2024	3	26	JEE-T3	SEAT NUMBER				

IIT INSPIRE
ACADEMY OF SCIENCE
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XI & XII Science (CBSE/state)
IIT- JEE (Mains + Advance)
NEET, MHT-CET , NDA
Mo. No. 9595445177/9021445177

Branches : Chhatrapati Sq., Mangalmurti Sq.

Day - 3

JEE TEST

Time : 3 Hrs.	(4 Pages)	Max. Marks: 300

Instructions

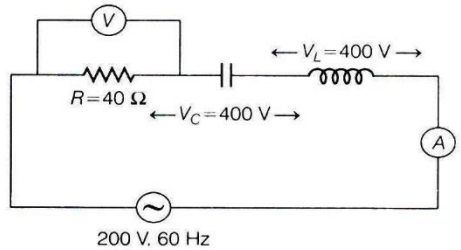
1. There are **three** parts in the questions paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
2. **Section A** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and – **1 mark** for wrong answer.
3. **Section B** This section contains 10 questions. In Section B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and – **1 mark** for wrong answer. For Section B, the answer should be rounded off to the nearest integer.

PHYSICS

Section A : Objective Type Questions

1. If kinetic energy of a particle remains constant, then
(a) the particle moves in a straight line
(b) the force acted on the particle is perpendicular to its velocity
(c) the force acted on the particle is perpendicular to its acceleration
(d) the object is stationary

2. In a given *L-C-R* circuit, resistance of the circuit is 40 Ω as given, then the voltage across terminals of resistance and current through ammeter will be



(a) 200 V, 5 A
(b) 400 V, 5 A
(c) 400 V, $5\sqrt{2}$ A
(d) 200 V, $5\sqrt{2}$ A

3. The approximate depth of an ocean is 5400 m. The compressibility of water is $45.4\times10^{-11}\ Pa^{-1}$ and density of water is $10^3\ kg/m^3$. What fractional compression of water will be obtained at the bottom of the ocean?
(a) 2.4×10^{-2}
(b) 1.2×10^{-2}
(c) 3.6×10^{-2}
(d) 2.4×10^{-5}

4. A horizontal force of 12 N acted on the block placed on a rough floor ($\mu = 0.3$) as shown in the figure

The frictional force between the block and the surface is
(a) 15 N
(b) 18 N
(c) 12 N
(d) 20 N

5. **Assertion** The rms speed of hydrogen (H_2) is greater than the rms speed of oxygen (O_2).
Reason The rms speed of a gas is inversely proportional to its molecular weight.
(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
(c) Assertion is correct, but Reason is incorrect.
(d) Assertion is incorrect, but Reason is correct.

6. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47th division on the circular scale coincides with reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm^2) of the wire in appropriate number of significant figures.

(a) 2 (b) 3 (c) 2.6 (d) 3.6

7. Two semi-circular ring charges with linear charge densities $+\lambda$ and $-\lambda$ are placed to form a ring of radius R. The electric field at the centre of the charged ring will be

(a) $\frac{\lambda}{2\pi\varepsilon_0 R}$ (b) zero (c) $\frac{2\lambda}{\pi\varepsilon_0 R}$ (d) $\frac{\lambda}{\pi\varepsilon_0 R}$

8. The wave nature of light was established by

(i) Maxwell's equations

(ii) de-Broglie relation

(iii) Hertz experiment

(iv) Einstein's theory

Choose the appropriate answer from the given options.

(a) (i) and (ii) only (b) (ii) and (iv) only
(c) (i) and (iii) only (d) (iii) and (iv) only

9. The coulomb force **F** between outermost electron and nucleus of Li^{2+} atom of radius r is

(a) $3K\frac{e^2}{r^2}\hat{\mathbf{r}}$ (b) $-K\frac{e^2}{r^3}\hat{\mathbf{r}}$

(c) $-3K\frac{e^2}{r^3}\mathbf{r}$ (d) $3K\frac{e^2}{r^3}\mathbf{r}$

10. Study the following statement regarding logic gates.

I. NOT gate produces an inverted version of input at its output.

II. If inputs of a NOR gate is 0 and 1, then its output will be 1.

III. AND gate produces same result, when both its input having same value.

Which of the following statement(s) is/are correct ?

(a) Both I and III (b) Only I
(c) Both I and II (d) All I, II, III

11. **Statement I** The equivalent emf of parallel combination of the cells is just the sum of their individual emf.

Statement II In a series combination of two cells, if the polarity of one cell is reversed, then their equivalent emf will be changed, while equivalent resistance remains unchanged.

Choose the correct option.

(a) Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I

(b) Both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I

(c) Statement I is true but Statement II is false.

(d) Statement I is false but Statement II is true.

12. Unpolarised light of intensity 40 Wm^{-2} passes through three polarisers such that transmission axes of the first and second polariser makes an angle 30° with each other and the transmission axis of the last polariser is crossed with that of the first. The intensity of final emerging light will be

(a) 2.25 W/m^2 (b) 8.35 W/m^2
(c) 3.75 W/m^2 (d) 15 W/m^2

13. Which of the following statement is correct ?

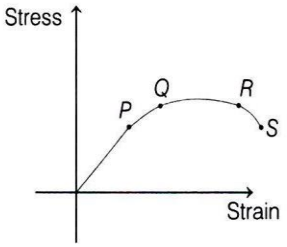
(a) The acceleration due to gravity depends on the mass of the body.

(b) Escape velocity of a body on the surface of the earth is independent of radius of the earth.

(c) Inside a uniform spherical shell, the gravitational potential is same everywhere.

(d) Speed remains constant in a planetary motion.

14.



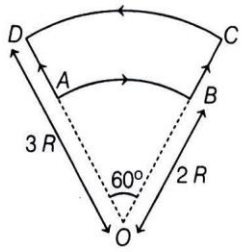
Now, match the terms in Column I with corresponding terms in Column II.

Column I		Column II	
A.	Yield point	1.	*P*
B.	Breaking point	2.	*Q*
C.	Ultimate strength point	3.	*R*
D.	Proportionally limit point	4.	*S*

Codes

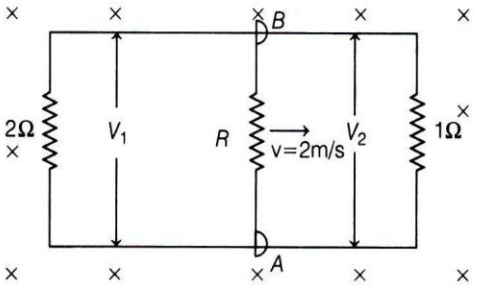
	A	B	C	D
(a)	1	2	3	4
(b)	2	1	3	4
(c)	2	4	3	1
(d)	1	4	3	2

15. The figure shows a current loop having two circular arcs joined by two radial lines. The magnetic field at O is



(a) $\frac{\mu_0 i}{36R}$ (b) $\frac{\mu_0 i}{24R}$ (c) $\frac{\mu_0 i}{72R}$ (d) $\frac{\mu_0 i}{48R}$

16. A rectangular loop has a sliding connector AB of length 1m and resistance 3Ω and it is moving with speed 2 m/s as shown. The set-up is placed in uniform magnetic field 5T.



Then, the values of V_1 and V_2 are

(a) $V_1 = \frac{20}{11}$, $V_2 = \frac{90}{11}$ V

(b) $V_1 = \frac{20}{13}$, $V_2 = \frac{21}{13}$ V

(c) $V_1 = V_2 = \frac{20}{11}$ V

(d) $V_1 = V_2 = \frac{21}{11}$ V

17. For the density measurement of a sphere, the mass and radius are measured as (15 ± 0.30) kg and (0.20 ± 0.01) m. The error in the measurement of density is

(a) $0.31\,\text{kgm}^{-3}$ (b) $0.17\,\text{kgm}^{-3}$

(c) $0.28\,\text{kgm}^{-3}$ (d) $0.27\,\text{kgm}^{-3}$

18. A plane electromagnetic wave of frequency 60 MHz travels in free space along positive x-direction. At a particular time in space, $\mathbf{B} = 2.1 \times 10^{-8}\,\hat{\mathbf{k}}$T, then the corresponding electric field at that point is

(a) $6.3\hat{\mathbf{j}}$ V/m (b) $-6.5\hat{\mathbf{j}}$ V/m

(c) $-6.3\hat{\mathbf{j}}$ V/m (d) $6.5\,\hat{\mathbf{j}}$ V/m

19. As the electron in Bohr orbit of hydrogen atom passes from state $n = 2$ to $n = 1$, the kinetic energy K and potential energy U changes as

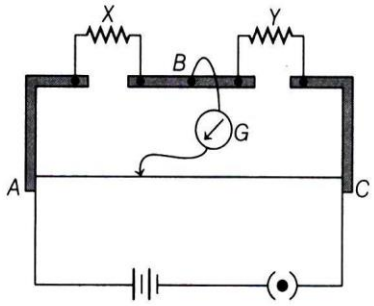
(a) K becomes one-fourth and U becomes four times

(b) K becomes four times, U becomes twice

(c) both K and U become four times

(d) both K and U become twice

20. The null point is found to be 60 cm away from the end A with resistances X and Y in position of a meter bridge as shown. When a resistance of 15 Ω is connected in series with Y, then the null point is found to shift by 10 cm towards the end A of the wire. If a resistance of 30 Ω wire connected in parallel with Y, then the position of null point is



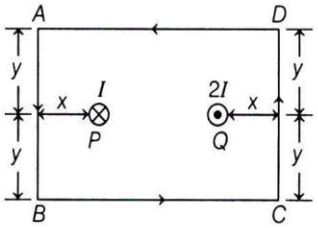
(a) 25 cm from end A (b) 75 cm from end A

(c) 15 cm from end C (d) 45 cm from end C

Section B : Numerical Value Type Questions

21. A body is executing SHM, when its displacement from the mean position is 4 cm and 5cm, the corresponding velocities of the body are 10 cm/s and 8 cm/s, respectively. Then, the time period of the body is .....s .

22. Let B_P and B_Q be the magnetic fields produced by the wires P and Q which are placed symmetrically in a rectangular loop $ABCD$ as shown in figure. Current in wire P is I directed inward and in Q is $2I$ directed outwards. If $\int_A^B \mathbf{B}_Q \cdot d\mathbf{l} = +2\mu_0$ T-m, $\int_D^A \mathbf{B}_P \cdot d\mathbf{l} = -2\mu_0$ T-m and $\int_A^B \mathbf{B}_P \cdot d\mathbf{l} = -\mu_0$T-m, then value of I will be ..... A.

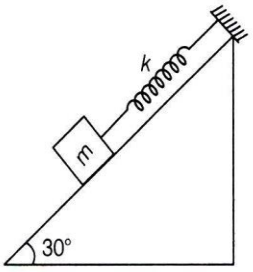


23. The refracting angle of a glass prism is 30°. A ray is incident onto one of the faces perpendicular to it. If the refractive index of glass is 1.5, then the angle between incident ray and emergent ray is (in degree) ..... .

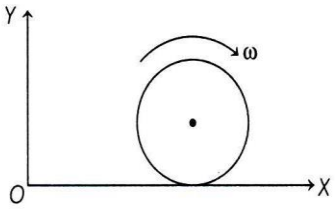
24. A capacitor of capacitance 10 μF is charged to 15 μC. If the plates are pulled apart to reduce the capacitance to 6 μF, then the external work done on the plates of capacitor is 7.5×10^{-x} J. The value of x is ..... .

25. The position of a particle varies as, $x = 4t - 2t^2$. The distance covered by particle in 4 s is ..... m.

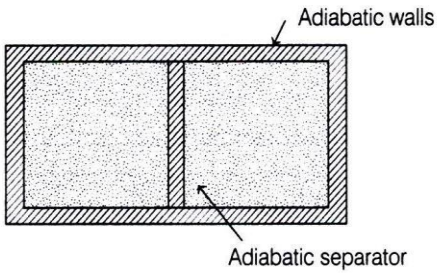
26. One end of a massless spring of relaxed length 50 cm and spring constant k is fixed on the top of a frictionless inclined plane of inclination $\theta = 30°$ as shown in figure below. When a mass $m = 1.5$ kg is attached at the other end, the spring extends by 2.5 cm. The mass is displaced slightly and released. The time period (in s) of the resulting oscillation will be ..... .



27. A disc of mass 2 kg and radius 5 cm is rolling with angular speed of 8 rad/s on a horizontal plane as shown in figure, then the magnitude of angular momentum of the disc about the origin O is ..... kg-m^2/s.

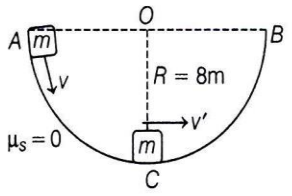


28. An ideal gas ($\gamma = 1.5$) is contained in equal amounts and in same state in two compartments of an adiabatic vessel separated by an adiabatic frictionless separator.

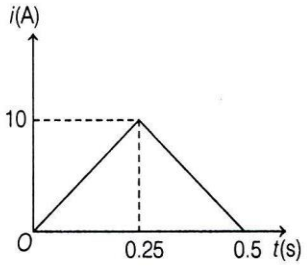


The separator is now sliding slowly towards right such that pressure in right compartment becomes 8 times that in left compartment. Ratio of internal energies of gas in left compartment to that in right compartment is $\frac{x}{2}$. The value of x is..... .

