2024	3	27	JEE-T4	SEAT NUMBER				

IIT INSPIRE
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XI & XII Science (CBSE/state)
IIT- JEE (Mains + Advance)
NEET, MH-CET , NDA
Mo. No. 9595445177/9021445177

Branches : Chhatrapati Sq., Mangalmurti Sq.

Day - 4

JEE TEST

Time : 3 Hrs.	(4 Pages)	Max. Marks : 300

Instructions

1. There are **three** parts in the questions paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
2. **Section A** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **– 1 mark** for wrong answer.
3. **Section B** This section contains 10 questions. In Section B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **– 1 mark** for wrong answer. For Section B, the answer should be rounded off to the nearest integer.

PHYSICS

Section A : Objective Type Questions

1. In a Wheatstone bridge, $P = 90\ \Omega$, $Q = 110\ \Omega$, $R = 40\ \Omega$ and $S = 60\ \Omega$ and a cell of emf 4V. Then, the potential difference between the diagonal along which a galvanometer is connected, is

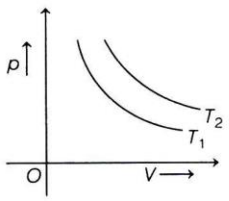
(a) – 0.2V (b) + 0.2V
(c) –1V (d) +1 V

2. A wall is made of equally thick layers A and B of different materials. Thermal conductivity of A is twice that of B. In the steady state, the temperature difference across the wall is 72°C. The temperature difference across the layer A is

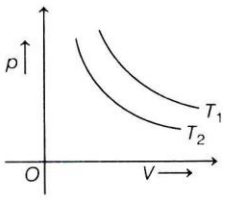
(a) 12° C (b) 36°C
(c) 24°C (d) 0° C

3. Identify the graph (s) which correctly represents an isotherm at temperatures T_1 and T_2 $(T_2 > T_1)$

(a)



(b)



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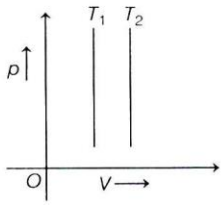

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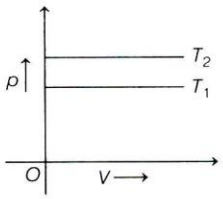
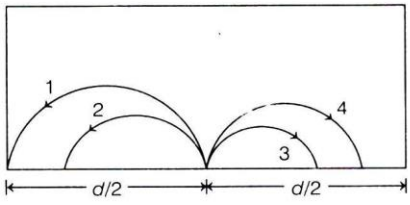


4. Binding energy per nucleon of helium and deuterium nuclei are 7 MeV and 1.1 MeV, respectively. When two deuterium nuclei undergo fusion to form helium nucleus, energy released in the process is

(a) 23.6 J (b) 37.76×10^{-13} MeV
(c) 23.6 eV (d) 37.76×10^{-13} J

5. A beam consisting of 4 different types of ions named α, β, γ and δ enters a region of magnetic field as shown in the figure below.



Magnetic field is perpendicular to the velocity of beam. All ions in the beam travels with same speed.

Mass and charge of ions are given as

Ion	Mass	Charge
α	$2m$	e
β	$4m$	e^-
γ	$2m$	e^-
δ	$3m$	e

Now, match the trajectories with its ions.

	Column I (Ion)		Column II (Path)
i.	α	p.	1
ii.	β	q.	2
iii.	γ	r.	3
iv.	δ	s.	4

Codes

	i	ii	iii	iv		i	ii	iii	iv
(a)	p	r	s	q	(b)	q	s	r	p
(c)	r	p	q	s	(d)	s	q	p	r

6. 5.6 L of helium gas at STP is compressed adiabatically to 0.7 L. If the initial temperature of the gas be T_1, the work done in the process is found to be $-\frac{m}{n}RT_1$, then the value of $m-n$ is

(a) 1 (b) 1.5 (c) 2 (d) 2.5

7. When an AC voltage of variable frequency is applied across the L-C-R circuit, the current in the circuit was found to be same at 8 MHz and 18 MHz. Then, the frequency at which current will be maximum is

(a) 24 MHz (b) 13 MHz
(c) 10 MHz (d) 12 MHz

8. Two charges are placed at a certain distance in air experience a force of F Newton. If both charges were placed in a liquid of dielectric constant K, at some distance, then the force between them will be

(a) FK (b) $F\sqrt{K}$ (c) $\frac{F}{K}$ (d) $\sqrt{FK}$

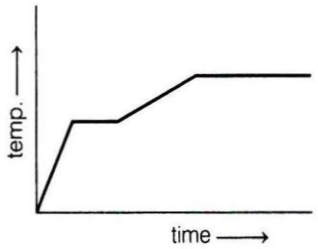
9. A cubical block of mass m and edge a slides down a rough inclined plane of inclination θ with a uniform velocity. The torque of the normal force on the block about its centre has a magnitude

(a) zero (b) mga (c) $mga\sin\theta$ (d) $\frac{mga\sin\theta}{2}$

10. Two particles execute SHM of the same amplitude and frequency along the same straight line. If they pass one another when going in opposite directions, each time their displacement is half of their amplitude, the phase difference between them is

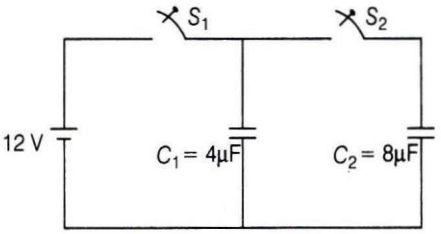
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

11. Heat is supplied to a certain homogeneous sample of matter, at a uniform rate. Its temperature is plotted against time, as shown. Which of the following conclusions can be drawn?



(a) Its specific heat capacity is greater in the solid state than in the liquid state
(b) Its specific heat capacity is greater in the liquid state than in the solid state
(c) Its latent heat of vaporisation is equal to its latent heat of fusion
(d) Its latent heat of vaporisation is smaller than its latent heat of fusion

12. The figure given below shows two capacitors $C_1 = 4\mu F$ and $C_2 = 8\mu F$ connected to a 12 V battery and two switches S_1 and S_2. Switch S_2 is initially kept open and S_1 is kept closed until the capacitance C_1 is fully charged. Switch S_1 is now opened and S_2 is closed until electrostatic conditions are restored. In this process, the percentage loss of energy stored in C_1 is



(a) 88.9% (b) 76.5%
(c) 63.4% (d) 55.0%

13. The maximum height attained by a projectile is increased by 10% by increasing its speed of projection without changing the angle of projection. The percentage increase in the horizontal range will be

(a) 10% (b) 20% (c) 15% (d) 25%

14. If proton and α-particle are accelerated by same potential V, then the ratio of their wavelengths $\lambda_p : \lambda_\alpha$ would be

(a) $2\sqrt{2}:1$ (b) $1:2\sqrt{2}$ (c) $2:1$ (d) $1:2$

15. Read the following statements

I. If the stone tied at the end of a string is whirled and the string breaks, the stone flies off tangentially.

II. The necessary centripetal force required to keep the stone moving in a circle is provided by tension in the string.

III. Centripetal force is a real force.

Choose the correct statements

(a) I only (b) II and III only
(c) I and III only (d) I, II and III

16. In a reverse biased p-n junction, when the applied bias voltage is equal to the breakdown voltage, then

(a) current remains constant, while voltage increases sharply
(b) voltage remains constant, while current increases sharply
(c) current and voltage increase
(d) current and voltage decrease

17. The dimensional formula of Young's modulus is $[ML^{-1}T^{-2}]$. Find its new dimensional formula in the form of velocity (v), acceleration (A) and Force (F), if these all are taken as fundamental quantities instead of mass (M), length (L) and time (T).

(a) $[F^2A^2v^{-2}]$ (b) $[FA^2v^{-4}]$
(c) $[FA^3v^{-2}]$ (d) $[F^2Av^{-5}]$

18. **Assertion** If the velocity of a revolving satellite is increased to $\sqrt{2}$ times, then it escapes out from the earth.

Reason Escape velocity does not depend on the direction of projection.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion
(c) Assertion is true and Reason is false
(d) Assertion is false and Reason is true

19. Diameter of a cylindrical thin wire measured by a screw gauge of least count 0.001 cm is 0.05 cm. The length measured by a scale of least count 0.1 cm is 110.0 cm. When a weight of 50 N is suspended from the wire, the extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. The maximum error in the measurement of Young's modulus of the material of the wire is

(a) $2.05\times10^{9}\,Nm^{-2}$ (b) $1.09\times10^{10}\,Nm^{-2}$
(c) $3.07\times10^{9}\,Nm^{-2}$ (d) $7.03\times10^{10}\,Nm^{-2}$

20. The two statements are given below.

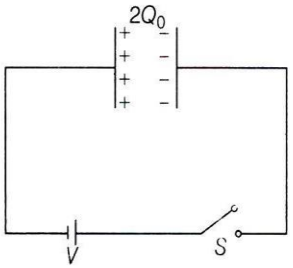
Statement I Electromagnetic waves consist of oscillating electric and magnetic fields, that are parallel to each other and perpendicular to the direction of propagation of EM waves.

Statement II Electromagnetic waves are transverse in nature and travel through vacuum with same speed of 3×10 m/s.

(a) Both Statement I and Statement II are correct and Statement II is the correct explanation of Statement I.
(b) Both Statement I and Statement II are correct and Statement II is not the correct explanation of Statement I.
(c) Statement I is true and Statement II is false
(d) Statement I is false and Statement II is true.

Section B : Numerical Value Type Questions

21. A capacitor of capacitance C having initial charge $2Q_0$, is connected to a battery of potential difference $V = \frac{Q_0}{C}$ as shown, then work done by the battery is found to be $\frac{aQ_0^2}{C}$. The value of a is ..... .



