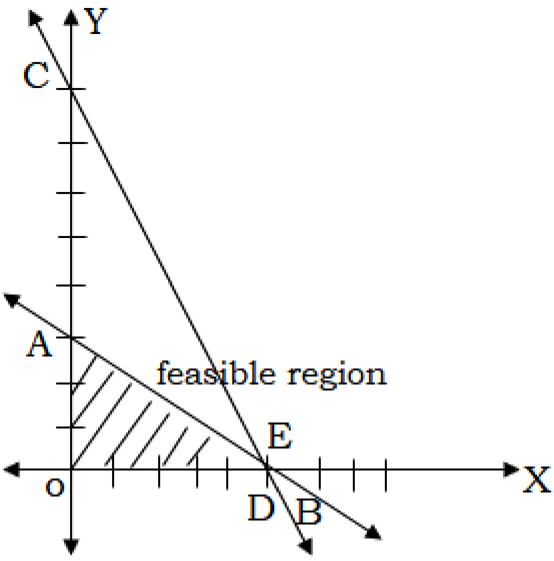
Put $x=0 \therefore y=2 \therefore A\equiv(0,2)$

Put $y=0 \therefore x=\frac{10}{3} \therefore B\equiv\left(\frac{10}{3},0\right)$

Consider $5x+3y=15 \rightarrow (2)$

Put $x=0 \therefore y=5 \therefore C\equiv(0,2)$

Put $x=0 \therefore x=3 \therefore D\equiv(3,0)$



$5\times(1)-3\times(2)$ gives $\quad -¿15x+25y=50$

$15x+9y=45$

$16y=5 \Rightarrow y=\frac{5}{16}$

Put $y=\frac{5}{16}\in(1), we\ get$

$3x+5\times\frac{5}{16}=10$

$\therefore\ 3x=10-\frac{25}{16}$

$\therefore 3x=\frac{135}{16} \Rightarrow x=\frac{45}{16}$

$\therefore (1)\wedge(2)\ intrsect\ at\ E\left(\frac{45}{16},\frac{5}{16}\right)$

From the graph feasible region is OAED.

Point on feasible region	$z=6x+10y$
$O(0,0)$	$z=0$
$A(0,2)$	$z=20$
E$\left(\frac{45}{16},\frac{5}{16}\right)$	$z=20$
$D(3,0)$	$z=18$

$\therefore Max.z=20$ Which occurs at points A & E.

$\therefore$ *given LPP* has infinitely many solutions between points A & E

Q.30 Show that: $2\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{24}{7}\right)$ **(4)**

Ans:

Let $\sin^{-1}\left(\frac{3}{5}\right)=\alpha. \Rightarrow \sin\alpha=\frac{3}{5}$

$\therefore \tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{3/5}{4/5}=\frac{3}{4}$

$\therefore \alpha=\tan^{-1}\left(\frac{3}{4}\right)$

$\therefore \sin^{-1}\left(\frac{3}{5}\right)=¿\tan^{-1}\left(\frac{3}{4}\right)¿$

$\therefore 2\sin^{-1}\left(\frac{3}{5}\right)=¿2\tan^{-1}\left(\frac{3}{4}\right)¿$

$¿\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{3}{4}\right)$

$$¿\tan^{-1}\left(\frac{\frac{3}{4}+3/4}{1-\frac{3}{4}\times\frac{3}{4}}\right) \qquad \therefore \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$¿\tan^{-1}\left(\frac{6/4}{1-\frac{9}{16}}\right)$$

$$¿\tan^{-1}\left(\frac{3/2}{7/16}\right)$$

$$¿\tan^{-1}\left(\frac{8+6\times 3}{7\times 2}\right)$$

$$¿\tan^{-1}\left(\frac{24}{7}\right)$$

$$\therefore 2\sin^{-1}\left(\frac{3}{5}\right)=¿\tan^{-1}\left(\frac{24}{7}\right)¿$$

Hence, proved.

Q.31 Sand is pouring at the rate of $12\,cm^3/sec$. The falling sand forms a cone on the ground in such a way that the height of the cone is always $\left(\frac{1}{6}\right)^{th}$ of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?(4)

Ans:

Given: - $h=\frac{1}{6}r. \Rightarrow r=6h \rightarrow (1)$

Volume of cone = $\frac{1}{3}\pi r^2 h$

$¿\frac{\pi}{3}(6h)^2 h$ (Using (1))

$¿\frac{\pi}{3}.36h^3$

$\therefore V=12\pi h^3$

$\therefore \frac{dv}{dt}=12\pi 3h^2\frac{dh}{dt}$

$\therefore 12=12\pi.3h^2\frac{dh}{dt}$

$\therefore \frac{dh}{dt}=\frac{1}{3\pi h^2}$

For $h=4\,cm, we\ get$

$\frac{dh}{dt}=\frac{1}{3\pi\times 4^2}=\frac{1}{48\pi}cm/sec$

Q.32 A body cools down according to Newton's law from 100^0C to 60^0C in 20 minutes. The temperature of the surrounding being 20^0C, how long will it take to cool down to 30^0C? (4)

Ans:

According to Newton's law of cooling,

$\frac{d\theta}{dt}\alpha(\theta-\theta_0)$

$\therefore \frac{d\theta}{dt}=-k(\theta-\theta_0)$

$\therefore \frac{d\theta}{\theta-\theta_0}=-k\,dt$

$\therefore \frac{d\theta}{\theta-20}=-k\,dt\,(\therefore \theta_0=20)$

Integrating both sides $\therefore \int \frac{d\theta}{\theta-20}=-k\int dt$

$\therefore \log(\theta-20)=e^{-kt+c}$

$\therefore \theta=20+e^{c}.e^{-kt}$

$\therefore \theta=20+a\ e^{-kt}\rightarrow(1), where\, a=e^{c}$

$For\, t=0, \theta=100$

$\therefore 100=20+a e^{0}$

$\therefore a=80$

$\therefore (1)\, becomes\, \theta=20+80e^{-kt}$

$For\, t=20, \theta=60$

$\therefore 60=20+80e^{-20k}$

$\therefore 60-20=80e^{-20k}$

$\therefore \frac{40}{80}=e^{-20k}$

$\therefore e^{-20k}=\frac{1}{2}\rightarrow(2)$

For $\theta=30^{\circ}$,

$30=20+80e^{-kt}$

$\therefore 30-20=80e^{-kt}$

$\therefore \frac{10}{80}=e^{-kt}$

$\therefore \frac{1}{8}=e^{-kt}$

$\therefore \frac{1}{2^3}=e^{-kt}$

$\therefore 2^{-3}=e^{-kt}$

$\therefore 2^{-3}=(e^{-20k})^{t/20}$

$\therefore 2^{-3}=\left(\frac{1}{2}\right)^{t/20}$

$\therefore 2^{-3}=2^{-t/20}\Rightarrow\frac{-t}{20}=-3$

$\therefore t=60\,min¿1\,hour.$

$\therefore$the body cools down to $30^{\circ}c$ after 1 hour

Q.33 Prove that: $\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c$ **(4)**

Ans:

$$I=\int \sqrt{x^2-a^2}\,dx \rightarrow (1)$$
$$¿\int \sqrt{x^2-a^2}\,.1\,dx$$
$$¿\sqrt{x^2-a^2}\,.\int 1\,dx-\int \left\{\frac{d}{dx}\left(\sqrt{x^2-a^2}\right).\int 1\,dx\right\}dx$$
$$¿\,x\sqrt{x^2-a^2}-\int \frac{1}{2\sqrt{x^2-a^2}}.2\,x.\,x\,dx$$
$$¿\,x\sqrt{x^2-a^2}-\int \frac{x^2}{\sqrt{x^2-a^2}}.\,dx$$
$$¿\,x\sqrt{x^2-a^2}-\int \frac{\left(x^2-a^2+a^2\right)}{\sqrt{x^2-a^2}}.\,dx$$
$$¿\,x\sqrt{x^2-a^2}-\int \frac{\left(x^2-a^2\right)}{\sqrt{x^2-a^2}}.\,dx-a^2\int \frac{1}{\sqrt{x^2-a^2}}dx$$
$$¿\,x\sqrt{x^2-a^2}-\int ¿\sqrt{x^2-a^2}\,dx-a^2.\log\left|x+¿\sqrt{x^2-a^2}\right|+C_1$$
$$\therefore I=x\sqrt{x^2-a^2}-I-a^2\log\left|x+¿\sqrt{x^2-a^2}\right|+C_1(using(1))$$
$$\therefore 2I=x\sqrt{x^2-a^2}-a^2\log\left|x+¿\sqrt{x^2-a^2}\right|+C_1(using(1))$$
$$\therefore I=\frac{x}{2}\sqrt{x^2-a^2}\;\frac{-a^2}{2}\log\left|x+¿\sqrt{x^2-a^2}\right|+\frac{C_1}{2}$$
$$\therefore \int \sqrt{x^2-a^2}\,dx=¿\frac{x}{2}\sqrt{x^2-a^2}\,¿\,\frac{-a^2}{2}\log\left|x+¿\sqrt{x^2-a^2}\right|+C$$
$$where,\,C=\frac{C_1}{2}$$

Hence, Proved.

Q.34 Evaluate: $\int_0^{\pi/2} \frac{1}{5+4\,cosx}dx$

(4)

Ans:

$$I=\int_0^{\pi/2}\frac{1}{5+4\cos x}dx$$

Put $\tan\left(\frac{x}{2}\right)=t$

$$\therefore dx=\frac{2dt}{1+t^2}$$

$$\cos x=\frac{1-t^2}{1+t^2}$$

X	T
0	0
$\pi/2$	1

$$\therefore I=\int_0^1 \frac{\frac{2dt}{1+t^2}}{5+4\left(\frac{1-t^2}{1+t^2}\right)}$$

¿ $2\int_0^1 \frac{\frac{1}{1+t^2}}{\frac{5\left(1+t^2\right)+4\left(1-t^2\right)}{\left(1+t^2\right)}}dt$

¿ $2\int_0^1 \frac{1}{5+5t^2+4-4t^2}dt$

¿ $2\int_0^1 \frac{1}{t^2+9}dt$

¿ $2.\frac{1}{3}\left[\tan^{-1}\left(\frac{t}{3}\right)\right]_0^1$

¿ $\frac{2}{3}\left[\tan^{-1}\left(\frac{1}{3}\right)-\tan^{-1}\left(\frac{1}{3}\right)-\tan^{-1}(0)\right]$

$\therefore I=\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\right) \qquad (\therefore \tan^{-1}(0)=0)$

"All the Best"

"If you are not willing to learn,

no one can help you.

If you are determined to learn,

no one can stop you."