29. A box of mass 2 kg is given a speed 4 m/s at point A of a hemispherical bowl, then the value of normal reaction at point C is xN. What is the value of x ? (Take, $g = 10\,m/s^2$)



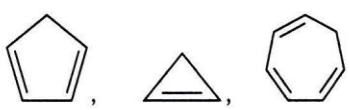
30. In a coil of resistance 200 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. Then the magnitude of change in flux through the coil is x Wb. What is the value of x ?



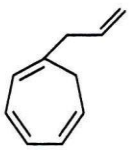
CHEMISTRY

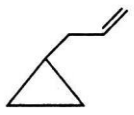
Section A : Objective Type Questions

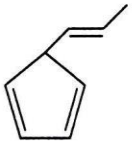
31. What will be the product formed when most acidic species among the following will react with 3-chloroprop-1-ene?



(a) 

(b) 

(c) 

(d) 

32. Nitrogen exist as N_2 but phosphorus exist as P_4 this is due to

(a) triple bond exist between phosphorus atom
(b) $p\pi$-$p\pi$ bond is weak in P_4
(c) $p\pi$-$p\pi$ bond is weak in N_2
(d) multiple bonds form easily

33. Which of the following pairs of complexes is isomeric with each other but their aqueous solutions exhibit different molar conductivities?

(a) $[Co(NH_3)_5NO_2]Cl_2$ and $[Co(NO_3)_5ONO]Cl_2$
(b) $[Co(NH_3)_5Br]SO_4$ and $[Co(NH_3)_5SO_4]Br$
(c) $[PtCl_2(NH_3)_4]Br_2$ and $[Pt(NH_3)_4Br_2]Cl_2$
(d) $[Co(NH_3)_4Cl_2]NO_2$ and $[Co(NH_3)_4ClNO_2]Cl$

34. Match List-I and List-II.

	List-I		List-II
A.	Alcoholic potassium hydroxide	I.	Strong reducing agent.
B.	$Pd/BaSO_4$	II.	Obtained by addition reaction
C.	BHC (Benzene hexachloride)	III.	Used for β-elimination reaction
D.	$LiAlH_4$	IV.	Lindlar's Catalyst

Choose the correct answer from the options given below.

(a) A-III, B-IV, C-II, D-I (b) A-III, B-I, C-IV, D-II
(c) A-II, B-III, C-I, D-IV (d) A-II, B-IV, C-III, D-I

35. Which carbonyl compound gives tertiary alcohol with Grignard reagent followed by acidic hydrolysis?

(a) Formaldehyde (b) Acetaldehyde
(c) Acetone (d) Methyl formate

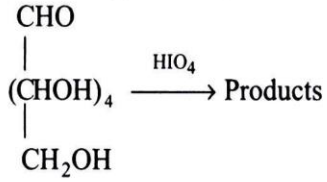
36. Match List-I with List-II.

	List-I		List-II
A.	$S_2O_3^{2-} + FeCl_3$ solution	I.	Green colouration
B.	$SO_3^{2-} + K_2Cr_2O_7/H^+$	II.	Green colour due to $[Cr(H_2O)_6]^{3+}$ ion
C.	Copper wire test for halogens	III.	Beilstein's test
D.	To test the reducing nature of sugar	IV.	Barfoed test

Choose the correct answer from the options given below.

	A	B	C	D		A	B	C	D
(a)	IV	III	II	I	(b)	I	II	III	IV
(c)	I	IV	II	III	(d)	IV	II	I	III

37. Consider the following reaction,



In the above reaction the products are

(a) $5HCOOH + HCHO$
(b) $4HCOOH + HCHO + CO_2$
(c) $3HCOOH + HCHO + CHO(COOH)$
(d) $4HCOOH + HCHO + CO_2 + H_2O$

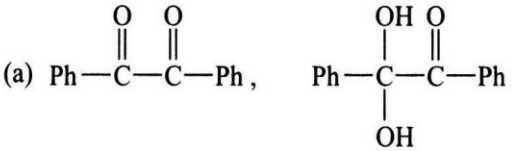
38. Λ°_m for NaCl, HCl and NaA are 126.4, 425.9 and 100.5 S cm^2 mol^{-1}, respectively. If the conductivity of 0.001 M HA is 5×10^{-5} S cm^{-1}. Then, what will be the degree of dissociation of HA?

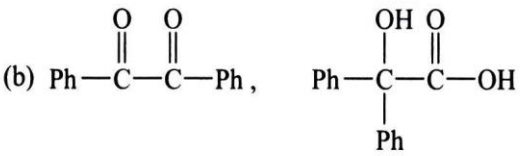
(a) 125×10^{-3} (b) 220×10^{-2}
(c) 225×10^{2} (d) 120×10^{-4}

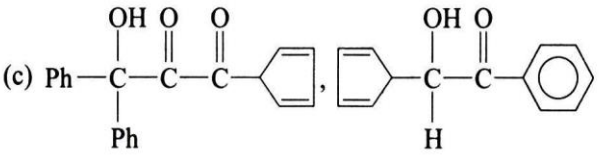
39. Which of the following set of compounds represents isostructural characteristics?

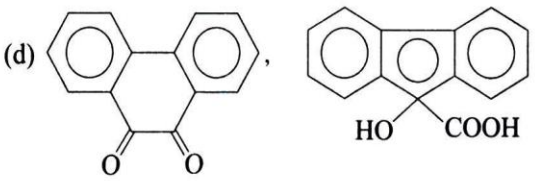
(a) SF_4, CH_4, NH_3 (b) NF_3, BCl_3, NH_3
(c) $BF_3, NF_3, AlCl_3$ (d) BF_3, BCl_3, BBr_3

40. An organic diketone (A) having molecular formula $C_{14}H_{10}O_2$ when undergo reaction with hydroxyl ion (OH^-) converted into benzylic acid (B) after acidification. The molecular structure of (A) and (B) are

(a) 

(b) 

(c) 

(d) 

41. Which of the following statements is incorrect?

(a) The electronic configuration of Cr is [Ar] $3d^5\,4s^1$ [Atomic number of Cr = 24]

(b) The magnetic quantum number may have a negative value.

(c) In silver atom, 23 electrons have a spin of one type and 24 of the opposite type. (Atomic number of Ag = 47)

(d) The oxidation state of nitrogen in HN_3 is –3.

42. When aqueous NaOH is added to an aqueous solution of chromium (III) ions, a green blue precipitate is formed first which re-dissolves to give a green solution. The green colour of the solution is due to the formation of

(a) $[Cr(H_2O)_6]^{3+}$ (b) $[Cr(OH)_4]^-$

(c) CrO_4^{2-} (d) $[Cr(OH)_3(H_2O)_3]$

43. Given below are two statements one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) BF_3 molecules has zero dipole moment.

Reason (R) BF_3 has sp^2-hybridisation and regular trigonal planar geometry.

In the light of the above statements, choose the correct answer from the options given below.

(a) Both A and R are correct and R is the correct explanation of A.

(b) Both A and R are correct but R is not the correct explanation of A.

(c) A is correct, but R is incorrect.

(d) A is incorrect, but R is correct.

44. Consider the following reaction,

$$C_2H_5NH_2 \xrightarrow{C_6H_5CHO} [A]$$
$$C_2H_5NH_2 \xrightarrow{NOCl} [B]$$
$$C_2H_5NH_2 \xrightarrow{C_6H_5SO_2Cl} [C]$$

Which product is identified as a Schiff's base?

(a) *A* (b) *B*

(c) *C* (d) Both *A* and *C*

45. Given below are the two statements.

Statement I Alkyl benzene is not prepared by Friedel-Crafts alkylation of benzene.

Statement II Alkyl halides are less reactive than acyl halides.

In the light of the above statements, choose the correct answer from the option given below.

(a) Both Statement I and Statement II are correct.

(b) Statement I is correct but Statement II is incorrect.

(c) Statement I is incorrect but Statement II is correct.

(d) Both Statement I and Statement II are incorrect.

46. Which of the following species does not exist?

(a) Be_2^+ (b) Be_2 (c) B_2 (d) N_2

47. Which of the following functional groups present in the structure of maltose?

(a) One ketal and hemiketal

(b) Two acetals

(c) One acetal and one hemiacetal

(d) One acetal and one ketal

48. Consider the following reaction,

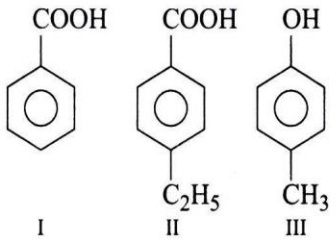
$$3B_2H_6 + 6NH_3 \xrightarrow[\text{temp.}]{\text{High}} 12H_2 + X,$$

the chemical formula of *X* is

(a) $B_3N_3H_6$ (b) $(BN)_X$

(c) B_2N (d) B_3H_4

49. Which of the following is the correct order of acidity of the given compounds?



(a) III > II > I (b) II > I > III

(c) I > II > III (d) III > I > II

50. There are some statements regarding the periodic classification of elements.

A. Non-metallic elements are less in number than metallic elements.

B. The first ionisation enthalpies of elements generally increases with an increase in atomic number as we go along a period.

C. For transition elements, 3*d*-orbitals are filled with electrons after 3*p*-orbitals and before 4*s*-orbitals.

Choose the correct statement from the options given below:

(a) A and B (b) A and C

(c) B and C (d) A, B and C

Section B : Numerical Value Type Questions

51. A 200 mL solution of I_2 is divided into two unequal parts, '*A*' and '*B*'. '*A*' part reacts with hypo solution in acidic medium, 30 mL of 0.2 M hypo was consumed. '*B*' part was added with 50 mL of 0.6 M NaOH solution. The initial concentration of I_2 was ……$\times 10^{-3}$ M.

52. Addition of 0.643 g of a compound to 50 mL of benzene (density = 0.879 g/mL) lowers the freezing point from 5.51°C to 5.03°C. If K_f for benzene is 5.12 K kg mol^{-1}. The molecular weight of the compound is ..... kg mol^{-1}.

53. Following equilibrium is studied by taking 1 mole of N_2 and 3 moles of H_2 in 1 L flask at a given temperature.

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$

$NH_3(g)$ formed at equilibrium is neutralised by 100 mL of 1 M HCl. The value of equilibrium constant, K_C will be .....$\times 10^{-6}$.

54. EMF of the cell,
$Ag \mid AgNO_3$ (0.1 M) || KBr (1 N), $AgBr(s) \mid Ag$ is –0.6 V at 298 K.
$AgNO_3$ is 80% and KBr is 60% dissociated. K_{sp} of AgBr at 298 K is .....$\times 10^{-7}$ M.

55. The gaseous decomposition reaction, $A(g) \longrightarrow 2B(g) + C(g)$ is observed to be first order over the excess of liquid water at 28°C. It is found that after 20 min the total pressure of the system is 200 torr and after a long time (suppose, all reactant is consumed) it is 380 torr. The rate constant of the reaction is ..... $\times 10^{-3}$ min^{-1}.
(Given, vapour pressure of H_2O = 20 torr, ln 2 = 0.3010 and ln 3 = 0.4771)

56. The density of a solution prepared by dissolving 120 g of urea (mol. mass = 60 u) in 1000 g of water is 1.15 g/mL. The molarity of solution is ..... M.
(Nearest integer).

57. 1.4 g of organic compound was digested by Kjeldahl method and the evolved ammonia was absorbed in 70 mL of *M*/10 sulphuric acid. The unreacted acid required 20 mL of *M*/10 sodium hydroxide for complete neutralisation. The percentage of nitrogen in the compound is ..... %.

58. The molarity of HNO_3 in a sample which has density 1.4 g/mL and mass percentage of 63% is ..... M. (Molecular weight of HNO_3 = 63).

59. MY_2 dissociates into M^{2+} and Y^- ions in an aqueous solution with a degree of dissociation, α, of 0.25. The ratio of the observed depression of freezing point of the aqueous solution to the value of the depression of freezing point in the absence of ionic dissociation is ..... $\times 10^{-1}$.

60. A piston filled with 0.04 mole of an ideal gas expands reversibly from 50 mL to 350 mL at a constant temperature of 310 K. It absorbs 208 J of heat during this process. The division of q and W for this process will be $-x$. The value of x is ..... .

MATHEMATICS

Section A : Objective Type Questions

61. The radius of the circle passing through foci of the ellipse $\frac{x^2}{4} + \frac{4}{7}y^2 = 1$ and having its centre at $\left(0, \frac{\sqrt{11}}{2}\right)$ is

(a) $\sqrt{5}$ (b) 3 (c) $\sqrt{12}$ (d) $\frac{7}{2}$

62. $\lim\limits_{x \to 0} \frac{\int_0^x \sin t dt}{x^2}$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{1}{4}$

63. Consider the function $f(x) = \begin{cases} x \sin(\log x^2), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then

(a) f is continuous and differentiable at $x = 0$.
(b) f is not continuous but differentiable at $x = 0$.
(c) f is neither continuous nor differentiable at $x = 0$.
(d) f is continuous but not differentiable at $x = 0$.