22. A solid sphere of radius r made of a material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid. When a mass m is placed on the piston to compress the liquid, the fractional change in the radius of the sphere $(\Delta r/r)$ is found to be $\frac{x\,mg}{Ka}$, then the value of x is ..... .

23. The first line of the Lyman series in a hydrogen spectrum has a wavelength of 2420Å. The corresponding line of a hydrogen like atom of $Z = 11$ is equal to ..... Å.

24. A cell of constant emf first connected to a resistance 18 Ω and then connected to a resistance 8 Ω. If the power delivered in both the cases is same, then the internal resistance (in Ω) of cell will be ..... .

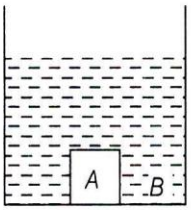
25. A block A is moving with a speed 10 m/s towards an another identical block, B, which was initially at rest and having mass 4 kg, then the velocity of block B after the collision will be ..... m/s.

26. A guitar of string length 180 cm vibrates with a fundamental frequency of 120 Hz. If the string produces fundamental frequency of 180 Hz, then length of the string is ..... cm.

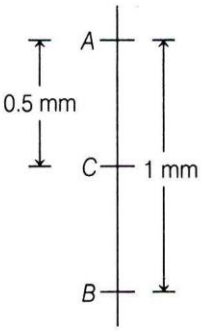
27. An L-C-R series circuit with a resistance of 100 Ω is connected to an AC source of 200 V (rms) and angular frequency of 300 rad/s. When only capacitor is removed, the current lags behind the voltage by 60°. When only inductor is removed, the current leads the voltage by 60°. Then, the power (in watt) dissipated in an L-C-R circuit will be ..... .

28. A juggler throws balls into air. He throws one whenever previous one is at its highest point. If he throws n balls per second, then the height to which each ball will rise is found to be $\frac{g}{2n^p}$, then the value of p will be ..... .

29. A parallel sided block of glass of refractive index 1.5, which is 36 mm thick rests on the bottom of a tank, filled with water (refractive index = 4 /3). The difference between the apparent depth of bottom at A and B, when seen from vertically above is found to be x mm, then the value of x will be ..... .



30. In Young's double slit experiment, the length of band AB is 1 mm. The fringe width is 0.021 mm. Find the number of fringes.



CHEMISTRY

Section A : Objective Type Questions

31. Glucose on treatment with conc. HCl produces

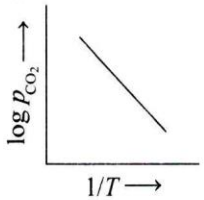
(a) formic acid (b) maltose + fructose

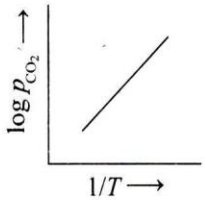
(c) fructose (d) levulinic acid

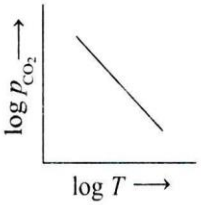
32. For the chemical equilibrium,

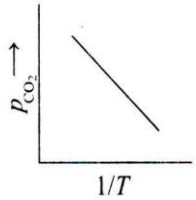
$$CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$$

ΔH_r° can be determined from which one of the following plots?

(a) 

(b) 

(c) 

(d) 

33. Which is the correct statement about birth control pills?

(a) Contain estrogen only.

(b) Contain progesterone only.

(c) Contain a mixture of estrogen and progesterone derivative.

(d) Progesterone enhances ovulation.

34. The correct order of ionic radii of Ce, La, Pm and Yb in +3 oxidation state is

(a) $Yb^{3+} < Pm^{3+} < Ce^{3+} < La^{3+}$

(b) $La^{3+} < Pm^{3+} < Ce^{3+} < Yb^{3+}$

(c) $La^{3+} < Ce^{3+} < Pm^{3+} < Yb^{3+}$

(d) $Yb^{3+} < Ce^{3+} < Pm^{3+} < La^{3+}$

35. An organic compound gave positive iodoform and Tollen's test. The compound is

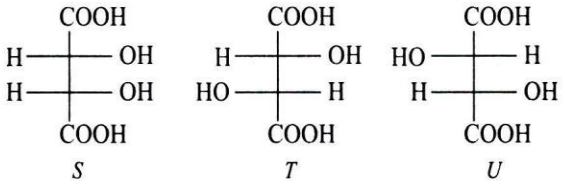
(a) CH_3CH_2OH

(b) $CH_3CH_2CH_2CHO$

(c) CH_3CHO

(d) CH_3COCH_3

36. P and Q are isomers of a dicarboxylic acid $C_4H_4O_4$. Both decolourise Br_2/H_2O. On heating P forms a cyclic anhydride. Upon treatment with dilute alkaline $KMnO_4$, P as well as Q could produce one or more than one form S, T and U.



S T U

(a) Optically active S and optically active pair (T, U).
(b) Optically inactive S and optically active pair (T, U).
(c) Optically active pair (T, U) and optically active S.
(d) Optically inactive pair (T, U) and optically inactive S.

37. Consider the following series of reactions,

$(A) \xrightarrow{KOH} (B)$ (gas turns red litmus blue)

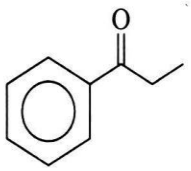
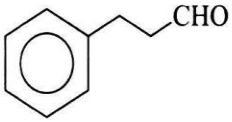
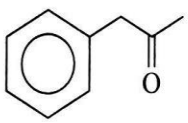
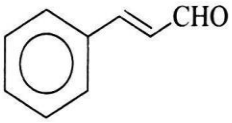
$(C) \xrightarrow{Zn + KOH} (B)$ (gas)

$(A) \xrightarrow{Heat}$ Gas (does not support combustion)

Identify A, B and C.

(a) $A = NH_4NO_3$, $B = N_2$, $C = KNO_3$
(b) $A = NH_4NO_2$, $B = NH_3$, $C = KNO_2$
(c) $A = (NH_4)_2SO_4$, $B = NH_3$, $C = KNO_3$
(d) $A = NH_4Cl$, $B = N_2$, $C = KNO_3$

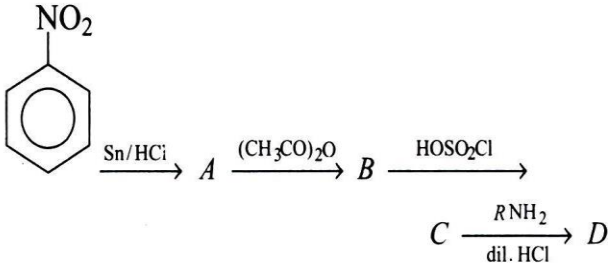
38. An organic compound (A) having molecular formula, $C_9H_{10}O$ is supplied to a student to determine the structural formula of compound, he observe that compound forms an orange red precipitate with 2,4-DNP reagent and yellow precipitate on heating with I_2 / NaOH. It neither reduces Tollen's reagent or Fehling solution nor it decolorises bromine water solution. On drastic oxidation with CrO_3, it produces a carboxylic acid having molecular formula $C_7H_6O_2$. Identify the correct compound A.

(a) 
(b) 
(c) 
(d) 

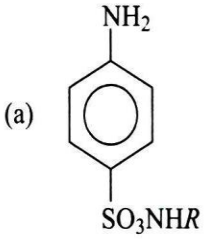
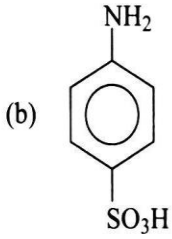
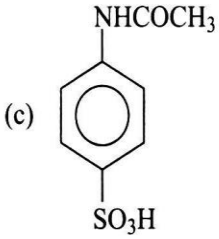
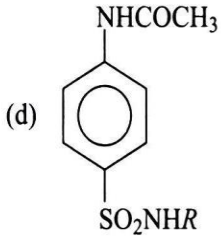
39. When an inorganic compound X having 3 centre-2-electron as well as $2C—2e^-$ bonds react with NH_3 gas at a certain temperature, gives a compound Y, isostructural with benzene. Compound X with ammonia at a high temperature produces a substance Z, then

(a) X is BH_3, Y is $B_2N_2H_3$, Z is inorganic benzene.
(b) X is B_2H_6, Y is $B_3N_3H_6$, Z is inorganic graphite.
(c) X is borax, Y is B_2O_3, Z is inorganic benzene.
(d) X is inorganic benzene, Y is diborane, Z is borax

40. Consider the following sequence of reactions,



The correct structure of product (D) is

(a) 
(b) 
(c) 
(d) 

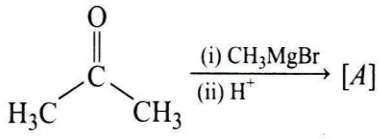
41. Match List-I with List-II.

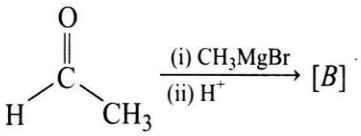
	List-I		List-I
A.	BrF_5	I.	bent
B.	$[CrF_6]^{3-}$	II.	square pyramidal
C.	O_3	III.	trigonal bipyramidal
D.	PCl_5	IV.	octahedral

Choose the correct answer from the options given below.