64. $\int x^{2/3}(1 + x^{1/2})^{-13/3} dx$

(a) $\frac{3}{5}(1 + x^{-1/2})^{-10/3} + C$ (b) $\frac{3}{5}(1 + x^{1/2})^{-10/3} + C$

(c) $\frac{1 + x^{1/2}}{13} + C$ (d) None of these

65. $\int \frac{dx}{x^2(x^n + 1)^{\frac{n-1}{n}}} = -[f(x)]^{1/n} + C$, then

(a) $f(x) = 1 + x^n$ (b) $f(x) = 1 + x^{-n}$

(c) $f(x) = x^n + x^{-n}$ (d) None of these

66. Let A be a square matrix of order 3 and matrices B, C and D are such that $B = \text{adj}(A), C = \text{adj}(\text{adj}\, A)$, $D = \text{adj}[\text{adj}(\text{adj}\, A)]$.
If $|\text{adj}\{\text{adj}(\text{adj}(\text{adj}\, ABCD)\}]| = |A|^k$, then k is equal to

(a) 128 (b) 265 (c) 256 (d) 240

67. The solution of differential equation $\frac{dy}{dx}=\frac{x(2\log x+1)}{\sin y+y\cos y}$ is

(a) $y\sin y=x\log x+C$
(b) $y^2\sin y=x^2\log x+C$
(c) $y\sin y=x^2\log x+\frac{x^2}{2}+C$
(d) $y\sin y=x^2\log x+C$

68. The line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersect the curve $xy=c^2, z=0$, if c is equal to

(a) $\pm\sqrt{5}$ (b) ±4 (c) $\pm\frac{1}{2}$ (d) $\pm\frac{1}{\sqrt{3}}$

69. If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are non-zero, non-coplanar vectors and

$\mathbf{b}_1=\mathbf{b}-\frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$, $\mathbf{b}_2=\mathbf{b}+\frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$

$\mathbf{c}_1=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}-\frac{\mathbf{c}\cdot\mathbf{b}}{|\mathbf{b}|^2}\mathbf{b}$,

$\mathbf{c}_2=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}-\frac{\mathbf{c}\cdot\mathbf{b}_1}{|\mathbf{b}|^2}\mathbf{b}_1$,

$\mathbf{c}_3=\mathbf{c}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}-\frac{\mathbf{a}\cdot\mathbf{b}_2}{|\mathbf{b}_2|^2}\mathbf{b}_2$, $\mathbf{c}_4=\mathbf{a}-\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$

Then, which of the following is a set of mutually orthogonal vectors?

(a) $\{\mathbf{a},\mathbf{b}_1,\mathbf{c}_1\}$ (b) $\{\mathbf{a},\mathbf{b}_1,\mathbf{c}_2\}$
(c) $\{\mathbf{a},\mathbf{b}_2,\mathbf{a}_3\}$ (d) $\{\mathbf{a},\mathbf{b}_2,\mathbf{c}_4\}$

70. $\int_{-4}^{-5}e^{(x+5)^2}dx+3\int_{\frac{1}{5}}^{\frac{2}{3}}e^{9\left(x-\frac{2}{3}\right)^2}dx$ is equal to

(a) 1 (b) –1 (c) 0 (d) –3

71. The sides of a rhombus $ABCD$ are parallel to the lines $x-y+2=0$ and $7x-y+3=0$. If the diagonals of the rhombus intersect at $P(1,2)$ and the vertex A (different from the origin) is on the Y-axis, then the ordinate of A is p/q. The value of $p+q$, is

(a) 7 (b) 9 (c) 5 (d) 3

72. **Statement I** If $ax^2+bx+c=0$ is a quadratic equation $(a,b,c\in R)$ such that its roots are α,β and $a+b+c<0$, $a-b+c<0$ and $c>0$, then $[\alpha]+[\beta]=-1$, (where $[\cdot]$ denotes greater integer function).

Statement II If for any two real numbers p and q continuous function $f(x)$ is such that $f(p)f(q)<0$ $\Rightarrow f(x)$ has at least one root lying in (p,q).

(a) Statement I is true, Statement II is true, Statement II is a correct explanation of Statement I.
(b) Statement I is true, Statement II is true, Statement II is not a correct explanation of Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

73. The largest term of the sequence $\frac{1}{503},\frac{4}{524},\frac{9}{581},\frac{16}{692},\ldots$ is

(a) 5th (b) 6th (c) 7th (d) 8th

74. If n is a positive integer, then $(\sqrt{5}+1)^{4n}-(\sqrt{5}-1)^{4n}$ is

(a) an odd positive integer
(b) an even positive integer
(c) a rational number other than a positive integer
(d) an irrational number

75. If $[\sqrt{2}\cos x]+[\sin x]=-3$, $x\in[0,2\pi]$

(where $[\cdot]$ denotes the greatest integer function), then x belongs to

(a) $\left(\pi,\frac{5\pi}{4}\right)$ (b) $\left[\pi,\frac{5\pi}{4}\right]$
(c) $\left(\frac{5\pi}{4},2\pi\right)$ (d) $\left[\frac{5\pi}{4},2\pi\right]$

76. The variance of first 25 even natural numbers is

(a) $\frac{202}{3}$ (b) 202 (c) $\frac{208}{3}$ (d) 208

77. If ${}^nP_r={}^nP_{r+1}$ and ${}^nC_r={}^nC_{r-1}$, then the value of r is equal to

(a) 1 (b) 2 (c) 3 (d) 4

78. Four unbiased dice A_1,A_2,A_3,A_4 each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that A_4 shows a number appearing on one of A_1,A_2 and A_3 is

(a) $\frac{108}{216}$ (b) $\frac{91}{216}$ (c) $\frac{125}{216}$ (d) $\frac{127}{216}$

79. L_1 and L_2 are two lines whose vector equations are

$L_1: r=\lambda[(\cos\theta+\sqrt{3})\hat{\mathbf{i}}+(\sqrt{2}\sin\theta)\hat{\mathbf{j}}+(\cos\theta-\sqrt{3})\hat{\mathbf{k}}]$

and $L_2: r=\mu(a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}})$,

where λ and μ are scalars and α is the acute angle between L_1 and L_2. If the angle α is independent of θ, then the value of α is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

80. Let $f(x) = ([\alpha]^2 - 5[\alpha] + 4)x^3 - (6\{\alpha\}^2 - 5\{\alpha\} + 1)x - (\tan x) \times \operatorname{sgn} x$ be an even function for all $x \in R$. Then, sum of all possible values of α is, (where $[\cdot]$ denotes greatest integer function and $\{\cdot\}$ denotes fractional part function).

(a) $\frac{17}{6}$

(b) $\frac{53}{6}$

(c) $\frac{31}{3}$

(d) $\frac{35}{3}$

Section B : Numerical Value Type Questions

81. Number of positive terms in the sequence $S_n = \frac{195}{4 \cdot {}^{n}P_n} - \frac{{}^{n+3}P_3}{{}^{n+1}P_{n+1}}, n \in N$ is ..... .

82. If z is any complex number satisfying $|z - 3 - 2i| \le 2$, then the maximum value of $|2z - 6 + 5i|$ is ..... .

83. Let $\cos^{-1} x + \cos^{-1} 2x + \cos^{-1} 3x = \pi$. If x satisfies the equation $ax^3 + bx^2 + c = 0$, then the value of $a + b + c$ is ..... .

84. Area bounded by the line $y = x$, curve $y = f(x), f(x) > x, \forall x > 1$ and the lines $x = 1, x = t$ is $[t + \sqrt{1 + t^2} - (1 + \sqrt{2})] \forall t > 1$. Then, $f(0)$ is equal to ..... .

85. Find the number of integral values of parameter 'a' for which three chords of the ellipse $\frac{x^2}{2a^2} + \frac{y^2}{a^2} = 1$ $\frac{x^2}{2a^2} + \frac{y^2}{a^2} = 1$ (other than its diameter) passing through the point $P\left(11a, \frac{-a^2}{4}\right)$ are bisected by the parabola $y^2 = 4ax$.

86. Let S_K denote the sum of infinite geometric series, where $K = 1, 2, \ldots, 100$ and first term is $\frac{K-1}{K!}$, common ratio is $\frac{1}{K}$, then the value of $\sum_{K=2}^{100} |(K^2 - 3K + 1)S_K|$ is $3 - \frac{\lambda^2}{99!}$, then λ is equal to ..... .

87. On the curve $y^3 = 27x$, find the total possible number of integral values of y for which the ordinate changes as faster rate than abscissa.

88. Let $\mathbf{a}$ and $\mathbf{b}$ be two unit vectors such that $|\mathbf{a} + \mathbf{b}| = \sqrt{2}$. If $\mathbf{c} = \mathbf{a} + 2\mathbf{b} + 3(\mathbf{a} \times \mathbf{b})$ and $2|\mathbf{c}|$ is equal to K, then $\sqrt{14}\,K$ is .....

89. If $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^8 = \lambda B$, then λ is equal to ..... .

90. The number of quadratic polynomial $ax^2 + 2bx + c$ which satify the following conditions is ..... .

(i) a, b, c are distinct

(ii) $a, b, c \in \{1, 2, 3, \ldots, 1002\}$

(iii) $x + 1$ divides $ax^2 + 2bx + c$

JEE PAPER-3

PHYSICS (Section A: Objective Type Questions)										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans	b	a	a	c	a	c	d	c	c	a
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans	d	c	c	c	c	c	b	a	c	b
	(Section B: Numerical Value Type Questions)									
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans	3.14	6	18.6	6	4	0.45	0.06	1	64	500
CHEMISTRY (Section A: Objective Type Questions)										
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans	a	b	b	a	c	b	a	a	d	b
Que.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
Ans	d	b	b	a	a	b	c	a	c	a
	(Section B: Numerical Value Type Questions)									
Que.	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans	90	156	455	22	14	2	12	14	15	1
MATHEMATICS (Section A: Objective Type Questions)										
Que.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
Ans	a	a	d	a	b	d	d	a	b	c
Que.	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
Ans	a	a	c	d	a	d	b	b	a	d
	(Section B: Numerical Value Type Questions)									
Que.	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
Ans	4	13	25	1	2	10	4	28	128	501000

DETAILED SOLUTIONS

Physics

1. *(b)* As, kinetic energy (K) = constant

$\therefore \quad \Delta K = K_f - K_i = 0$

$\therefore$ From work-energy theorem,

$$W_{net} = \Delta K = 0$$

$$\mathbf{F} \cdot d\mathbf{r} = 0$$

$\therefore \quad \theta = 90°$

$\therefore$ Force is perpendicular to displacement or the velocity of the particle.

2. *(a)* Here, $V_L = V_C = 400\,\text{V}$

$\therefore$ Current passing through inductor and capacitor is same, then $X_L = X_C$

The voltage across resistor,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$\because \quad X_L - X_C = 0$

$\therefore \quad Z = R$

and $V_R = V = 200\,\text{V}$

Now, voltage across resistor is given by

$$V = I\,Z$$

$$200 = I \times 0 \qquad [\because Z = R = 40\Omega]$$

$\therefore \quad I = \dfrac{200}{40} = 5\text{A}$

Therefore, the voltage across terminals of resistance and current through ammeter are 200 V and 5A, respectively.

3. *(a)* Given, $d = 5400\,\text{m}$

$$\rho = 10^3\,\text{kg/m}^3$$

Compressibility $= 45.4 \times 10^{-11}\ \text{Pa}^{-1}$

The pressure at the bottom of the ocean is given by

$$p = \rho g d = 10^3 \times 10 \times 5400 = 54 \times 10^6\,\text{Pa}$$

So, fractional compression = compressibility × pressure

$$= 45.4 \times 10^{-11} \times 54 \times 10^6$$

$$= 2451.6 \times 10^{-5}$$

$$= 2.4516 \times 10^{-2} = 2.4 \times 10^{-2}$$

4. *(c)* As, $f_{\lim} = \mu mg = 0.3 \times 5 \times 10 = 15\,\text{N}$

Applied force (F) = 12 N < $f_{\lim}$

$\therefore$ The friction between the surfaces of block and floor is static friction which is equal to applied force.

$\therefore$ Frictional force (f) = 12 N

5. *(a)* The rms speed of a gas, $v_{rms} = \sqrt{\dfrac{3RT}{M}}$

$$\Rightarrow \quad v_{rms} \propto \frac{1}{\sqrt{M}}$$

As $\quad M_{H_2} < M_{O_2}$

$$\Rightarrow \quad v_{rms,H_2} > v_{rms,O_2}$$

6. *(c)* Least count of the screw gauge,

$$= \frac{1\,\text{mm}}{100} = 0.01\,\text{mm}$$

Diameter of wire = $1 \times 1 + 47 \times 0.01$

$$= 1.47\,\text{mm} = 0.147\,\text{cm}$$

Curved surface area of wire (A) = $2\pi rL$

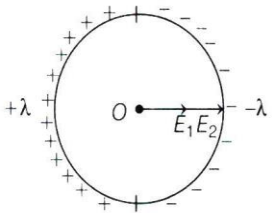
$$= 2\pi \frac{D}{2} \times L = 3.14 \times 0.147 \times 5.6$$

$$= 2.5848\,\text{cm}^2$$

As length of wire is upto one decimal place,

$\therefore \quad A = 2.6\,\text{cm}^2$

7. *(d)* The given arrangement of charged ring can be shown as



The electric field due to a charged semi-circular ring ($+\lambda$) is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}$$

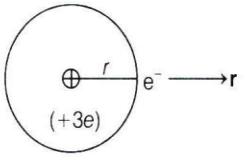
$\therefore$ Net electric field at the centre O is

$$E_{net} = E_1 + E_2 = 2E_1 \qquad \left[\because E_1 = E_2 = \frac{\lambda}{2\pi\varepsilon_0 R}\right]$$

$$= 2 \times \frac{\lambda}{2\pi\varepsilon_0 R} = \frac{\lambda}{\pi\varepsilon_0 R}$$

8. *(c)* The Maxwell's equations of electromagnetism and Hertz experiment on the generation and detection of electromagnetic waves in 1887, strongly established the wave nature of light.

9. *(c)* Li^{2+} atom is shown as



The coulomb force **F** between nucleus and outermost e^-,

$$\mathbf{F} = K\frac{(3e)(e)}{r^2}(-\hat{\mathbf{r}})$$

$$\Rightarrow \quad \mathbf{F} = -3K\frac{e^2}{r^3}\cdot\mathbf{r} \qquad \left[\because \hat{\mathbf{r}} = \frac{\mathbf{r}}{r}\right]$$

10. *(a)* For NOT gate, $Y = \overline{A}$

At $A = 0, Y = 1$ (invertion of input A)

At $A = 1, Y = 0$ (invertion of input A)

For AND gate, $Y = A.B$

At $A = 0, B = 0, Y = 0$ (same as that of A, B)

At $A = 1, B = 1, Y = 1$ (same as that of A, B)

11. *(d)* The equivalent emf of series combination of cells is the sum of their individual emf.

Statement I is false.

For series combination of two cells of emf E_1 and E_2, internal resistance r_1 and r_2, if polarity of E_1 is reversed, then

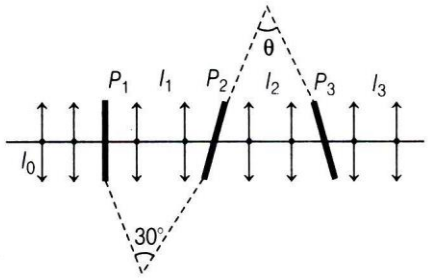
Equivalent emf $= |E_1 - E_2|$ (Affected)

Equivalent resistance $= r_1 + r_2$ (Unaffected)

$\therefore$ Statement II is true.

12. *(c)* Given, angle between P_1 and $P_2 = 30°$

Angle between P_2 and $P_3 = \theta = 90° - 30° = 60°$



The intensity of light transmitted by P_1,

$$I_1 = \frac{I_0}{2} = \frac{40}{2} = 20\frac{\text{W}}{\text{m}^2}$$

According to Malus law, the intensity of light transmitted by P_2 is

$$I_2 = I_1 \cos^2 30° = 20\left(\frac{\sqrt{3}}{2}\right)^2 = 15\frac{\text{W}}{\text{m}^2}$$

Similarly, intensity of light transmitted by P_3,

$$I_3 = I_2 \cos^2\theta = 15\cos^2 60°$$

$$= 15\times\left(\frac{1}{2}\right)^2 = 3.75\ \text{W/m}^2$$

13. *(c)* $g = \frac{GM}{R^2}$, M = Mass of the earth.

The acceleration due to gravity depends on the mass of the Earth and is independent of mass of the body.

The escape velocity of a body on the Earth surface is

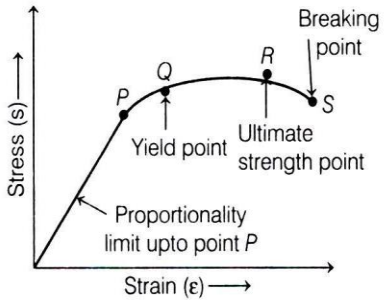
$$v_e = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2gR_e}$$

which is dependent on the radius of the Earth.

Inside a uniform spherical shell, the gravitational field is zero everywhere, i.e. gravitational potential is same everywhere.

The speed of a planet in planetary motion varies due to elliptical shape of the planet's orbit.

14. *(c)* Typical graph of metal under tension is



15. *(c)* Magnetic field at point O due to wires CB and DA will be zero.

So, magnetic field due to wire AB will be

$$B_1 = \left(\frac{\theta}{2\pi}\right)\left[\frac{\mu_0 I}{2(2R)}\right]$$

$$= \frac{\pi}{2\pi}\left[\frac{\mu_0 I}{4R}\right] = \frac{\mu_0 I}{24R}$$

Direction of field B_1 is going into the plane of paper.

Similarly, magnetic field due to wire CD will be

$$B_2 = \frac{\mu_0 I}{36R}$$

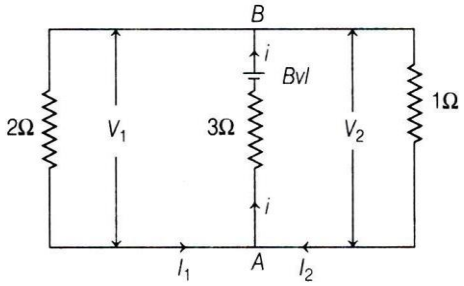
Direction of field B_2 is coming out of the plane of paper.

The resultant field at point O,

$$B = B_1 - B_2$$

$$= \frac{\mu_0 I}{24R} - \frac{\mu_0 I}{36R} = \frac{3\mu_0 I - 2\mu_0 I}{72R} = \frac{\mu_0 I}{72R}$$

16. *(c)* The circuit diagram is shown as



Equivalent resistance between points A and B,.

$$R_{AB} = \frac{2\times 1}{2+1} + 3 = \frac{11}{3}\Omega$$

$$\therefore \quad i = \frac{Bvl}{R_{AB}} = \frac{5\times 2\times 1}{\frac{11}{3}} = \frac{30}{11}\text{A}$$

$$\therefore \quad V_1 = V_2 = -i\times 3 + Bvl$$

$$= -\frac{30}{11}\times 3 + 5\times 2\times 1 = \frac{20}{11}\text{V}$$

17. *(b)* Volume of sphere, $V = \frac{4}{3}\pi r^3$

$\therefore$ Density of sphere, $d = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3}$

$\therefore \quad \frac{\Delta d}{d} = \frac{\Delta m}{m} + 3\left(\frac{\Delta r}{r}\right)$

$= \frac{0.30}{15} + 3\times\frac{0.01}{0.20} = 0.17\,\text{kgm}^{-3}$

18. *(a)* $\mathbf{B} = 2.1\times10^{-8}\,\hat{\mathbf{k}}$

$\therefore\ E = cB = 3\times10^8\times2.1\times10^{-8} = 6.3$

As EM wave is in $+x$-direction, **B** is in $+z$-direction.
Hence, **E** will be in $+y$-direction.

Here, $\mathbf{E} = 6.3\hat{\mathbf{j}}\,\text{V/m}$

19. *(c)* As, we know that, potential energy, $U = 2E$, kinetic energy, $K = -E$

and total energy of electron, $E = -\frac{13.6}{n^2}$

For state $n = 1$, $E_1 = \frac{-13.6}{(1)^2} = -13.6$

For state $n = 2$, $E_2 = \frac{-13.6}{(2)^2} = \frac{E_1}{4} \Rightarrow E_1 = 4E_2$

But $K = -E$, so $K_1 = 4K_2$

Similarly, for state $n = 1$, potential energy,

$$U_1 = 2E_1 = 2\times\frac{-13.6}{(1)^2} = -2\times13.6$$

For state $n = 2$, potential energy,

$$U_2 = 2E_2 = 2\times\frac{-13.6}{(2)^2} = \frac{-2\times13.6}{4}$$

$$U_2 = \frac{U_1}{4} \Rightarrow U_1 = 4U_2$$

Thus, both kinetic energy K and potential energy U become four times.

20. *(b)* In first case, $\frac{X}{Y} = \frac{60}{40}$ or $\frac{X}{Y} = \frac{3}{2}$ ...(i)

In second case,

$$\frac{X}{Y+15} = \frac{60-10}{40+10} = \frac{50}{50} = 1 \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{X}{Y}\times\frac{Y+15}{X} = \frac{3}{2}\times1$$

or $\quad 1+\frac{15}{Y} = \frac{3}{2}$

$$Y = 30\Omega$$

or $\quad X = \frac{3}{2}Y = \frac{3}{2}\times30 = 45\Omega$

When a resistance of 30Ω is connected in parallel with Y, then the resistance in the right gap becomes

$$Y' = \frac{30Y}{30+Y} = \frac{30\times30}{30+30} = 15\Omega$$

Suppose, the null point occurs at l cm from end A.

Then, $\frac{X}{Y'} = \frac{l}{100-l}$

$$\frac{45}{15} = \frac{l}{100-l}$$

$$300-3l = l \Rightarrow 4l = 300$$

$$l = 75\,\text{cm}$$

21. *(3.14)* As we know,

$$v = A\omega\cos\omega t$$

$$v = A\omega\sqrt{1-\sin^2\omega t}$$

$$\Rightarrow\quad v = A\omega\sqrt{1-\frac{x^2}{A^2}}$$

$$\Rightarrow\quad v = A\omega\frac{1}{A}\sqrt{A^2-x^2}$$

$$\Rightarrow\quad v = \omega\sqrt{A^2-x^2}$$

Here, $v_1 = 10\,\text{cm/s}$, $x_1 = 4\,\text{cm}$ and $v_2 = 8\,\text{cm/s}$, $x_2 = 5\,\text{cm}$. Then,

$$v_1 = \omega\sqrt{A^2-x_1^2} \quad \text{... (i)}$$

and $\quad v_2 = \omega\sqrt{A^2-x_2^2} \quad \text{... (ii)}$

From Eqs. (i) and (ii), we get

$$\frac{v_1}{v_2} = \sqrt{\frac{A^2-x_1^2}{A^2-x_2^2}}$$

$$\left(\frac{v_1}{v_2}\right)^2 = \frac{A^2-x_1^2}{A^2-x_2^2}$$

$$\frac{100}{64} = \frac{A^2-16}{A^2-25}$$

$$\frac{25}{16} = \frac{A^2-16}{A^2-25}$$

$$\Rightarrow\quad A^2 = 41\,\text{cm}$$

From Eq. (i),

$$\omega = \frac{v_1}{\sqrt{A^2-x_1^2}}$$

$$= \frac{10}{\sqrt{41-16}}$$

$$\omega = \frac{10}{5} = 2$$

$$\frac{2\pi}{T} = 2$$

$$\Rightarrow\quad T = 3.14\,\text{s}$$

22. *(6)* From given diagram in question,

$$\int_A^B \mathbf{B}_P d\mathbf{l} + \int_B^C \mathbf{B}_P d\mathbf{l} + \int_C^D \mathbf{B}_P d\mathbf{l} + \int_D^A \mathbf{B}_P d\mathbf{l} + \int_A^B \mathbf{B}_Q d\mathbf{l} + \int_B^C \mathbf{B}_Q d\mathbf{l} + \int_C^D \mathbf{B}_Q d\mathbf{l} + \int_D^A \mathbf{B}_Q d\mathbf{l} = \mu_0 (2I - I)$$

$$\Rightarrow \quad (-\mu_0 - 2\mu_0 - \mu_0 - 2\mu_0) + (2\mu_0 + 4\mu_0 + 2\mu_0 + 4\mu_0) = \mu_0 I$$

$$\Rightarrow \quad 6\mu_0 = \mu_0 I$$

$$\Rightarrow \quad I = 6\,\text{A}$$

23. *(18.6)* Given, $A = 30°$, $\mu = 1.5$ and $i_1 = 0°$

Since, $i_1 = 0°$, therefore, r_1 is also equal to 0°.

Further, since, $r_1 + r_2 = A$

$\therefore \quad r_2 = A = 30°$

Using, $\mu = \dfrac{\sin i_2}{\sin r_2}$

we have, $1.5 = \dfrac{\sin i_2}{\sin 30°}$

or $\quad \sin i_2 = 1.5 \sin 30° = 1.5 \times \dfrac{1}{2} = 0.75$

$\therefore \quad i_2 = \sin^{-1}(0.75) = 48.6°$

Now, the deviation,

$$\delta = (i_1 + i_2) - A = (0° + 48.6°) - 30°$$

or $\quad \delta = 18.6°$

24. *(6)* The potential energy of parallel plate capacitor,

$$U = \frac{q^2}{2C}$$

$$\therefore (\text{WD})_{\text{external}} = \Delta U = U_f - U_i$$

$$= \frac{q^2}{2}\left(\frac{1}{C_f} - \frac{1}{C_i}\right)$$

$$= \frac{(15 \times 10^{-6})^2}{2}\left(\frac{1}{6} - \frac{1}{10}\right) \times 10^6$$

$$= 7.5 \times 10^{-6}\ \text{J}$$

According to question,

$$7.5 \times 10^{-x} = 7.5 \times 10^{-6}$$

$\therefore \quad x = 6$

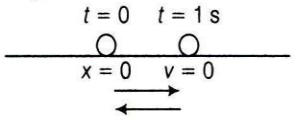
25. *(4)* In expression, $x = 4t - 2t^2$ the x represents displacement not distance.

$$x = 4t - 2t^2$$

Taking differentiation w.r.t. t,

$$v = 4(1 - t)$$

At $t = 1, v = 0$



The velocity of the particle become zero at $t = 1$ s and then it moves towards the negative direction.