(a) A-II, B-IV, C-I, D-III (b) A-III, B-I, C-II, D-IV
(c) A-II, B-IV, C-III, D-I (d) A-I, B-III, C-II, D-IV

42. Consider the following reactions





The reagent used to differentiate between *A* and *B* is

(a) $FeCl_3$ (b) active metal Na

(c) Lucas reagent (d) Ninhydrin Test

43. The coordination number of Ni^{2+} is 4.

$NiCl_2$ + KCN (excess) ⟶ *A* (Cyano complex)

$NiCl_2$ + Conc. HCl (excess) ⟶ *B* (Chloro complex)

The IUPAC name of *A* and *B* are

(a) potassiumtetracyanonickelate(II), potassiumtetrachloronickelate(II)

(b) tetracyanopotassiumnickelate (II), tetrachloropotasium-nickelate(II)

(c) tetracyanonickel(II), tetrachloronickel(II)

(d) potassiumtetracyanonickel(II), potassium tetrachloronickel(II)

44. Given below are two statements.

Statement I The graph of molar conductivity for KI increases steeply with dilution.

Statement II The graph of molar conductivity for carbonic acid increases slowly with dilution.

In the light of the above statements, choose the correct answer from the options given below.

(a) Both Statement I and Statement II are correct.

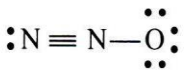
(b) Both Statement I and Statement II are incorrect.

(c) Statement I is correct and Statement II is incorrect.

(d) Statement I is incorrect and Statement II is correct.

45. Given below are two statements one is labelled as Assertion (A) and other is labelled as Reason (R).

Assertion (A) Following structure is acceptable.



Reason (R) Octet rule is followed for each atom, central N has positive charge and oxygen has negative charge.

In the light of the above statements. Choose the correct answer from the options given below.

(a) Both (A) and (R) are true, but (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

46. The IUPAC name of $[Fe(O_2)(CN)_4Cl]^{4-}$ is

(a) chlorotetracyanodioxoferrate (II) ion

(b) chlorotetracyanoperoxoferrate (II) ion

(c) chlorotetracyanosuperoxoferrate (II) ion

(d) chlorotetracyanideoxoferrate(II) ion

47. The chlorobenzene is generally obtained from a corresponding diazonium salt by reacting it with

(a) Cu_2Cl_2 (b) $CuSO_4$

(c) Cu (d) $[Cu(NH_3)_4]^{2+}$

48. Sodium sulphate is soluble in water whereas barium sulphate is sparingly soluble because

A. the hydration energy of sodium sulphate is less than its lattice energy.

B. the lattice energy of barium sulphate is more than its hydration energy.

C. the lattice energy decrease down the period.

D. the hydration energy of sodium sulphate is more than its lattice energy.

Choose the correct reasons regarding the above statement.

(a) A and B (b) C and D

(c) A and C (d) B and D

49. When N_2 is converted into N_2^+ and O_2 is converted into O_2^+, then the bond strength, respectively

(a) increases, increases (b) decreases, decreases

(c) increases, decreases (d) decreases, increases

50. Match the List-I with List-II.

	List-I		List-II
A.	$[B_4O_7]^{2-} \longrightarrow [B(OH)_3]$	I.	Heat
B.	$[AlO_2]^- \longrightarrow Al(OH)_3$	II.	Hydrolysis
C.	$[SiO_4]^{4-} \longrightarrow [Si_2O_7]^{6-}$	III.	Acidification
D.	$Bi^{3+} \longrightarrow (BiO)^+$	IV.	Dilution by water

Choose the correct answer from the options given below.

	A	B	C	D		A	B	C	D
(a)	I	III	IV	II	(b)	IV	II	III	I
(c)	III	IV	I	II	(d)	II	IV	I	III

Section B : Numerical Value Type Questions

51. The number of structural isomers for C_6H_{14} is .....

52. 1 mole of carbon dioxide gas at 300 K is expanded under adiabatic condition such that its volume becomes 27 times. The work done during the process is + ... cal
(Given, $\gamma = 1.33$ and $C_V = 6$ cal mol^{-1} for CO_2)

53. 5.7g of bleaching powder was suspended in 500 mL of water. 25 mL of this solution on treatment with KI in the presence of HCl liberated iodine which reacted with 24.35 mL of $N/10$ $Na_2S_2O_3$. The percentage of available chlorine in the bleaching powder is ..... %. [Nearest integer]

54. Photoelectrons are liberated by ultraviolet light of wavelength 3000 Å from a metallic surface for which the photoelectric threshold is 4000 Å. The de-Broglie wavelength of electron emitted with maximum kinetic energy is ..... 10^{-9} m.
[Nearest integer] [Given: $h = 6.626 \times 10^{-34}$ Js]

55. Reaction, $A + B \longrightarrow C + D$, follows rate law; $R = k[A]^{1/2}[B]^{1/2}$ starting with 1 M of each A and B. The time taken for concentration of A become 0.1 M will be .....s.
[Given, $k = 4.606 \times 10^{-4}$ s^{-1}]

56. The vapour pressure of two pure liquids, A and B which form an ideal solution are 1000 and 1600 torr, respectively at 400 K. A liquid solution of A and B for which the mole fraction of A is 0.60 is contained in a cylinder by a piston on which the pressure can be varied. The solution is slowly vapourised at 400 K by decreasing the applied pressure. The value of mole fraction of B is .....$\times 10^{-1}$.

57. The potential of an electrode which originally contained 0.1 M NO_3^- and 0.4 M H^+ and which has been treated by 6% of the cadmium necessary to reduce all the NO_3^- to $NO(g)$ at 1 atm will be .....$\times 10^{-2}$ V.
[Given, $NO_3^- + 4H^- + 3e^- \longrightarrow NO + 2H_2O$, $E^\circ = 0.95$ V and $\log 2 = 0.3010$]

58. The volume in mL of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 50 mL of 0.01 M solution of $[Cr(H_2O)_5Cl]Cl_2$ as silver chloride, is nearest to ..... .

59. 0.45 g of an organic compound, in a quantitative analysis yielded 0.5 g of the barium sulphate. The percentage of sulphur in the substance is ..... .
[Nearest integer]

60. The standard reduction potentials are 1.51 V for MnO_4^- / Mn^{2+}, 1.36 V for Cl_2/Cl^-, 1.07 V for Br_2/Br^-, and 0.54 V for I_2/I^- at 298 K.
At pH = 3, the number of species expected to oxidise by permanganate is/are .....
$\left(\frac{RT}{F} = 0.059 \text{ V}\right)$

MATHEMATICS

Section A : Objective Type Questions

61. If $\int \frac{2\cos x - \sin x + \lambda}{\cos x + \sin x - 2} dx$
$= A \ln|\cos x + \sin x - 2| + Bx + C$,
then ordered triplet A, B and λ is
(a) $\left(\frac{1}{2}, \frac{3}{2}, -1\right)$ (b) $\left(\frac{3}{2}, \frac{1}{2}, -1\right)$
(c) $\left(\frac{1}{2}, -1, -\frac{3}{2}\right)$ (d) $\left(\frac{3}{2}, -1, \frac{1}{2}\right)$

62. The area bounded by the curve $y = \log x$, $y = \log|x|$, $y = |\log x|$ and $y = |\log|x||$ is
(a) 4 sq units (b) 6 sq units
(c) 10 sq units (d) None of these

63. The value of (a, b), so that the function
$$f(x) = \begin{cases} x + a\sqrt{2}\sin x & , \quad 0 \le x < \frac{\pi}{4} \\ 2x\cot x + b & , \quad \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x & , \quad \frac{\pi}{2} < x \le \pi \end{cases}$$
is continuous for $x \in [0, \pi]$
(a) $\left(\frac{\pi}{6}, \frac{\pi}{12}\right)$ (b) $\left(\frac{\pi}{6}, -\frac{\pi}{12}\right)$ (c) $\left(-\frac{\pi}{4}, \frac{\pi}{12}\right)$ (d) $\left(\frac{\pi}{4}, \frac{\pi}{6}\right)$

64. The value of K for which
$\cos^3 2\theta + 3\cos 2\theta = k(\cos^6\theta - \sin^6\theta)$
(a) 1 (b) 0 (c) 2 (d) 4

65. All the three vertices of an equilateral triangle lie on the parabola $y = x^2$ and one of its sides has a slope of 2. Then, sum of the x-coordinates of three vertices is

(a) $\frac{3}{11}$ (b) $\frac{2}{7}$

(c) $\frac{13}{17}$ (d) $\frac{4}{7}$

66. If $f(x) = \min\{2\sin x, 1-\cos x, 1\}$, then $\int_0^{\pi} f(x)\,dx$ is equal to

(a) $\sqrt{3}-1+\frac{5\pi}{6}$ (b) $\sqrt{3}-1+\frac{2\pi}{3}$

(c) $1-\sqrt{3}+\frac{2\pi}{3}$ (d) $1-\sqrt{3}+\frac{5\pi}{6}$

67. Sum of the given series,

$$\cot^{-1}\left(1^2+\frac{3}{4}\right)+\cot^{-1}\left(2^2+\frac{3}{4}\right)+\cot^{-1}\left(3^2+\frac{3}{4}\right)+\ldots \infty \text{ is}$$

(a) $\frac{\pi}{4}$ (b) $\tan^{-1} 2$

(c) $\tan^{-1} 3$ (d) None of these

68. Let $P(\alpha, \beta)$ lies in the first quadrant. If the two circles passes through P touches the coordinate axes cut each other orthogonally, then

(a) $\alpha^2+\alpha\beta+\beta^2=0$ (b) $\alpha^2-4\alpha\beta-\beta^2=0$

(c) $\alpha^2-4\alpha\beta+\beta^2=0$ (d) $\alpha^2+4\alpha\beta+\beta^2=0$

69. If the point $P(z), Q(-z)$ and $R(1-z)$ are the vertices of an equilateral ΔABC, then Re (z) is equal to

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{1}{4}$

70. Let $f_1 = \sum_{i=1}^{10} i(i-1)\,{}^{10}C_i$,

$f_2 = \sum_{i=1}^{10} i\,{}^{10}C_i$ and $f_3 = \sum_{i=1}^{10} i^2\,{}^{10}C_i$

Statement I $f_3 = 55\times 2^9$

Statement II $f_1 = 90\times 2^8$ and $f_2 = 10\times 2^8$,

Which of the following is correct?