$\Rightarrow$ x_1 for 1st second

$$= 4 - 2 \times 1 = +2\,\text{m}$$

$\Rightarrow$ x_2 for 2nd second $= 4 \times 2 - 2 \times 4$

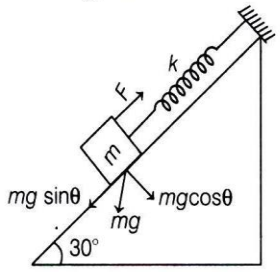
$$= 8 - 8 = 0$$

So, first particle has covers 2 m and then it comes back 2 m.

So, the total distance covered = 2 + 2 = 4 m

26. *(0.45)* The force required to displace the mass by 2.5 m is equal to the restoring force F.

$$F = mg \sin\theta$$



$\therefore$ Spring constant,

$$k = \frac{F}{x} = \frac{mg \sin\theta}{x}$$

New time period,

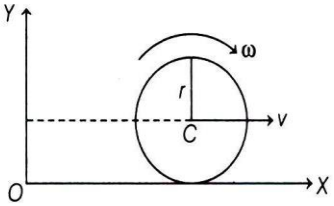
$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{m}{mg \sin\theta / x}}$$

$$= 2\pi\sqrt{\frac{x}{g \sin\theta}} = 2\pi\sqrt{\frac{2.5 \times 10^{-2}}{9.8 \times \sin 30°}}$$

$$= 2\pi \times \frac{5}{7} \times \frac{1}{10} = 0.45\,\text{s}$$

27. *(0.06)*



The total angular momentum of disc about O is given by

$$L_{\text{net}} = L_T + L_R$$

$$= mv \times r + I_{\text{CM}}\omega$$

$$= m(r\omega) \cdot r + \frac{1}{2}mr^2 \cdot \omega$$

$$= \frac{3}{2}mr^2\omega$$

$$= \frac{3}{2} \times 2 \times (5 \times 10^{-2})^2 \times 8 = 0.06\ \text{kg-m}^2/\text{s}$$

28. (1) In an adiabatic process,

$$T \propto p^{1-\frac{1}{\gamma}}$$

$$\Rightarrow \quad T \propto p^{\frac{1}{3}} \qquad [\because \gamma = 1.5]$$

If T_L and T_R are final temperatures of gases in left and right compartments respectively, p_L and p_R are respective pressures, then

the ratio of internal energy U_L and U_R is

$$\frac{U_L}{U_R} = \frac{\frac{f}{2}nRT_L}{\frac{f}{2}nRT_R} = \frac{T_L}{T_R}$$

Using Eq. (i), then

$$\Rightarrow \quad \frac{U_L}{U_R} = \left(\frac{p_L}{p_R}\right)^{\frac{1}{3}} = \left(\frac{1}{8}\right)^{\frac{1}{3}} = \frac{1}{2} = \frac{x}{2} \qquad \text{(given)}$$

$$\therefore \quad x = 1$$

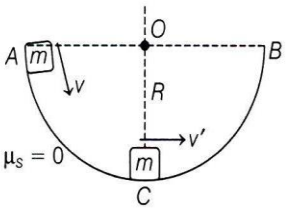
29. (64) At the lowest point,

$$N_C = mg + \frac{mv'^2}{R} \qquad \ldots(i)$$

From mechanical energy conservation,

$$\frac{1}{2}mv^2 + mgR = \frac{1}{2}mv'^2$$

$$\Rightarrow \quad \frac{mv^2}{R} + 2mg = \frac{mv'^2}{R} \qquad \ldots(ii)$$



From Eqs. (i) and (ii), we get

$$N_C = mg + \left(\frac{mv^2}{R} + 2mg\right)$$

$$= 3mg + \frac{mv^2}{R}$$

$$= 3\times 2\times 10 + \frac{2\times 4\times 4}{8} = 64\ \text{N}$$

30. (500) According to Faraday's law of electromagnetic induction,

$$e = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad iR = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad \int d\phi = \int iR dt = R\int i dt$$

$$\Rightarrow \quad \Delta\phi = R \times \text{Area under the curve of } i \textit{ versus } t \text{ graph}$$

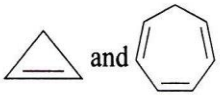
$$= 200\times\frac{1}{2}\times 0.5\times 10$$

$$= 500\ \text{Wb}$$

$$\therefore \quad x = 500$$

Chemistry

31. (*a*) **Acidic character** The species which easily donate its hydrogen and produces stable conjugate base is acid. The species which produces more stable conjugate base is more stronger acid.

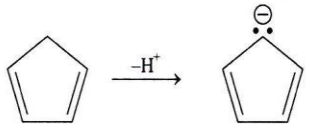
 and do not lose H^+.

Hence, these are not acidic.

 loses the H^+ ion but does not produces a stable aromatic ion. i.e. 

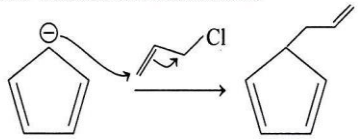
(Non-aromatic)

 loses the H^+ ion easily and produces stable aromatic cyclopentadienyl anion.



Non-aromatic — Aromatic (Cyclopentadienyl anion)

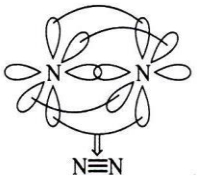
Now, cyclopentadienyl anion on reaction with 3-chloroprop-1-ene produces the product *via* nucleophilic substitution reaction.

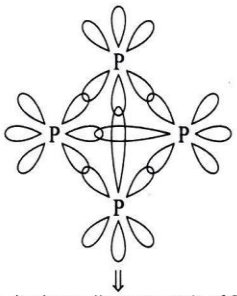


Cyclopentadienyl anion — Product

32. (*b*) Existence of nitrogen as N_2 is due to strong $p\pi$- $p\pi$ bonding between smaller sized *p*-orbitals of N.

Existence of phosphorus as P_4 is due to existence of weak $p\pi$- $p\pi$ bonding due to large size of *p*-orbital of phosphorous atom.





Thus, it show discrete unit of P_4, (due to large size of *p*-orbital or phosphorus)

33. (*b*) This pair of complex shows ionisation isomerism as follows :

$[Co(NH_3)_5Br]SO_4 \longrightarrow [Co(NH_3)_5Br]^{2+} + SO_4^{2-}$

$[Co(NH_3)_5SO_4]Br \longrightarrow [Co(NH_3)_5SO_4]^{+} + Br^{-}$

Study Tactics

Greater the charge on ions, produced by coordination compound on dissociation, greater will be its molar conductivities. Molecule having different charge on ions have different molar conductivity.

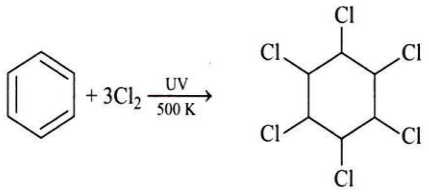
34. (*a*) The correct match is

A → (III), B → (IV), C → (II), D → (I).

- Alcoholic KOH is used for β-elimination reaction.

$$\overset{+-}{KOH} + CH_3CH_2OH \longrightarrow \underset{\text{Acts as a base to abstract β-hydrogen}}{CH_3CH_2\bar{O}} + H_2O$$

- $Pd/BaSO_4$ known as Lindlar's catalyst.
- Benzene hexachloride (BHC) is obtained by addition reaction.



Benzene hexachloride (BHC)

- $LiAlH_4$ is a strong reducing agent.

35. (*c*) The reaction between Grignard reagent and ketone produces tertiary alcohol as a product.

The reaction is carried out in a single step and involves the formation of a six-membered transition state, which is formed when the Grignard reagent attacks the carbonyl centre of the ketone.

36. (*b*) The correct match is A-I, B-II, C-III, D-IV

A. $FeCl_3 + 2S_2O_3^{2-} \longrightarrow \underset{\text{Violet solution}}{[Fe(S_2O_3)_2]^{-}} + 3Cl^{-}$

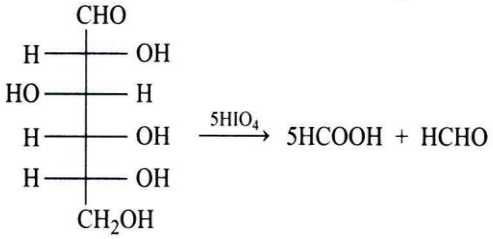
$$[Fe(S_2O_3)_2]^{-} + Fe^{3+} \longrightarrow \underset{\text{(Green)}}{2Fe^{2+}} + S_4O_6^{2-}$$

B. $Cr_2O_7^{2-} + 8H^{+} + 3SO_3^{2-} \longrightarrow \underset{\text{(Green)}}{2[Cr(H_2O)_6]^{3+}} + 3SO_4^{2-} + 4H_2O$

C. The Beilstein test is a simple qualitative chemical test for organic halides.

D. Barfoed's test is a chemical test used for detecting the presence of monosaccharides. It is based on the reduction of copper (II) acetate to copper (I) oxide (Cu_2O), which forms a brick red precipitate.

37. (*a*) Glucose reacts with excess of HIO_4 to form 5 moles of formic acid and 1 mole of formaldehyde.



38. (*a*) $\Lambda^{\circ}_m(NaCl) = 126.4\ S\ cm^2\ mol^{-1}$

$\Lambda^{\circ}_m(HCl) = 425.9\ S\ cm^2\ mol^{-1}$

$\Lambda^{\circ}_m(NaA) = 100.5\ S\ cm^2\ mol^{-1}$

$\Lambda^{\circ}_m(HA) = \Lambda^{\circ}_m(HCl) + \Lambda^{\circ}_m(NaA) - \Lambda^{\circ}_m(NaCl)$

$= 425.9 + 100.5 - 126.4$

$= 400\ S\ cm^2\ mol^{-1}$

$$\Lambda_m = \frac{\kappa(HA)\times 1000}{\text{Molarity of } HA} = \frac{5\times 10^{-5}\times 1000}{0.001}$$

$= 50\ S\ cm^2\ mol^{-1}$

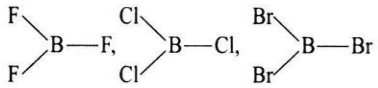
$$\alpha = \frac{\Lambda_m(HA)}{\Lambda^{\circ}_m(HA)} = \frac{50}{400} = 0.125$$

$= 125\times 10^{-3}$

39. (*d*) As, trihalides of boron have same structure due to same hybridisation

$$H = \frac{V + M - C + A}{2} = \frac{3+3}{2} = 3$$

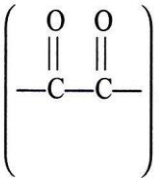
Hybridisation = sp^2



40. (*b*) **Molecular structure determination** Molecular structure of compound having molecular formula $C_{14}H_{10}O_2$ is determined by calculating degree of unsaturation.

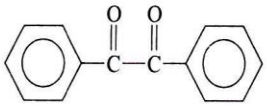
$$u = (C+1) - \frac{H}{2} - \frac{X}{2} + \frac{N}{2} = (14+1) - \frac{10}{2}$$

$= 15 - 5 = 10$

Degree of unsaturation is 10 in which 2 units are already considered to be used as diketo group  .

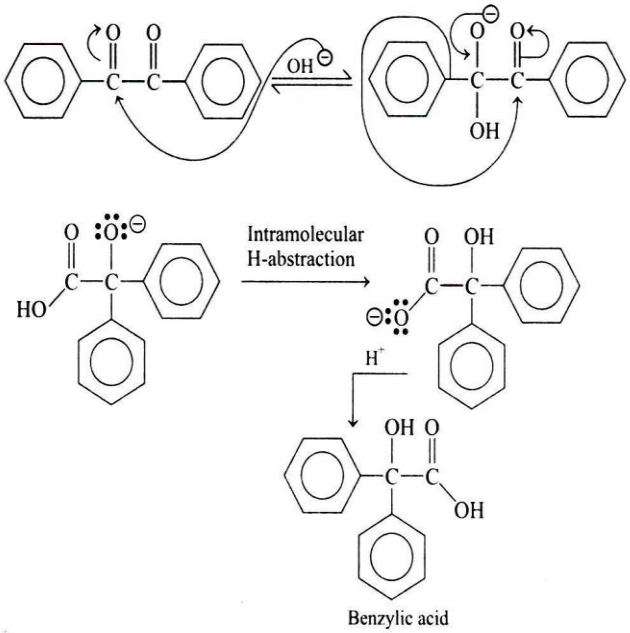
Rest 8 unit of unsaturation is satisfied by two phenyl ring each having, $u = 4$.

Hence, correct structure is



Benzilic acid rearrangement

Conversion of benzil to benzylic acid in presence of base is known as benzilic acid rearrangement. In general benzilic acid is α-hydroxycarboxylic acid. The reaction is believed to occur as



Benzylic acid

Study Tactics

This problem include conceptual mixing of molecular structure determination and benzilic acid rearrangement. This problem can be solved by using following sequential step.

I. Calculate the degree of unsaturation and then determine the appropriate molecular structure.

II. Complete the reaction using the concept of benzilic acid rearrangement in which diketone undergo benzilic acid rearrangement in presence of base to produce corresponding benzilic acid.