(a) Statement I is false and Statement II is true.
(b) Statement I is true and Statement II is false.
(c) Statement I and Statement II both are true.
(d) Statement I and Statement II both are false.

71. Let $f(x) = \frac{x-3}{x+1}, x \neq -1$, then $f^{2100}(2100)$ is equal to, where $f^n(x) = f_o f_o \ldots of(x)$ (n times) is

(a) 2100 (b) 4200

(c) 1050 (d) 1

72. The radius of the base of a cone is increasing at the rate of 3 cm/min and the altitude is decreasing at the rate of 4 cm/min. The rate of change of lateral surface area when the radius is 7 cm and altitude is 24 cm is

(a) 50 cm^2/min (b) 54 π cm^2/min

(c) 62π cm^2/min (d) 66 π cm^2/min

73. If p, q and r are three complex numbers such that

$p^2+q^2+r^2=0$ and

$$\Delta = \begin{vmatrix} q^2+r^2 & pq & pr \\ pq & r^2+p^2 & qr \\ pr & qr & p^2+q^2 \end{vmatrix} = \left(\frac{K}{2}\right)^2 p^2q^2r^2,$$

then K is equal to

(a) ± 1 (b) ± 2

(c) ± 4 (d) ± 6

74. If t is any fixed real number such that $\frac{t-p}{ap}=\frac{t-q}{bq}=\frac{t-r}{cr}$, if a, b and c are in AP, then p, q and r are in

(a) arithmetic progression.
(b) geometric progression.
(c) harmonic progression.
(d) Both (a) and (b)

75. If ${}^{m}C_{p-1} = (\lambda^2-8)\,{}^{m+1}C_p$, then λ belongs to

(a) $[-3, -2\sqrt{2})$ (b) $[-3, -2\sqrt{2}]$

(c) $(-3, -2\sqrt{2})$ (d) $(-3, 3)$

76. There are exactly two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose distances from its centre are same and are equal to $\sqrt{\frac{(2a^2+3b^2)}{\sqrt{3}}}$. Then, the eccentricity of the ellipse is

(a) $\frac{1}{3}$ (b) $\sqrt{\frac{2}{3}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2\sqrt{3}}$

77. The straight lines whose direction cosines are given by $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular to each other, then the value of $\left(\frac{f}{a} + \frac{g}{b} + \frac{h}{c}\right)! + 2$ is equal to

(a) 2 (b) 3
(c) 9 (d) 8

78. If $\mathbf{r}$ is the position vector of a point which is equidistant form the points $\mathbf{a}$ and $\mathbf{b}$, then $\left|\mathbf{r} - \frac{\mathbf{a}+\mathbf{b}}{2}\right| \cdot (\mathbf{a} - \mathbf{b})$ is equal to

(a) $|\mathbf{a}|$ (b) $|\mathbf{b}|$
(c) 1 (d) 0

79. A set X has 5 elements. A_1 and A_2 are subsets of X. The probability that A_1 and A_2 are not disjoint, is

(a) $\frac{781}{1024}$ (b) $\frac{243}{1024}$
(c) $\frac{32}{1024}$ (d) $\frac{275}{1024}$

80. If Z_n, $n = 1, 2, 3, \ldots$ are the solutions of the system of equations $\left|\frac{z-12}{z-8i}\right| = \frac{5}{3}; \left|\frac{z-4}{z-8}\right| = 1$, then $\Sigma |z_n|$ is equal to

(a) $10 - \sqrt{325}$ (b) $5 + \sqrt{325}$
(c) $10 - 5\sqrt{13}$ (d) $10 + 5\sqrt{13}$

Section B : Numerical Value Type Questions

81. Consider the three points $A(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$, $B(-\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ and $C(\lambda\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \mu\hat{\mathbf{k}})$ are vertices of a triangle and its median through A is equally inclined to the positive direction of the axes. The value of $2\lambda - \mu$ is equal to ..... .

82. If $x \sin x \frac{dy}{dx} + (x + x\cos x + \sin x)y = \sin x$, $y\left(\frac{\pi}{2}\right) = 1 - \frac{2}{\pi}$, then $3 \lim\limits_{x \to 0} y(x)$ is equal to ..... .

83. If for $0 < x < \frac{\pi}{2}$, $\exp[(\sin^2 x + \sin^4 x + \sin^6 x + \ldots \infty) \log_e 2]$ satisfies the quadratic equation $x^2 - 9x + 8 = 0$. If the value of $\frac{\sin x - \cos x}{\sin x + \cos x}$ is $a - \sqrt{b}$, then $a + b$ equals ..... .

84. Let $\theta = \frac{\pi}{7}$ and $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, if $B = A + A^6$, then $\det(B)$ lies in (α, β) (where, α and β are numerical integer values), then $|\alpha + \beta|$ is equal to

85. If $\sum\limits_{i=1}^{n} (x_i + 1)^2 = 9n$ and $\sum\limits_{i=1}^{n} (x_i - 1)^2 = 5n$, then the variance of $x_1, x_2, \ldots, x_n$ is ..... .

86. If sum of the last three digits of 17^{256} is x, then x is equal to ..... .

87. If $\log_4\left(\frac{2f(x)}{1 - f(x)}\right) = x$, then the value of $f(20) + f(-19)$ is equal to ..... .

88. The length of perpendicular drawn from point $(3, -1, 11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is $\sqrt{k}$, then the value of k..... .

89. If $x + \frac{1}{x} = 1$, $p = x^{2005} + \frac{1}{x^{2005}}$ and q is the last digit of the number $2^{2^n} - 2$, where $n \in Z$, $n > 1$, then $p^2 + q^2$ is .....

90. The interior angles of a regular polygon measure 150° each. The number of diagonals of the polygon are ..... .

JEE PAPER-4

PHYSICS (Section A: Objective Type Questions)										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans	a	c	a	d	c	a	d	c	d	d
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans	b	a	a	a	d	b	b	b	b	d
	(Section B: Numerical Value Type Questions)									
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans	3	0.34	20	12	5	120	400	2	3	49
CHEMISTRY (Section A: Objective Type Questions)										
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans	d	a	c	a	c	d	b	c	b	a
Que.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
Ans	a	c	a	b	a	c	a	d	d	c
	(Section B: Numerical Value Type Questions)									
Que.	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans	5	1200	30	1	5000	5	86	10	15	2
MATHEMATICS (Section A: Objective Type Questions)										
Que.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
Ans	b	a	b	d	a	d	b	c	d	b
Que.	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
Ans	a	b	c	c	a	b	b	d	a	d
	(Section B: Numerical Value Type Questions)									
Que.	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
Ans	4	1	5	1	5	15	1	53	17	54

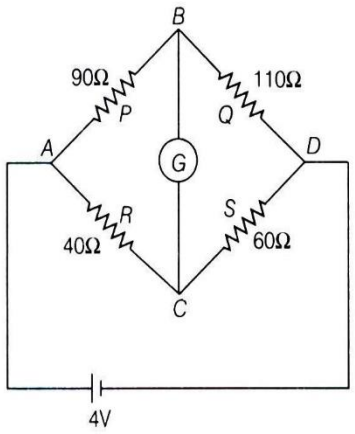
DETAILED SOLUTIONS

Physics

1. *(a)* Current through resistance P and Q,

$$i_1 = \frac{4}{90+110} = \frac{1}{50}\text{ A}$$

$$V_A - V_B = Pi_1 = 90\times\frac{1}{50} = 1.8\text{ V}$$



Current through resistance R and S,

$$i_2 = \frac{4}{40+60} = \frac{1}{25}\text{ A}$$

$$V_A - V_C = Ri_2 = 40\times\frac{1}{25} = 1.6\text{ V}$$

$$V_B - V_C = (V_A - V_C) - (V_A - V_B)$$
$$= 1.6 - 1.8 = -0.2\text{ V}$$

2. *(c)* Given, $K_A = 2K_B$

$$T_A - T_B = 72^\circ\text{C}$$

Let T be the temperature at the junction.

As, $\left(\frac{\Delta\theta}{\Delta t}\right)_A = \left(\frac{\Delta\theta}{\Delta t}\right)_B$

$$\therefore \quad \frac{K_A A(T_A - T)}{x} = \frac{K_B A(T - T_B)}{x}$$

or $\quad 2K_B(T_A - T) = K_B(T - T_B)$

or $\quad 2(T_A - T) = T - T_B$

Add $(T_A - T)$ on both sides, we get

$$3(T_A - T) = T_A - T + T - T_B$$

or $\quad 3(T_A - T) = T_A - T_B$

or $\quad T_A - T = \frac{T_A - T_B}{3}$

$$= \frac{72}{3} = 24^\circ\text{C}$$

$\therefore$ Temperature difference across the layer A

$$= T_A - T = 24^\circ\text{C}$$

3. *(a)* For isothermal process, pV = constant.

Therefore, p-V graph is a rectangular hyperbola.

Since, $T_2 > T_1 \Rightarrow (pV)_{\text{at } T_2} > (pV)_{\text{at } T_1}$

Thus, the graph given in option (a) correctly represents the given situation.

4. *(d)* Reaction under consideration is

$${}_1^2\text{H} + {}_1^2\text{H} \longrightarrow {}_2^4\text{He} + Q$$

Here, $BE_{\text{H}} = 1.1\text{ MeV}$ and $BE_{\text{He}} = 7\text{ MeV}$

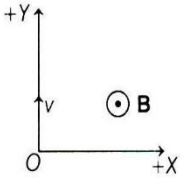
As Q value $= \sum BE_{\text{products}} - \sum BE_{\text{reactants}}$

$$Q = 4\times 7 - (2\times 1.1 + 2\times 1.1)$$
$$= 23.6\text{ MeV} = 23.6\times 10^6 \times 1.6\times 10^{-19}\text{ J}$$
$$= 37.76\times 10^{-13}\text{ J}$$

5. *(c)* $\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$

Let $\mathbf{B}$ be directed out of the plane of the paper.

For positive charges, i.e. α and δ, force experienced will be in the direction of $\mathbf{v}\times\mathbf{B}$, i.e. towards $+x$-direction.



For negative charge, i.e. β and γ, force will be along $-x$-direction.

Now, $r = \frac{mv}{Bq} = \frac{v}{B\left(\frac{q}{m}\right)}$

$\Rightarrow r \propto \frac{1}{s}$ where, $s = \frac{q}{m}$

Since, magnitude of charge is same

$\Rightarrow r \propto m$

Hence, (i) - (r), (ii)-(p), (iii)-(q), (iv)-(s)

6. *(a)* At STP, 22.4 L of any gas is 1 mole

$$\therefore \quad 5.6\text{ L} = \frac{5.6}{22.4} = \frac{1}{4}\text{ moles} = n$$

In adiabatic process, $TV^{\gamma-1}$ = constant

$$\therefore \quad T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$$

or $\quad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$$= T_1\left(\frac{5.6}{0.7}\right)^{5/3-1} \qquad \left[\because \gamma = \frac{C_p}{C_V} = \frac{5}{3}\right]$$

$$= 4T_1$$

Further, in adiabatic process, $Q = 0$

$\therefore W + \Delta U = 0$

$\therefore \quad W = -\Delta U = -nC_V \Delta T$

$$= -n\left(\frac{R}{\gamma - 1}\right)(T_2 - T_1)$$

$$= -\frac{1}{4}\left(\frac{R}{\frac{5}{3} - 1}\right)(4T_1 - T_1)$$

$$= -\frac{9}{8}RT_1$$

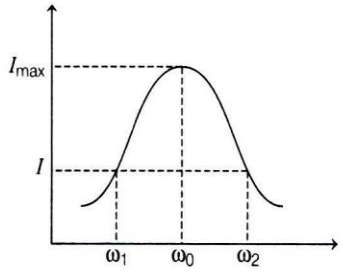
$$= -\frac{m}{n}RT_1 \text{ (given)}$$

$\therefore \quad m = 9, n = 8$

Hence, $m - n = 9 - 8 = 1$

7. ***(d)*** In a given *L-C-R* circuit, current will be same at different frequencies if impedance Z is same. Current in the circuit will be maximum, if circuit is in resonance.

Typical graph of current *versus* frequency of *L-C-R* circuit is



Now, as resistance is independent of frequency. So, current will same when impedance Z at two frequencies is same. At lower frequency, capacitive reactance is more but at higher frequencies, inductive reactance dominates.

So, $$Z_{1 \text{ at } \omega_1} = \frac{1}{\omega_1 C} - \omega_1 L$$

$$Z_{2 \text{ at } \omega_2} = \omega_2 L - \frac{1}{\omega_2 C}$$

For current to be same

$$Z_1 = Z_2$$

$$\Rightarrow \quad \frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$\Rightarrow \quad \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)\frac{1}{C} = (\omega_1 + \omega_2)L$$

$$\Rightarrow \quad \frac{\omega_1 + \omega_2}{\omega_1 \omega_2} = (\omega_1 + \omega_2)LC$$

Now, current is maximum at $\omega_0 = \frac{1}{\sqrt{LC}}$

$\therefore$ Resonant frequency, $\omega_0 = \sqrt{\omega_1 \omega_2}$

For given problem,

$$\omega_0 = \sqrt{8 \times 18} = 12 \text{ MHz}$$

8. ***(c)*** $F_{air} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$

$$F_{liquid} = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{q_1 q_2}{r^2} = \frac{F_{air}}{K} = \frac{F}{K}$$

9. ***(d)*** Here, rigid body is moving with constant velocity, i.e. it is in mechanical equilibrium.

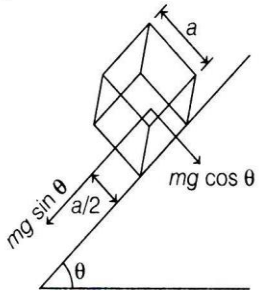
$\Rightarrow \quad \sum F_{ex} = 0$, i.e. $\sum F_x = 0$,

$\sum F_y = 0, \sum F_z = 0$

and $\quad \sum \tau_x = 0, \sum \tau_y = 0,$

$\sum \tau_z = 0$

Since, the cubical block slides with a uniform velocity and does not topple.



$\therefore$ Torque due to normal force = Torque due to weight

= Component of weight parallel to plane

$\times$ Perpendicular distance from lower face = $(mg \sin\theta)\frac{a}{2}$

10. ***(d)*** Equation of simple harmonic wave is given as

$$y = A \sin(\omega t + \phi)$$

Here, $y = \frac{A}{2} \Rightarrow A \sin(\omega t + \phi) = \frac{A}{2}$

So, $\quad \delta = \omega t + \phi = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$

So, the phase difference of the two particles when they are crossing each other at $y = A/2$ in opposite direction are

$$\delta = \delta_1 - \delta_2 = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$$

11. ***(b)*** From graph, we can say temperature is increasing then become constant which suggests phase transition and once its done, its temperature further increases and we see another phase transition.

Here, we can say initially substance was in solid state then liquid and gas at the end. As we see, the slope of solid state is more than liquid state, so specific heat is greater in liquid state. The latent heat of vaporisation is more that its latent heat of fusion.

12. ***(a)*** When the capacitor is fully charged, the voltage V_1 across it = voltage of the battery = 12 V

The charge on C_1, $Q = C_1 V_1 = 4 \times 10^{-6} \times 12$

$$= 48 \times 10^{-6} = 48 \mu C$$

When S_1 is opened and S_2 is closed, charge will flow from C_1 and C_2 until they have the same potential V which is given by $V = \frac{Q}{C_1 + C_2} = \frac{48}{4+8}$

$$= \frac{48}{12} = 4\text{ V}$$

Since, C_1 and C_2 are parallel, therefore they have the same, i.e. common potential.

Now, $U_i = \frac{1}{2} C_1 V_1^2$

$$= \frac{1}{2} \times 4 \times 10^{-6} \times (12)^2$$

$$= 288 \times 10^{-6}\text{ J}$$

$$U_f = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (4)^2$$

$$= \frac{1}{2} \times 4 \times 10^{-6} \times 4 \times 4 = 32 \times 10^{-6}\text{ J}$$

$\therefore$ Loss of energy stored in C_1

$$= 288 \times 10^{-6} - 32 \times 10^{-6} = 256 \times 10^{-6}\text{ J}$$

The percentage loss of energy is $\frac{256 \times 10^{-6}}{288 \times 10^{-6}} \times 100$

$$= 88.88\% \approx 88.9\%$$

13. ***(a)*** The maximum height,

$$h = \frac{u^2 \sin^2 \theta}{2g} \quad \ldots(\text{i})$$

The change in height is given by differentiating this expression. Thus,

$$dh = \frac{2u du \sin^2 \theta}{2g} \quad \ldots(\text{ii})$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{dh}{h} = \frac{2du}{u}$$

$$\Rightarrow \quad \frac{2du}{u} = 0.1 \qquad \left[\because \frac{dh}{h} = 10\%\right]$$

Now, the range, $R = \frac{u^2 \sin 2\theta}{g} \quad \ldots(\text{iii})$

$$\therefore \quad dR = \frac{2u\, du \sin 2\theta}{g} \quad \ldots(\text{iv})$$

On dividing Eq. (iv) by Eq. (iii), we get

$$\Rightarrow \quad \frac{dR}{R} = \frac{2du}{u} = 0.1$$

Hence, range is also increased by 10%.

14. ***(a)*** KE of proton and α-particle shall be 1 eV and 2 eV, respectively.

Now, momentum, $p = \sqrt{2m\,\text{KE}}$

$\therefore$ Momentum of proton, $p_p = \sqrt{2 m_p \times 1}$

$\therefore$ Momentum of α-particle whose mass $m_\alpha = 4 m_p$ is

$$p_\alpha = \sqrt{8 m_p \times (2)}$$

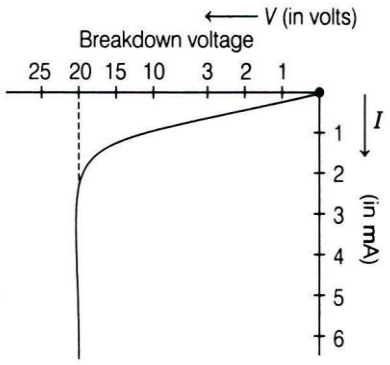
$$= \sqrt{16 m_p}$$

$\therefore$ Now, $\lambda = \frac{h}{p}$ [de-Broglie relation]

$$\Rightarrow \quad \frac{\lambda_p}{\lambda_\alpha} = \frac{p_\alpha}{p_p} = \frac{\sqrt{16 m_p}}{\sqrt{2 m_p}} = 2\sqrt{2} : 1$$

15. ***(d)*** All the statements I, II and III are correct. When a stone is moving around a circular path, its velocity acts tangentially to the circle. When the string breaks, the centripetal force will not act. Due to inertia, the stone continues to move along the tangent to the circular path and stone flies off tangentially. The necessary centripetal force required to keep the stone moving in a circle is provided by tension in the string. Hence, the centripetal force is a real force.