41. (*d*) • $_{24}Cr = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^5, 4s^1$

$= [Ar]\, 3d^5, 4s^1$

- For magnetic quantum number (m), negative values are possible.

  For, s-subshell, $l = 0$, hence $m = 0$.

  For p-subshell, $l = 1$, hence $m = -1, 0, +1$

- $_{47}Ag = 1s^2, 2s^2\, 2p^6\, 3s^2\, 3p^6\, 3d^{10}, 4s^2\, 4p^6\, 4d^{10}\, 5s^1$

Hence, 23 electrons have a spin of one type and 24 of the opposite type.

- oxidation state of N in N_3H is $-1/3$.

42. (*b*) When aqueous solution of NaOH is added to aqueous solution of Cr (III) ion it produces $[Cr(H_2O)_3(OH)_3]$.

$$[Cr(H_2O)_6]^{3+} + NaOH \longrightarrow \underset{\text{Green blue ppt.}}{[Cr(OH)_3(H_2O)_3]} + H_2O$$

Which on further redissolves in *aq.* NaOH (in excess) to produce dark green solution due to formation of $[Cr(OH)_4]^-$

$$[Cr(OH)_3\ (H_2O)_3] + \underset{\text{Excess}}{OH^-} \longrightarrow \underset{\text{Dark green solution}}{[Cr(OH)_4]^-}$$

43. (*b*) Both A and R are correct but R is not the correct explanation of A.

44. (*a*) The reactions of $C_2H_5NH_2$ with given reagents are given below.

$$C_2H_5NH_2 \xrightarrow{C_6H_5CHO} \underset{(A)}{C_2H_5{-}N{=}CHC_6H_5}$$

$$C_2H_5NH_2 \xrightarrow{NOCl} \underset{(B)}{C_2H_5Cl} + N_2 + H_2O$$

$$C_2H_5NH_2 \xrightarrow{C_6H_5SO_2Cl} \underset{(C)}{C_6H_5SO_2NHC_2H_5} + HCl$$

Here, in the first reaction, (A) is Schiff's base because it resembles with general structure of Schiff's base, i.e. $R_2C = NR'$.

45. (*a*) Due to $+I$-effect of the alkyl group, electron density increases in alkylbenzene relative to benzene and hence further substitution occurs to form a poly alkyl substituted benzene. Thus, Statement I is correct.

Now, the C-atoms of alkyl halides are less electrophilic than the C-atom of carbonyl.

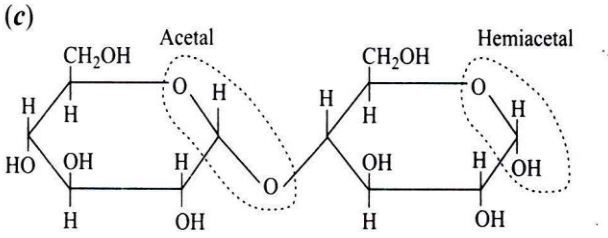
So, alkyl halides are less reactive than acyl halides.

46. (*b*) $Be_2(8) = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2$

$$\text{Bond order} = \frac{4-4}{2} = 0$$

So, no bond is formed between two atoms of Be. Hence, Be_2 does not exist.

47. (*c*)



One acetal and one hemiacetal group is present in maltose.

48. (*a*) The reaction is represented as

$$3B_2H_6 + 6NH_3 \longrightarrow 3[BH_2(NH_3)_2]^+[BH_4]^- \longrightarrow \underset{(X)}{2B_3N_3H_6} + 12H_2$$

The chemical formula of X is $B_3N_3H_6$.

49. (*c*) The decreasing order of acidic strength of given compound is I > II > III.

The benzoate ion is more stabilised because the negative charge is on the more electronegative oxygen atom, whereas in phenoxide ion, it is on the less electronegative carbon atoms. This causes benzoic acid to be a stronger acid than phenol.

The alkyl group attached to *para* position to carboxylic group in aromatic carboxylic acid reduces the acidity. Similarly, electron releasing group, i.e. —CH_3 present in the para position to —OH group decreases its acidity.

Study Tactics

The more stable is the conjugate base the stronger is the acid. Also, ERG decreases the acidity while EWG increases the acidity.

50. (*a*) In case of transition elements (or any other elements), the order of filling of electrons in various orbital is $3p > 4s > 3d$.

So, according to Aufbau principle, 3*d*-orbital is filled when 4*s*-orbital gets completely filled.

Hence, among the given statements, only (C) is incorrect while (A) and (B) are correct.

51. (*90*) $I_2 + 2Na_2S_2O_3 \longrightarrow 2NaI + Na_2S_4O_6$ ...(i)

Moles of $Na_2S_2O_3$ consumed

$= 0.03 \times 0.2 = 0.006$ moles

Moles of I_2 consumed $= \frac{0.006}{2} = 0.003$ moles

$3I_2 + 6NaOH \longrightarrow 5NaI + NaIO_3 + 3H_2O$ ...(ii)

Moles of I_2 reacted with NaOH,

$$\frac{3 \times 0.05 \times 0.6}{6} = 0.015 \text{ moles}$$

Total moles of I_2 consumed in reactions (i) and (ii)

$= 0.003 + 0.015 = 0.018$ moles

Molarity of $I_2 = \frac{0.018}{0.2} = 0.09$ M

$= 90 \times 10^{-3}$ M

52. (*156*) $K_f = 5.12$ K kg mol^{-1}

$W_{solute} = 0.643$ g

$\Delta T_f = T_0 - T_f$

$= 5.51 - 5.03 = 0.48$ K

$W_{solvent} = 43.95$ g

$\therefore \left[d = \frac{M}{V}\right] = (V \times d = 50 \text{ mL} \times 0.879)$

$$M_{solute} = \frac{K_f \times w_{solute} \times 1000}{\Delta T_f \times W_{solvent}}$$

$$= \frac{5.12 \times 0.643 \times 1000}{0.48 \times 43.95} = 156 \text{ kg mol}^{-1}$$

53. (*455*) $NH_3(g) + HCl(aq) \rightleftharpoons NH_4Cl(aq)$

$\Rightarrow$ 1 mole of NH_3 = 1 mole of HCl

100 mL of 1 M HCl ≡ 100 millimole of HCl

= 0.1 mole of HCl

= 0.1 mole of NH_3

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$

Initially	1	3	0
At equilibrium	$1-x$	$3-3x$	$2x$

$\Rightarrow \quad 2x = 0.1,$

i.e. $x = \frac{0.1}{2} = 0.05$ mol

and $V = 1$ L

Therefore, $K_C = \frac{[NH_3]^2}{[N_2][H_2]^3}$

$$= \frac{\left(\frac{2x}{V}\right)^2}{\left(\frac{1-x}{V}\right)\left(\frac{3-3x}{V}\right)^3}$$

$$= \frac{4x^2V^2}{27(1-x)^4}$$

$$= \frac{4 \times (0.05)^2 \times (1)^2}{27(1-0.05)^4}$$

$= 4.55 \times 10^{-4}$

$= 455 \times 10^{-6}$

54. (22) The given cell is a concentration cell.

$\therefore \quad E^{\ominus}_{cell} = 0.0$ V

The half-cell reactions are

Anode reaction

$$Ag \text{ (anode)} \rightleftharpoons Ag^{\oplus}(0.1 \text{ M}) + e^-$$

Cathode reaction

$Ag^{\oplus}$ (cathode) $+ e^- \rightleftharpoons Ag(s)$(cathode)

Cell reaction

$Ag^{\oplus}$ (cathode) $\rightleftharpoons Ag^{\oplus}$ (0.1 M) anode

Since, $AgNO_3$ is 80% ionised.

$\therefore [Ag^{\oplus}]_a = 0.1 \times \frac{80}{100} = 0.08$ M

$\therefore \quad E_{cell} = E^{\ominus}_{cell} - \frac{0.06}{1} \log \frac{[Ag^{\oplus}]_a}{[Ag^{\oplus}]_c}$ (Take $0.059 \approx 0.06$)

$-0.6 \text{ V} = 0 - 0.06 \log \frac{0.08}{[Ag^{\oplus}]_c}$

$\log [Ag^{\oplus}]_c = \log 0.08 - 10$

$= (\log 2^3 - \log 100) - 10$

$= 0.9 - 2 - 10 = -11.1$

$\therefore \quad [Ag^{\oplus}]_c = \text{Antilog}(-11.1)$

$= 7.9 \times 10^{-12}$

$\approx 8 \times 10^{-12}$ M

Since, KBr is 60% dissociated.

$$\therefore \quad [Br^{\ominus}] = 1 \times \frac{60}{100} = 0.6\ M$$

$$\therefore \quad K_{sp} = [Ag^{\oplus}][Br^{\ominus}]$$

$$= 8 \times 10^{-12} \times 0.6$$

$$= 4.8 \times 10^{-12}\ M^2$$

$$\text{Solubility } (S) = \sqrt{K_{sp}} = \sqrt{4.8 \times 10^{-12}\ M^2}$$

$$= 2.190 \times 10^{-6}\ M$$

$$\simeq 2.2 \times 10^{-6}\ M$$

$$\simeq 22 \times 10^{-7}\ M$$

55. *(14)*

$$A(g) \longrightarrow 2B(g) + C(g)$$

After 20 mins	$p_0 - x$	$2x$	x
After long time, $t \to \infty$	0	$2p_0$	p_0

As given, $p_0 - x + 2x + x +$ vapour pressure of water $= 200$

$$p_0 + 2x + 20 = 200$$

$$p_0 + 2x = 180 \qquad \ldots(i)$$

and $\quad 2p_0 + p_0 + 20 = 380$

$$3p_0 = 360$$

$$p_0 = 120\ \text{torr} \qquad \ldots(ii)$$

On substituting in Eq, (i) we get

$$120 + 2x = 180$$

$$2x = 60$$

$$x = 30\ \text{torr}$$

$$k = \frac{1}{t} \ln \left(\frac{p_0}{p_0 - x} \right) = \frac{1}{20} \ln \left(\frac{120}{90} \right)$$

$$k = \frac{1}{20} (\ln 12 - \ln 9)$$

$$= \frac{1}{20} (\ln 4 + \ln 3 - 2 \ln 3)$$

$$= 0.014\ \text{min}^{-1}$$

$$= 14 \times 10^{-3}\ \text{min}^{-1}$$

56. *(2)* $\text{Molarity} = \dfrac{\text{Moles of solute}}{\text{Volume of solution (in L)}}$

Mass of solution $= 1000 + 120 = 1120$ g

$$V = \frac{\text{Mass}}{\text{Density}}$$

$$\text{Molarity} = \frac{120 \times 1.15}{60 \times 1120} \times 1000 = 2.05\ M \approx 2M$$

57. *(12)* The formula used for the estimation of N percentage in an organic compound is

$$\%\ \text{of N} = \frac{1.4 \times \text{m eq. of acid}}{\text{mass of organic compound}}$$

$$\text{meq. of sulphuric acid} = 70 \times \frac{M}{10} \times 2 = 14$$

$$\text{meq. of sodium hydroxide} = 20 \times \frac{M}{10} = 2$$

Since, out of 14g, 2g of acid was unreacted, so the total gram equivalent of ammonia $= 14 - 2 = 12$ g

Now, % of N $= (1.4 \times 12)/1.4 = 12\%$

58. *(14)* For 100 g of sample of the solution,

Mass of $HNO_3 = 63$ g

$$\therefore \quad n_{HNO_3} = \frac{63}{63} = 1\ \text{mol}$$

$$\text{Volume of the solution} = \frac{\text{Mass}}{\text{Density}}$$

$$= \frac{100\ \text{g}}{1.4\ \text{g/mL}}$$

$$= \frac{100}{1.4}\ \text{mL} = \frac{1}{14}\ \text{L}$$

$$\text{Molarity} = \frac{n}{V(\text{L})} = \frac{1}{(1/14)} = 14\ M$$

59. *(15)* The given compound disssociates as follows

$$\underset{(1-\alpha)}{MY_2} \rightleftharpoons \underset{\alpha}{M^{2+}} + \underset{2\alpha}{2Y^-}$$

Now, total number of moles (after dissociation)

$$= 1 - \alpha + \alpha + 2\alpha$$

$$= 1 + 2a = 1 + 2(0.25)$$

$$= 1 + 0.5 = 1.5$$

Thus, total number of moles after dissociation = 1.5 and number of moles before dissociation of $MY_2 = 1$

$$\therefore \quad i = \frac{\text{Total no. of moles after dissociation}}{\text{No. of moles before dissociation}}$$

$$= \frac{1.5}{1} = 1.5$$

$$= 15 \times 10^{-1}$$

60. *(1)* As we know, $\Delta U = q + W$, and the process is reversibly isothermal and expansion occurs, here.

Therefore,

$$q = -W$$

and $\Delta U = 0$

$$W = -2.303\, nRT \log \frac{V_2}{V_1}$$

$$= -2.303 \times 0.04 \times 8.314 \times 310 \times \log \frac{350}{50}$$

$$= -237.42 \times \log 7$$

$$= -200.64$$

So, $\quad q = +200.64$

The division of q and $W = \dfrac{-200.64}{200.64} = -1$

Mathematics

61. *(a)* Given equation of ellipse is $\frac{x^2}{4}+\frac{y^2}{7/4}=1$

Here, $a^2=4$ and $b^2=\frac{7}{4}$

$\therefore \quad b^2=a^2(1-e^2)$

$\Rightarrow \quad \frac{7}{4}=4(1-e^2)$

$\Rightarrow \quad e^2=1-\frac{7}{16}=\frac{9}{16}\Rightarrow e=\frac{3}{4}$

Thus, foci are $\left(\pm\frac{3}{2},0\right)$.

The radius of required circle

$$=\sqrt{\left(\pm\frac{3}{2}-0\right)^2+\left(0-\frac{\sqrt{11}}{2}\right)^2}=\sqrt{5}$$

62. *(a)* $\lim_{x\to 0}\frac{\int_0^x \sin t\,dt}{x^2}$ $\left[\frac{0}{0}\text{ form}\right]$

$=\lim_{x\to 0}\frac{\sin x}{2x}$ [using L-Hospital's rule]

$=\frac{1}{2}\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=\frac{1}{2}\times 1=\frac{1}{2}$

63. *(d)* Given, $f(x)=\begin{cases} x\sin(\log x^2), & x\neq 0 \\ 0, & x=0 \end{cases}$

Test for continuity

LHL $=\lim_{h\to 0} f(0-h)$

$=\lim_{h\to 0}(-h)\sin(\log(-h)^2)=-\lim_{h\to 0} h\sin(\log h^2)$

As $h\to 0$, $\log h^2\to -\infty$

$\Rightarrow \sin(\log h^2)$ oscillates between -1 and 1.

So, LHL $=0\times$ a number between -1 and 1

$=0$

RHL $=\lim_{h\to 0} f(0+h)$

$=\lim_{h\to 0} h\sin(\log h^2)$

$=0\times$ a number between -1 and $1=0$

Also, $f(0)=0$ [given]

$\Rightarrow$ LHL = RHL = $f(0)$

$f(x)$ is continuous at $x=0$

Test of differentiability

$$Lf'(0)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

$$=\lim_{h\to 0}\frac{-h\sin[\log(-h)^2]-0}{h}$$

$$=\lim_{h\to 0}[-\sin(\log h^2)]$$

= a number between -1 and 1

$\Rightarrow$ Limit does not exist.

$\therefore$ Left hand derivation is not defined.

$f(x)$ is not differentiable at $x=0$.

64. *(a)* Let $I=\int x^{2/3}(1+x^{1/2})^{-13/3}\,dx$

$=\int x^{2/3}x^{-13/6}(x^{-1/2}+1)^{-13/3}\,dx$

$=\int x^{-3/2}(x^{-1/2}+1)^{-13/3}\,dx$

Put $x^{-1/2}+1=t\Rightarrow x^{-3/2}dx=-2dt$

$\therefore \quad I=-2\int t^{-13/3}\,dt$

$=-2\frac{t^{-10/3}}{\left(\frac{-10}{3}\right)}=\frac{3}{5}(x^{-1/2}+1)^{-10/3}+C$

65. *(b)* Let $I=\int\frac{dx}{x^2(x^n+1)^{\frac{n-1}{n}}}$

$=\int\frac{dx}{x^2x^{n\frac{(n-1)}{n}}(1+x^{-n})^{\frac{n-1}{n}}}$

$=\int\frac{dx}{x^{n+1}(1+x^{-n})^{\frac{n-1}{n}}}$

Put $1+x^{-n}=t\Rightarrow x^{-(n+1)}dx=\frac{dt}{-n}$

$\Rightarrow \quad \frac{dx}{x^{n+1}}=\frac{-dt}{n}$

$\therefore \quad I=\int\frac{-dt}{nt^{\left(\frac{n-1}{n}\right)}}=-\frac{1}{n}\int t^{\left(\frac{1}{n}-1\right)}dt$

$=-\frac{1}{n}\frac{t^{1/n}}{\frac{1}{n}}+C=-t^{1/n}+C$

$=-(1+x^{-n})^{1/n}+C$

Thus, $f(x)=1+x^{-n}$

66. *(d)* Here, A is a matrix of order 3 and $B=\text{adj}(A)$, $C=\text{adj}(\text{adj}\,A)$, $D=\text{adj}[\text{adj}(\text{adj}\,A)]$

Let $ABCD=E$, then

$|\text{adj}[\text{adj}\{\text{adj}(\text{adj}\,E)\}]|=|E|^{(3-1)^4}=|E|^{16}$

$|ABCD|^{16}=|A|^{16}|B|^{16}|C|^{16}|D|^{16}$

$=|A|^{16}\cdot(|A|^2)^{16}\cdot(|A|^4)^{16}\cdot|(A)^8|^{16}$

$=|A|^{16+32+64+128}=|A|^{240}$

$\Rightarrow \quad |A|^k=|A|^{240}\Rightarrow k=240$

67. *(d)* We have, $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$

$\Rightarrow (\sin y + y\cos y)\,dy = x(2\log x + 1)\,dx$

$\Rightarrow -\cos y + \{(y\sin y) + \cos y\} = 2\left\{\frac{x^2}{2}\log x - \frac{1}{2}\int x\,dx + \frac{x^2}{2}\right\}$

$\Rightarrow \quad y\sin y = x^2\log x + C$

68. *(a)* For the points, where the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the curve $xy = c^2$, $z = 0$, we have $z = 0$

$\therefore \quad \frac{x-2}{3} = \frac{y+1}{2} = \frac{0-1}{-1}$

$\Rightarrow \quad \frac{x-2}{3} = 1$ and $\frac{y+1}{2} = 1$

$\Rightarrow \quad x = 5, y = 1$

On putting the values of x and y in $xy = c^2$, we get

$$c = \pm\sqrt{5}$$

69. *(b)* Since, $\mathbf{b}_1 = \mathbf{b} - \frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$, $\mathbf{b}_2 = \mathbf{b} + \frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$

and $\mathbf{c}_1 = \mathbf{c} - \frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a} - \frac{\mathbf{c}\cdot\mathbf{b}}{|\mathbf{b}|^2}\mathbf{b}$,

$\mathbf{c}_2 = \mathbf{c} - \frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a} - \frac{\mathbf{c}\cdot\mathbf{b}_1}{|\mathbf{b}|^2}\mathbf{b}_1$,

$\mathbf{c}_3 = \mathbf{c} - \frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a} - \frac{\mathbf{a}\cdot\mathbf{b}_2}{|\mathbf{b}_2|^2}\mathbf{b}_2$, $\mathbf{c}_4 = \mathbf{a} - \frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$

which shows $\mathbf{a}\cdot\mathbf{b}_1 = 0 = \mathbf{a}\cdot\mathbf{c}_2 = \mathbf{b}_1\cdot\mathbf{c}_2$

So, $\{\mathbf{a}, \mathbf{b}_1, \mathbf{c}_2\}$ are mutually orhtogonal vectors.

70. *(c)* Let $I = \int_{-4}^{-5} e^{(x+5)^2}\,dx$

Put $x + 5 = K \Rightarrow dx = dK$

$\therefore \quad I_1 = \int_1^0 e^{K^2}\,dK$

Let $I_2 = 3\int_{\frac{1}{3}}^{\frac{2}{3}} e^{(3x-2)^2}\,dx$

Put $3x - 2 = K_1 \Rightarrow dx = \frac{dK_1}{3}$

$\therefore \quad I_2 = \int_{-1}^{0} e^{K_1^2}\,dK_1$

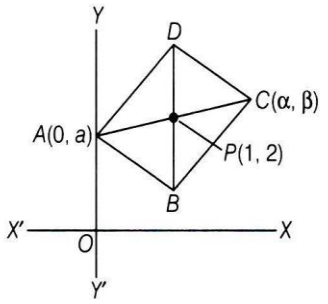
Put $K_1 = -K_2 \Rightarrow I_2 = -\int_1^0 e^{K_2^2}\,dK_2 = -I_1$

$\therefore \quad I_1 + I_2 = 0$

71. *(a)* Let $A(0,a)$ on the Y-axis and opposite vertex $C(\alpha,\beta)$, then mid-point of AC is $P(1,2)$.

$\Rightarrow \frac{\alpha+0}{2} = 1$ and $\frac{a+\beta}{2} = 2$

$\Rightarrow \quad \alpha = 2$ and $a + \beta = 4$



Sides of rhombus are parallel to the lines

$$x - y + 2 = 0 \text{ and } 7x - y + 3 = 0$$

Equation of lines parallel to diagonal is given by

$$\frac{x-y+2}{\sqrt{2}} = \pm\frac{7x-y+3}{5\sqrt{2}}$$

$\Rightarrow \quad 2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$

So, the slope of AC is $-\frac{1}{2}$ or 2.

$\therefore$ Slope of AP = Slope of AC

$\Rightarrow \quad \frac{2-a}{1-0} = \frac{-1}{2}$ or 2

$\Rightarrow \quad 2 - a = \frac{-1}{2}$ or 2

$\Rightarrow \quad a = \frac{5}{2}$ or 0

$\therefore A\left(0, \frac{5}{2}\right)$

Ordinate of $A = \frac{5}{2} = \frac{p}{q}$

Hence, $p + q = 5 + 2 = 7$

72. *(a)* Let $f(x) \equiv ax^2 + bx + c$

$f(0) = c > 0$

$f(1) = a + b + c < 0$

$f(-1) = a - b + c < 0$

Hence, one root lies in $(-1,0)$ and the other lies in $(0,1)$.

$\therefore \quad [\alpha] = -1$ and $[\beta] = 0 \Rightarrow [\alpha] + [\beta] = -1$

Therefore, Statement I is true.

Statement II is true and it is a correct explanation of Statement I.

73. *(c)* Given sequence $\frac{1}{503}, \frac{4}{524}, \frac{9}{581}, \frac{16}{692}, \ldots$

General term is

$$T_n = \frac{n^2}{500 + 3n^3}$$

For largest term, $\frac{dT_n}{dn} = 0$

$\Rightarrow \quad \frac{n(1000-3n^3)}{(500+3n^3)^2} = 0$

$\Rightarrow \quad n(1000-3n^3) = 0$

$\Rightarrow \quad n = 0 \text{ or } 1000-3n^3 = 0$

$\Rightarrow \quad n = \left(\frac{1000}{3}\right)^{1/3} \Rightarrow 6 < n < 7$

$\Rightarrow$ 7th term is largest.

74. ***(d)*** Given, $(\sqrt{5}+1)^{4n} - (\sqrt{5}-1)^{4n}$

Using Binomial Expansion,

$[(x+y)^n - (x-y)^n$

$= 2[C_1 x^{n-1} y + C_3 x^{n-3} y^3 + C_5 x^{n-5} y^5 + \ldots]]$

$= 2[{}^{4n}C_1 (\sqrt{5})^{4n-1} + {}^{4n}C_3 (\sqrt{5})^{4n-3}$

$+ {}^{4n}C_5 (\sqrt{5})^{4n-5} + \ldots + {}^{4n}C_{4n-1} (\sqrt{5})]$

$= 2\sqrt{5}[{}^{4n}C_1 (5)^{2n-1} + {}^{4n}C_3 (5)^{2n-2} + \ldots + {}^{4n}C_{4n-1}]$

This is an irrational number.

75. ***(a)*** Given, $[\sqrt{2}\cos x] + [\sin x] = -3$

$\Rightarrow [\sqrt{2}\cos x] = -2 \text{ and } [\sin x] = -1$

$\Rightarrow \quad -2 \le \sqrt{2}\cos x < -1 \text{ and } -1 \le \sin x < 0$

$\Rightarrow -\sqrt{2} \le \cos x < \frac{-1}{\sqrt{2}} \text{ and } -1 \le \sin x < 0$

$\Rightarrow \quad -1 \le \cos x < -\frac{1}{\sqrt{2}} \text{ and } -1 \le \sin x < 0$

$\Rightarrow \quad x \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \text{and } x \in (\pi, 2\pi)$

$\Rightarrow \quad x \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \cap (\pi, 2\pi) \Rightarrow x \in \left(\pi, \frac{5\pi}{4}\right)$

76. ***(d)*** First 25 even natural numbers are 2, 4, 6, ... 50.

$\text{Variance} = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$= \frac{2^2 + 4^2 + 6^2 + \ldots + 50^2}{25} - \left(\frac{2+4+6+\ldots+50}{25}\right)^2$

$= \frac{22100}{25} - (26)^2 = 884 - 676 = 208$

Time Saver Tip

$\because$ Variance of first n even natural numbers $= \frac{n^2-1}{3}$

$\therefore$ Variance of first 25 even natural numbers

$= \frac{25^2-1}{3} = \frac{624}{3} = 208$

77. ***(b)*** Since, ${}^nP_r = {}^nP_{r+1}$

$\Rightarrow \quad \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$

$\Rightarrow \frac{n!}{(n-r)(n-r-1)!} = \frac{n!}{(n-r-1)!}$

$\Rightarrow \quad n-r = 1 \qquad \ldots(i)$

and ${}^nC_r = {}^nC_{r-1}$

$\Rightarrow \quad \frac{n!}{r!(n-r)!} = \frac{n!}{(r-1)!(n-r+1)!}$

$\Rightarrow \frac{n!}{r(r-1)!(n-r)!} = \frac{n!}{(r-1)!(n-r+1)(n-r)!}$

$\Rightarrow \quad n-r+1 = r$

$\Rightarrow \quad 1+1 = r \qquad [\because n-r=1]$

$\Rightarrow \quad r = 2$

78. ***(b)*** For given condition, we have three cases

I. All dice shows same number.

II. Number appearing on A_4 appears on any one of A_1, A_2, A_3.

III. Number appearing on A_4 appears on any two A_1, A_2, A_3.

Now, sample space $|S| = 6\times6\times6\times6 = 6^4$

For Case I

Select one number on A_4 which appears on all number of favourable cases $= {}^6C_1 \times 1 = 6$

For Case II

For A_4 number of ways ${}^6C_1$, for other two there are 5×5 ways.