16. ***(b)*** In a reverse biased *p-n* junction, when the applied bias voltage is equal to the breakdown voltage, then voltage remains constant, while current increases sharply. Typical voltage-current graph of a reversed biased *p-n* diode is



17. ***(b)*** The dimensional formula of Young's modulus is $[ML^{-1}T^{-2}]$.

Now, the dimensions of v, A and F in terms of M, L and T are

$$[v] = [LT^{-1}], [A] = [LT^{-2}]$$

and $[F] = [MLT^{-2}]$

Let $Y = [v^a A^b F^c]$

$$[M^1 L^{-1} T^{-2}]$$

$$= [LT^{-1}]^a \times [LT^{-2}]^b \times [MLT^{-2}]^c$$

$$[M^1 L^{-1} T^{-2}] = [M^c L^{a+b+c} T^{-a-2b-2c}]$$

On equating powers of M, L and T, we have $c = 1$,

$a + b + c = -1, -a - 2b - 2c = -2$

On solving above equations, we get

$$a = -4, b = 2$$

Thus, new dimensional formula of

$$[Y] = [FA^2 v^{-4}]$$

18. *(b)* Escape velocity does not depend on the direction of projection of the body from the surface of the earth.
If the velocity of a revolving satellite around the earth is increased to $\sqrt{2}$ times or 41.4%, then it escapes out from the earth because in this condition, its velocity is greater than the escape velocity.

19. *(b)* We know that Young's modulus of elasticity is given by

$$Y = \frac{FL}{lA} = \frac{FL}{l\left(\frac{\pi d^2}{4}\right)}$$

Substituting the values given, we get

$$Y = \frac{50 \times 1.1 \times 4}{(1.25 \times 10^{-3}) \times \pi \times (5.0 \times 10^{-4})^2}$$

$$= 2.24 \times 10^{11}\ \text{Nm}^{-2}$$

Again, $\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + \frac{\Delta l}{l} + 2\frac{\Delta d}{d}$

$$= \frac{0.1}{110} + \frac{0.001}{0.125} + 2 \times \frac{(0.001)}{0.05} = 0.0489$$

$\Rightarrow \quad \Delta Y = 0.0489\, Y$

$$= (0.0489)(2.24 \times 10^{11})\text{Nm}^{-2}$$

$$= 1.09 \times 10^{10}\ \text{Nm}^{-2}$$

20. *(d)* Electromagnetic waves consist of oscillating electric and magnetic field that are perpendicular to each other and also perpendicular to the direction of propagation of EM waves. Thus, electromagnetic waves are transverse in nature and travel through vacuum with same speed of 3×10^8 m/s.

21. (3) Charge through the battery $= Q_0 - (-2Q_0) = 3Q_0$

Work done by battery $= \frac{(3Q_0)Q_0}{C} = \frac{3Q_0^2}{C}$

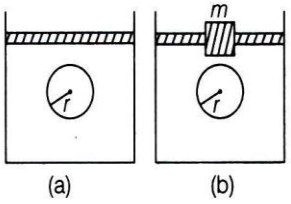
As compared with given equatio, $= \frac{aQ_0^2}{c}$

$\therefore \qquad a = 3$

22. (0.34) The volume of sphere in liquid,

$$V = \frac{4}{3}\pi r^3$$

When mass m is placed on the piston, the increased pressure, $p = \frac{mg}{a}$



Since, this increased pressure is equally applicable to all directions on the sphere, so there will be decrease in volume of sphere, due to decrease in its radius.

$$\Delta V = \frac{4\pi}{3} \times 3r^2 \Delta r = 4\pi r^2 \Delta r$$

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{4\pi r^2 \Delta r}{\frac{4}{3}\pi r^3} = \frac{3\Delta r}{r}$$

$\therefore$ Bulk modulus, $K = \frac{pV}{\Delta V} = \frac{mg}{a} \times \frac{r}{3\Delta r}$

$$\therefore \qquad \frac{\Delta r}{r} = \frac{mg}{3Ka}$$

On comparing with the given equation,

$$= \frac{x\,mg}{Ka}$$

$$\therefore \qquad x = 0.34$$

23. (20) By Bohr's formula, $\frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For first line of Lyman series, $n_1 = 1, n_2 = 2$

$$\therefore \qquad \frac{1}{\lambda} = Z^2 R \frac{3}{4}$$

In the case of hydrogen atom, $Z = 1$

$$\frac{1}{\lambda} = R\frac{3}{4}$$

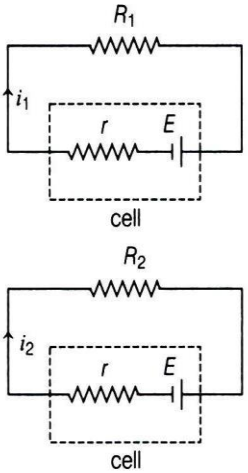
For hydrogen like atom, $Z = 11$

$$\frac{1}{\lambda'} = 121R\frac{3}{4}$$

$$\frac{\lambda'}{\lambda} = \frac{3R}{4} \times \frac{4}{121R \times 3} = \frac{1}{121}$$

$$\lambda' = \frac{\lambda}{121} = \frac{2420}{121} = 20\,\text{Å}$$

24. *(12)* Consider two cases,



Power delivered in both the cases is same.

Thus, $\qquad i_i^2 R_1 = i_2^2 \times R_2$

$$\Rightarrow \quad \left(\frac{E}{R_1+r}\right)^2 R_1 = \left(\frac{E}{R_2+r}\right)^2 R_2$$

$$\Rightarrow \quad (R_2+r)^2 \cdot R_1 = (R_1+r)^2 \cdot R_2$$

$$\Rightarrow \quad (R_1-R_2)r^2 = (R_1-R_2)(R_1R_2)$$

$$\Rightarrow \quad r = \sqrt{R_1R_2}$$

Put, $R_1 = 18\Omega$ and $R_2 = 8\Omega$

Then, $r = \sqrt{144} = 12\Omega$

25. (5) At maximum compression, the velocity of both the blocks will be same.

From conservation of momentum theorem,

$$4 \times 10 = (4+4) \times v' \text{ or } v' = 5 \text{ m/s}$$

26. (120) Fundamental frequency, $f = \frac{v}{2L}$

According to first condition,

$$120 = \frac{v}{2L_1} \quad \ldots(i)$$

According to second condition,

$$180 = \frac{v}{2L'} \quad \ldots(ii)$$

From Eq. (i) and (ii), we have

$$\frac{L'}{L_1} = \frac{120}{180} = \frac{2}{3}$$

$$\Rightarrow \quad L' = \frac{2}{3}L_1 = \frac{2}{3} \times 180 = 120 \text{ cm}$$

27. (400) $\tan\phi = \frac{|X_L - X_C|}{R}$

***Case* I** When capacitor is removed, then

$$\tan 60 = \frac{X_C}{R}$$

$$\tan 60° = \frac{X_L}{R} \Rightarrow X_L = \sqrt{3}\,R$$

***Case* II** When inductor is removed, then

$$\tan 60° = \frac{X_C}{R}$$

$$X_C = \sqrt{3}\,R$$

So, $Z = \sqrt{R^2 + (\sqrt{3}R - \sqrt{3}R)^2} = R$

So, *L-C-R* would acts as pure resistor and thus power factor $\cos\phi = 1$

Power dissipated, $P = \frac{V^2}{R} = \frac{200 \times 200}{100} = 400 \text{ W}$

28. (2) Time taken by each ball to reach the highest point, $t = \frac{1}{n}$ s

As the juggler throws second ball, when the first ball is at its highest point, so $v = 0$

Using $v = u - gt$, we have

$$0 = u - g \times 1/n \Rightarrow u = g/n$$

Also, $v^2 = u^2 + 2(-g)h$

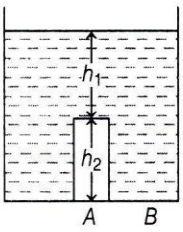
$$0 = (g/n)^2 + 2(-g) \times h \text{ or } h = \frac{g}{2n^2}$$

Comparing with given equation

$$h = \frac{g}{2n^2} = \frac{g}{2n^p}$$

$\therefore \quad p = 2$

29. (3) Apparent depth of *A*,



$$d_1 = \frac{h_1}{4/3} + \frac{h_2}{3/2} = \frac{3}{4}h_1 + \frac{2}{3}h_2 \quad \ldots(i)$$

Apparent depth of *B*,

$$d_2 = \frac{(h_1+h_2)}{4/3} = \frac{3}{4}(h_1+h_2) \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\therefore \quad d_2 - d_1 = \left(\frac{3}{4} - \frac{2}{3}\right)h_2 = 3 \text{ mm} \quad (\text{where, } h_2 = 36 \text{ mm})$$

$$\Rightarrow \quad x = 3$$

30. (49) The number of fringes on either side of *C* of screen,

$$n_1 = \left[\frac{AC}{\beta}\right] = \left[\frac{0.5}{0.021}\right] = [23.8] \approx 24$$

Total number of fringes = $2n_1$ + fringe at centre

$$= 2n_1 + 1 = 2 \times 24 + 1 = 48 + 1 = 49$$

Chemistry

31. (*d*) Glucose on reacting with conc. HCl forms levulinic acid.

$$C_6H_{12}O_6 \xrightarrow{\text{Conc. HCl}} \underset{\text{levulinic acid}}{CH_3COCH_2CH_2COOH} + HCOOH + H_2O$$

32. (*a*) It is plot of $\ln p_{CO_2}$ *v/s* $\frac{1}{T}$

$$K_p = Ae^{-\Delta H_r^\circ/RT}$$

$$\ln K_p = \ln A - \Delta H_r^\circ / RT$$

$$K_p = [p_{CO_2}]$$

$$\ln [p_{CO_2}] = \ln A - \Delta H_r^\circ / RT$$

$$\text{Slope} = -\Delta H_r^\circ / R$$

Therefore, option (a) is correct.

33. (*c*) Birth control pills are made up of a mixture of estrogen and progesterone derivative. Both of them are hormones.

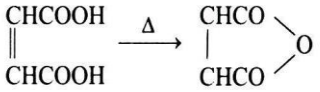
34. (*a*) Ionic radii decrease as we move across lanthanide series due to lanthanide contraction.

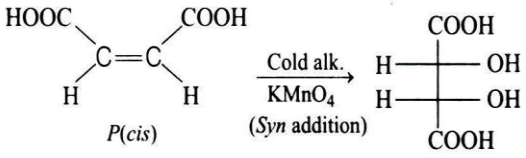
Hence, the correct order is $Yb^{3+} < Pm^{3+} < Ce^{3+} < La^{3+}$.

35. (*c*) The compound which contains —CHO group, gives positive Tollen's test and the compound with $CH_3-\overset{\overset{O}{\|}}{C}-$ group gives iodoform test.

Thus, the organic compound is $CH_3-\overset{\overset{O}{\|}}{\underset{\underset{H}{|}}{C}}$

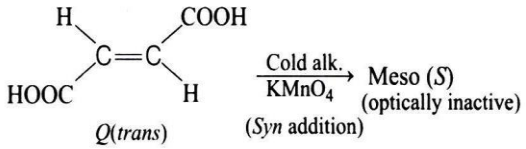
36. (*d*) Formation of anhydride from dicarboxylic acid indicates *cis* isomers.





P(*cis*)

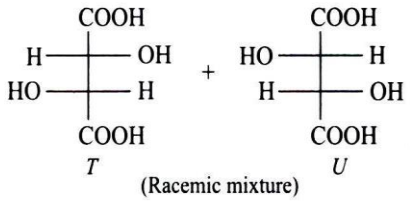
(*Syn* addition)



Q(*trans*)

(*Syn* addition)

Meso (*S*) (optically inactive)



T + *U*

(Racemic mixture)

Racemic mixture is optically inactive.

37. (*b*) The given reaction are represented as

$$\underset{(A)}{NH_4NO_2} + KOH \longrightarrow \underset{(B)}{NH_3}\uparrow + \underset{(C)}{KNO_2} + H_2O$$

$$\underset{(C)}{KNO_2} + 3\,Zn + 5\,KOH + 5H_2O \longrightarrow 3K_2[Zn(OH)_4] + \underset{(B)}{NH_3}\uparrow$$

$$\underset{(A)}{NH_4NO_2} \xrightarrow{Heat} N_2\uparrow + 2\,H_2O$$

(Does not support combustion)

38. (*c*)

- **2, 4-DNP test** Compound that gives positive 2,4-DNP test means the compound contain carbonyl group.
- **Iodoform test** A compound show iodoform test means it must contain keto group (not aldehydic) $-\overset{\overset{O}{\|}}{C}-CH_3$.

This is also confirmed by negative Tollen's test and Fehling test.

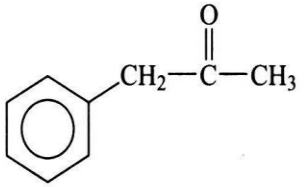
- **Baeyer's test** If compound does not decolourises bromine water mean there is no any free double bond or triple bond.

Molecular formula = $C_9H_{10}O$

$$u = (9+1) - \frac{10}{2} = 5$$

Degree of unsaturation is 5 and no any double or triple bond thus there must be a benzene ring and one C=O group.

Out of given four choices only option (c) is satisfying all criterion.



Hence, (c) is the correct choice.

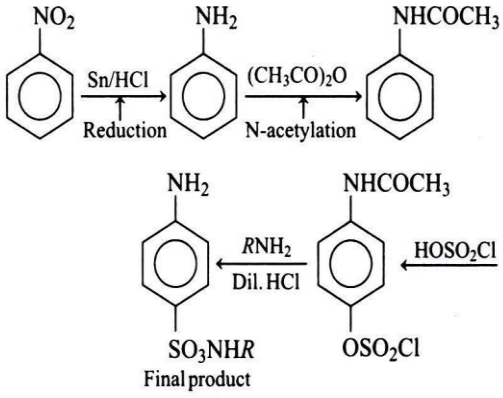
39. (*b*) The given reaction are represented as

$$\underset{\substack{(X)\\(3C-2e^-,\\2C-2e^-\text{ bonds})}}{B_2H_6} + 2NH_3 \xrightarrow{\text{Low temperature}} B_2H_6\cdot 2NH_3 \xrightarrow[\downarrow]{200^\circ C} \underset{(Y)}{B_3N_3H_6}$$

Y has structure similar to benzene (inorganic benzene).

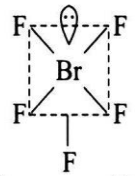
$$B_2H_6 + 2NH_3 \xrightarrow[\text{temperature}]{\text{High}} \underset{\substack{(Z)\\\text{Inorganic}\\\text{graphite}}}{2(BN)} + 6\,H_2$$

40. (*a*) The reaction sequence involves various processes are shown as

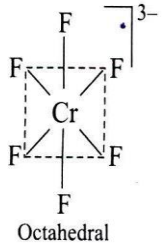


41. (*a*) The correct match is A-II, B-IV, C-I, D-III

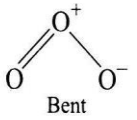
A. $BrF_5 \equiv$



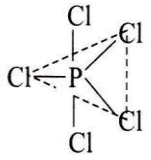
Square pyramidal

B. $[CrF_6]^{3-} \equiv$ 

Octahedral

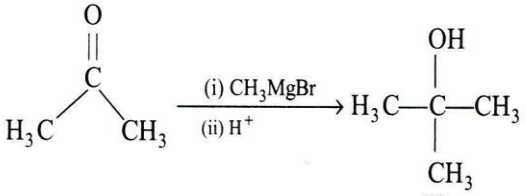
C. $O_3 \equiv$ 

Bent

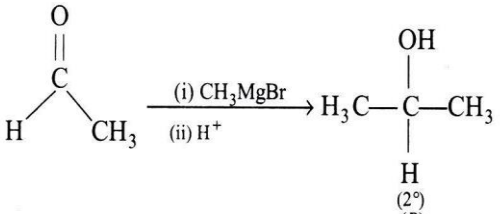
D. $PCl_5 \equiv$ 

Trigonal bipyramidal

42. (*c*) Secondary and tertiary alcohols are differentiated by Lucas reagent. The reactions are represented as



(3°)
(*A*)



(2°)
(*B*)

43. (*a*) Potassiumtetracyanonickelate(II), potassiumtetrachloronickelate(II).

The reactions are as follows:

$$NiCl_2 + \underset{\text{(excess)}}{4KCN} \longrightarrow K_2[Ni(CN)_4] + 2KCl$$

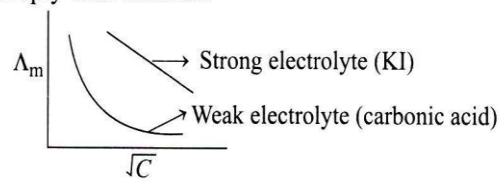
A is $K_2[Ni(CN)_4]$ also called potassiumtetracyanonickelate (II).

$$K_2[Ni(CN)_4] + 4HCl(\text{conc.}) \longrightarrow K_2[Ni(CN)_4] + 4HCN$$

B is $K_2[NiCl_4]$ also called potassium tetrachloronickelate(II).

44. (*b*) Both Statement I and Statement II are incorrect.

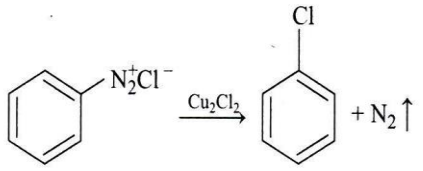
For KI, molar conductivity increases slowly with dilution. For carbonic acid, molar conductivity increases steeply with dilution.



45. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

46. (*c*) The correct IUPAC name of $[Fe(O_2)(CN)_4Cl]^{4-}$ is chlorotetracyanosuperoxoferrate (II) ion.

47. (*a*) Chlorobenzene is formed by reacting diazonium salt with Cu_2Cl_2. The reaction is represented as



48. (*d*) Due to high lattice energy of barium sulphate and high hydration energy of sodium sulphate, barium sulphate is sparingly soluble while sodium sulphate is completely soluble in water.

49. (*d*) $BO = \dfrac{N_b - N_a}{2}$

Bond strength ∝ bond order

M.O.E.C of $N_2 = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^2 = \pi 2p_y^2, \sigma 2p_z^2$

$$\text{Bond order} = \frac{10-4}{2} = 3$$

M.O.E.C of $N_2^+ = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^2 = \pi 2p_y^2, \sigma 2p_z^1$

$$\text{Bond order} = \frac{9-4}{2} = 2.5$$

Here, on converting N_2 to N_2^+ bond order decreases, so bond strength decreases.

M.O.E.C of $O_2 = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^1 = \pi^* 2p_y^1$

$$\text{Bond order} = \frac{10-6}{2} = 2$$

M.O.E.C of $O_2^+ = \sigma 1s^2\ \sigma^* 1s^2, \sigma 2s^2\ \sigma^* 2s^2, \sigma 2p_z^2 \quad \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^1 = \pi^* 2p_y^0$

$$\text{Bond order} = \frac{10-5}{2} = 2.5$$

On conversion of O_2 to O_2^+ bond order increases and hence bond strength also increases.

Study Tactics

To solve this question, remember that molecular orbital theory consider the intraction of atomic orbitals to form molecular orbitals and more electrons in bonding molecular orbitals lead to stronger bonds.