Number of favourable ways $= {}^6C_1 \times {}^3C_1 \times 5 \times 5$

For Case III

For A_4 there are ${}^6C_1$ ways.

It appears on any two of A_1, A_2, A_3 is ${}^3C_2 \times 1^2$ ways for remaining one there are 5 ways.

Number of favourable ways $= {}^6C_1 \times {}^3C_2 \times 1^2 \times 5$

Required probability

$= \frac{{}^6C_1 + {}^6C_1 \times {}^3C_1 \times 5^2 + {}^6C_1 \times {}^3C_2 \times 5}{6^4}$

$= \frac{6+6\times75+6\times15}{6^4}$

$= \frac{91}{216}$

79. ***(a)*** Both the lines pass through origin. Line L_1 is parallel to the vector

$V_1 = (\cos\theta + \sqrt{3}\,\hat{\mathbf{i}}) + (\sqrt{2}\sin\theta)\,\hat{\mathbf{j}} + (\cos\theta - \sqrt{3})\,\hat{\mathbf{k}}$

and L_2 is parallel to the vector

$V_2 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$

$$\therefore \cos\alpha = \frac{V_1 \cdot V_2}{|V_1|\,|V_2|}$$

$$= \frac{a(\cos\theta+\sqrt{3})+(b\sqrt{2})\sin\theta+c(\cos\theta-\sqrt{3})}{\sqrt{a^2+b^2+c^2}\sqrt{(\cos\theta+\sqrt{3})^2+2\sin^2\theta+(\cos\theta-\sqrt{3})^2}}$$

$$= \frac{(a+c)\cos\theta+b\sqrt{2}\sin\theta+(a-c)\sqrt{3}}{\sqrt{a^2+b^2+c^2}\sqrt{2+6}}$$

In order that $\cos\alpha$ in the independent of θ.

$$a+c=0 \text{ and } b=0$$

$$\therefore \cos\alpha = \frac{2a\sqrt{3}}{a\sqrt{2}\cdot 2\sqrt{2}} = \frac{\sqrt{3}}{2}$$

$$\alpha = \frac{\pi}{6}$$

80. *(d)* Let $f(x) = px^3 - qx - (\tan x)\operatorname{sgn} x$

$\because f(x)$ is even function.

$\therefore f(-x) = f(x)$

$\Rightarrow -px^3 + qx - (\tan x)\operatorname{sgn} x = px^3 - qx - (\tan x)\operatorname{sgn} x$

$\Rightarrow -2px^3 + 2qx = 0, \forall x \in R$

$\Rightarrow 2x(-px^2+q) = 0, \forall x \in R$

$\Rightarrow p = 0$ and $q = 0$

$\therefore [\alpha]^2 - 5[\alpha] + 4 = 0$ and $6\{\alpha\}^2 - 5\{\alpha\} + 1 = 0$

$\Rightarrow ([\alpha]-4)([\alpha]-1) = 0$ and $(3\{\alpha\}-1)(2\{\alpha\}-1) = 0$

$\Rightarrow [\alpha] = 4, [\alpha] = 1$ and $\{\alpha\} = \frac{1}{3}, \{\alpha\} = \frac{1}{2}$

$\Rightarrow \alpha = 1+\frac{1}{3}, 1+\frac{1}{2}, 4+\frac{1}{3}, 4+\frac{1}{2}$

Sum of values of $\alpha = \frac{35}{3}$

81. *(4)* We have,

$$S_n = \frac{195}{4^n P_n} - \frac{^{n+3}P_3}{^{n+1}P_{n+1}} = \frac{195}{4n!} - \frac{(n+3)!}{n!(n+1)!}$$

$$= \frac{195}{4n!} - \frac{(n+3)(n+2)(n+1)!}{(n+1)!n!}$$

$$= \frac{195}{4n!} - \frac{(n+3)(n+2)}{n!} = \frac{171-4n^2-20n}{4n!} > 0$$

$\Rightarrow 4n^2 + 20n - 171 < 0$ which is true for $n = 1, 2, 3, 4$

Number of positive terms = 4

82. *(13)* $\because |z-3-2i| \le 2 \Rightarrow |z-(3+2i)| \le 2$

$\therefore z$ lies on or inside the circle of radius 2 having centre at (3, 2).

$$\therefore |2z-6+5i|_{\max} = 2\left|z-3+\frac{5}{2}i\right|_{\max}$$

$$= 2\left|(z-(3+2i))+\frac{9}{2}i\right|_{\max}$$

$$= 2\left(|z-(3+2i)|_{\max} + \left|\frac{9}{2}i\right|\right)$$

$$= 2\left(2+\frac{9}{2}\right) = 4+9 = 13$$

83. *(25)* $\cos^{-1}x + \cos^{-1}2x + \cos^{-1}3x = \pi$

or $\cos^{-1}2x + \cos^{-1}3x = \pi - \cos^{-1}x$

$= \cos^{-1}(-x)$

$\Rightarrow \cos^{-1}[(2x)(3x) - \sqrt{1-4x^2}\sqrt{1-9x^2}] = \cos^{-1}(-x)$

$\Rightarrow 6x^2 - \sqrt{1-4x^2}\sqrt{1-9x^2} = -x$

$\Rightarrow (6x^2+x)^2 = (1-4x^2)(1-9x^2)$

$\Rightarrow x^2 + 12x^3 = 1 - 13x^2$

or $12x^3 + 14x^2 - 1 = 0$ ...(i)

On comparing Eqs. (i) with $ax^3 + bx^2 + c = 0$, we get

$a = 12, b = 14$ and $c = -1$

So, $a+b+c = 12+14-1 = 25$

84. *(1)* The area bounded by $y = f(x)$ and $y = x$ between the lines $x = 1$ and $x = t$ is $\int_1^t [f(x)-x]\,dx$.

But it is equal to $[t+\sqrt{1+t^2} - (1+\sqrt{2})]$

$\therefore \int_1^t [f(x)-x]\,dx = (t+\sqrt{1+t^2}) - (1+\sqrt{2})$

On differentiating both sides w.r.t. t, we get

$$f(t) - t = 1 + \frac{t}{\sqrt{1+t^2}} \Rightarrow f(t) = 1 + t + \frac{t}{\sqrt{1+t^2}}$$

$$\therefore \quad f(0) = 1 + 0 + \frac{0}{\sqrt{1+0}} \Rightarrow f(0) = 1$$

85. *(2)* Let point on parabola be $(at^2, 2at)$.

It is also mid-point of chord.

$\therefore$ Equation of chord : $T = S_1$

$$\frac{at^2 \cdot x}{2a^2} + \frac{2aty}{a^2} - 1 = \frac{a^2t^4}{2a^2} + \frac{4a^2t^2}{a^2} - 1$$

$$xt + 4y = at^3 + 8at$$

It passes $\left(11a, \frac{-a^2}{4}\right)$.

$$11at - a^2 = at^3 + 8at$$

$$t^3 - 3t + a = 0 \quad \ldots(i)$$

From Eq. (i), we must have three values of t.

Let $f(t) = t^3 - 3t + a$

$f'(t) = 3t^2 - 3 = 3(t^2-1)$

$f(1)\,f(-1) < 0$

$(a-2)(a+2) < 0$

$a \in (-2, 2) - \{0\}$ [a cannot be zero]

Hence, $a = -1$, 1 two values.

86. *(10)* First term $(a)=\dfrac{K-1}{K!}$

Common ratio $(r)=\dfrac{1}{K}$

Sum of infinite geometric series $=\dfrac{a}{1-r}$

$$\Rightarrow \quad S_K=\frac{\frac{K-1}{K!}}{1-\frac{1}{K}}=\frac{1}{(K-1)!}$$

$$\text{Now, } (K^2-3K+1)S_K=\frac{(K^2-2K+1)-K}{(K-1)!}$$

$$=\frac{(K-1)^2-K}{(K-1)!}$$

$$=\frac{K-1}{(K-2)!}-\frac{K}{(K-1)!}$$

$$\therefore \sum_{K=2}^{100}|(K^2-3K+1)S_K|=\sum_{K=2}^{100}\left|\frac{K-1}{(K-2)!}-\frac{K}{(K-1)!}\right|$$

$$\Rightarrow \quad 3-\frac{\lambda^2}{99!}=\left|\frac{1}{0!}-\frac{2}{1!}\right|+\left|\frac{2}{1!}-\frac{3}{2!}\right|+\left|\frac{3}{2!}-\frac{4}{3!}\right|+\ldots+\left|\frac{99}{98!}-\frac{100}{99!}\right|$$

$$=\frac{2}{1!}-\frac{1}{0!}+\frac{2}{1!}-\frac{3}{2!}+\frac{3}{2!}-\frac{4}{3!}+\ldots+\frac{99}{98!}-\frac{100}{99!}$$

$$=1+\frac{2}{1!}-\frac{100}{99!}$$

$$\Rightarrow \quad 3-\frac{\lambda^2}{99!}=3-\frac{100}{99!}$$

$$\Rightarrow \quad \lambda^2=100$$

$$\Rightarrow \quad \lambda=10$$

87. *(4)* Given, $y^3=27x$

On differentiating w.r.t. x, we get

$$3y^2\frac{dy}{dx}=27x$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{9}{y^2} \qquad \ldots(i)$$

Now, ordinate changes as a faster rate than the abscissa, then

$$\left|\frac{dy}{dt}\right|>\left|\frac{dx}{dt}\right|$$

$$\Rightarrow \quad \left|\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|>1\Rightarrow\left|\frac{dy}{dx}\right|>1$$

$$\Rightarrow \quad \left|\frac{9}{y^2}\right|>1\Rightarrow\frac{9}{y^2}>1$$

$$\Rightarrow \quad 1-\frac{9}{y^2}<0$$

$$\Rightarrow \quad \frac{(y+3)(y-3)}{y^2}<0$$

$$\therefore \quad y\in(-3,3)-\{0\}$$

Hence, the total possible number of integral values of y for which the ordinates changes as faster rate than abscissa = 4.

88. *(28)* Given that $|\mathbf{a}+\mathbf{b}|=\sqrt{2}$ …(i)

On squaring Eq. (i) on both sides, we get

$$|\mathbf{a}+\mathbf{b}|^2=2$$

$$|\mathbf{a}|^2+|\mathbf{b}|^2-2\mathbf{a}\cdot\mathbf{b}=2$$

$$1+1+2\cos\theta=2$$

$$\cos\theta=0$$

$$\theta=90^\circ$$

$\mathbf{a}\times\mathbf{b}$ is perpendicular to plane containing $\mathbf{a}$ and $\mathbf{b}$.

$$\mathbf{c}=\mathbf{a}+2\mathbf{b}+3(\mathbf{a}\times\mathbf{b}) \qquad \ldots(ii)$$

On squaring both sides, we get

$$|\mathbf{c}|^2=|\mathbf{a}|^2+4|\mathbf{b}|^2+9|\mathbf{a}\times\mathbf{b}|^2+4(\mathbf{a}\cdot\mathbf{b})$$

$$+6\mathbf{a}\cdot(\mathbf{a}\times\mathbf{b})+|12\mathbf{b}\cdot(\mathbf{a}\times\mathbf{b})|$$

$$=1+4+9\sin^2\theta+4\cos\theta+0+0$$

$$=1+4+9(1)^2+4(0)$$

$$\Rightarrow \quad |\mathbf{c}|^2=14$$

$$\Rightarrow \quad 2|\mathbf{c}|=2\sqrt{14}=K \qquad \text{(given)}$$

So, $\sqrt{14}\,K=28$

89. *(128)* Given, $A=\begin{bmatrix}i & -i\\ -i & i\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

Now, $\quad A=i\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}=iB$

$$A^2=(iB)^2=i^2B^2=-B^2$$

$$=-\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}=-\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}=-2B$$

$$\Rightarrow \quad A^2=-2B$$

$$\Rightarrow \quad A^4=(-2B)^2=4B^2=4(2B)=8B$$

$$\therefore \quad A^8=(8B)^2=64B^2=64(2B)=128B=\lambda B$$

$$\Rightarrow \quad \lambda=128$$

90. *(501000)* $\because\ (x+1)$ divides $ax^2+2bx+c$.

$\therefore\ x=-1$ is the root of quadratic equation.

$\Rightarrow a-2b+c=0$

$\Rightarrow \quad 2b=a+c$

Then, a and c both must be even or odd.

Then, number of such polynomials $=({}^{501}C_2)\times2\times2$

$$=\frac{501\times500}{2}\times2\times2$$

$$=501000$$