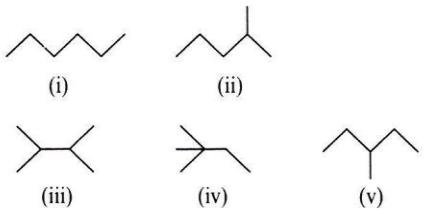
50. (*c*) A. III. On acidification $(B_4O_7)^{2-}$ gives $B(OH)_3$ or (H_3BO_3)

B. IV. AlO_2^- on dilution yields a white ppt. of $Al(OH)_3$.

C. I. When heated, $(SiO_4)^{4-}$ changes to $(Si_2O_7)^{6-}$.

D. II. Bi^{3+} hydrolyses to yield $(BiO)^+$ ion.

51. (*5*) The structural isomers are represented as



52. (*1200*) For adiabatic condition,

$$\frac{T_2}{T_1}=\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

or $$\frac{T_2}{T_1}=\left(\frac{1}{27}\right)^{1.33-1}=\left(\frac{1}{27}\right)^{0.33}=\left(\frac{1}{27}\right)^{1/3}=\frac{1}{3}$$

or $$T_2=\frac{1}{3}\times T_1$$

$$T_2=\frac{1}{3}\times 300=100\text{ K}$$

From first law of thermodynamics,

$$\Delta E=q+W$$

$$W=-\Delta E \quad (\because q=0 \text{ for adiabatic expansion})$$

$$=-C_V\,(T_2-T_1)$$

$$=-6\,(100-300)=+1200\text{ cal}$$

53. (*30*) Millieq. of $Na_2S_2O_3 = 24.35\times 1/10 = 2.435$

This would be the milliequivalent of I_2 and therefore that of Cl_2 (which liberates I_2 from KI).

Milliequivalents of Cl_2 in 500 mL $= 2.435\times 20 = 48.7$

Meq. of Cl_2 = Millieq. of bleaching powder = Millieq. of available Cl_2 in the bleaching powder.

$$\text{Percentage of chlorine}=\frac{48.7}{1000}\times\frac{35.5}{5.7}\times 100$$

$$=30.33\%\approx 30\%$$

54. (*1*) KE = quantum energy – threshold energy

$$=\frac{6.626\times 10^{-34}\times 3\times 10^{8}}{3000\times 10^{-10}}-\frac{6.626\times 10^{-34}\times 3\times 10^{8}}{4000\times 10^{-10}}$$

$$=6.626\times 10^{-19}-4.9695\times 10^{-19}$$

$$=1.6565\times 10^{-19}\text{ J}$$

$$\text{KE}=1.6565\times 10^{-19}=\frac{1}{2}mv^2$$

$$\therefore\ m^2v^2=2\times 1.6565\times 10^{-19}\times 9.1\times 10^{-31}$$

$$mv=5.49\times 10^{-25}$$

$$\text{de-Broglie wavelength, }\lambda=\frac{h}{mv}=\frac{6.626\times 10^{-34}}{5.49\times 10^{-25}}$$

$$=1.2\times 10^{-9}\text{ m}\approx 1\times 10^{-9}\text{ m}$$

55. (*5000*) For the reaction,

	A	+ B	$\longrightarrow$	C	+ D
At $t=0$	1	1		0	0
At $t=t$	$1-x$	$1-x$		x	x

$$R=k[A]^{1/2}\,[B]^{1/2}$$

$$\frac{dx}{dt}=k\,(1-x)^{1/2}\,(1-x)^{1/2}$$

$$\frac{dx}{dt}=k(1-x)$$

Integrating on both sides,

$$k=\frac{2.303}{t}\log\frac{1}{1-x}$$

$$4.606\times 10^{-4}=\frac{2.303}{t}\log\frac{1}{0.1}$$

$$2\times 10^{-4}=\frac{1}{t}\text{ or }t=\frac{10^4}{2}=\frac{10000}{2}=5000\text{ s}$$

56. (*5*) $p_A^\circ = 1000$ torr and $p_B^\circ = 1600$ torr

$$x_A=0.6,\ x_B=0.4$$

$$p_{\text{Total}}=p_A^\circ x_A+p_B^\circ x_B$$

$$=1000\times 0.6+1600\times 0.4$$

$$=1240$$

$$y_A=\frac{x_A\,p_A^\circ}{p_{\text{Total}}}$$

$$=\frac{1000\times 0.6}{1240}=0.483$$

$$y_B=1-0.48$$

$$=0.517$$

$$\approx 5\times 10^{-1}$$

57. (*86*) **After addition of Cd, it oxidises into** Cd^{2+}.

$$\underset{0.1-X}{NO_3^-}(aq)+\underset{0.4-4X}{4H^+}(aq)+3e^-\longrightarrow NO(g)+2H_2O(l)$$

$X = 0.06$ [Given]

$[NO_3^-]$ remaining $= 0.1-0.06 = 0.04$ M

$[H^+]$ remaining $= 0.4-4\times 0.06$

$$=0.4-0.24$$

$$=0.16\text{ M}$$

$$E_{NO_3^-/\,NO}=E^\ominus_{NO_3^-/NO}-\frac{0.0591}{3}\log\frac{1}{[NO_3^-][H^+]^4}$$

$$=0.95-\frac{0.0591}{3}\log\frac{1}{(0.04)\,(0.16)^4}$$

$$=0.95-\frac{0.0591}{3}\log\frac{1}{4\times 10^{-2}\times 10^{-8}\times 2^{16}}$$

$$=0.95-\frac{0.0591}{3}\log\frac{10^{10}}{2^{18}}$$

$$=0.86\text{ V}\approx 86\times 10^{-2}\text{ V}$$

58. *(10)* The number of ionisable Cl^- in $[Cr(H_2O)_5Cl]Cl_2$ is 2.

$\therefore$ Millimoles of $Cl^- = 50 \times 0.01 \times 2 = 1$

$\therefore$ Millimoles of Ag^+ required = 1

Now, molarity $= \dfrac{\text{no. of millimoles}}{V\ (\text{in mL})} \Rightarrow 0.1 = \dfrac{1}{V}$

$V = 10\,\text{mL}$

59. *(15)* The percentage of sulphur

$$= \frac{32}{233} \times \frac{\text{Weight of } BaSO_4}{\text{Weight of organic compound}} \times 100$$

$$= \frac{32}{233} \times \frac{0.5}{0.45} \times 100 = 15.2\% \approx 15\%$$

60. *(2)* $MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$

$$E = 1.51 - \frac{0.059}{5} \log \frac{[Mn^{2+}]}{[MnO_4^-][H^+]^8}$$

Taking Mn^{2+} and MnO_4^- in standard state, i.e. 1M,

$$E = 1.51 - \frac{0.059}{5} \times 8 \log \frac{1}{[H^+]}$$

$$E = 1.51 - \frac{0.059}{5} \times 8 \times 3 = 1.2268\ \text{V}$$

Hence, at this pH, MnO_4^- will oxidise only Br^- and I^- as standard reduction potential of Cl_2/Cl^- is 1.36 V which is greater than that for MnO_4^-/Mn^{2+}.

Mathematics

61. *(b)* Given, $A \ln|\cos x + \sin x - 2| + Bx + C = \int \dfrac{2\cos x - \sin x + \lambda}{\cos x + \sin x - 2}\,dx$

On differentiating both sides, we get

$$\frac{d}{dx}[A \ln|\cos x + \sin x - 2| + Bx + C] = \frac{2\cos x - \sin x + \lambda}{\cos x + \sin x - 2}$$

$$\Rightarrow \frac{A(\cos x - \sin x)}{\cos x + \sin x - 2} + B = \frac{2\cos x - \sin x + \lambda}{\cos x + \sin x - 2}$$

$$\Rightarrow \frac{(A+B)\cos x + (B-A)\sin x - 2B}{\cos x + \sin x - 2} = \frac{2\cos x - \sin x + \lambda}{\cos x + \sin x - 2}$$

On comparing both sides, we get

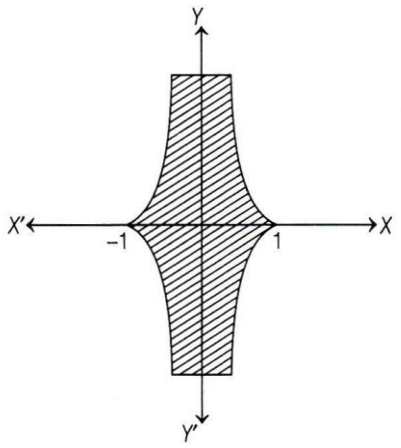
$A + B = 2,\ B - A = -1$ and $\lambda = -2B$

On solving, we get

$$A = \frac{3}{2},\ B = \frac{1}{2} \text{ and } \lambda = -1$$

62. (a) $\log x$ is defined for $x > 0$,

$|\log x| \geq 0$ and $|\log|x|| \geq 0$



$$\therefore \quad A = 4\int_0^1 |\log x|\,dx = -4\int_0^1 \log x\,dx$$

$$= -4[x\log x - x]_0^1 = -4(-1) = 4 \text{ sq units}$$

63. *(b)* $f(x) = \begin{cases} x + a\sqrt{2}\sin x, & 0 \leq x < \dfrac{\pi}{4} \\ 2x\cot x + b, & \dfrac{\pi}{4} \leq x \leq \dfrac{\pi}{2} \\ a\cos 2x - b\sin x, & \dfrac{\pi}{2} < x \leq \pi \end{cases}$

At $x = \dfrac{\pi}{4}$, LHL $= \lim\limits_{x \to \frac{\pi^-}{4}} f(x)$

$$= \lim_{x \to \frac{\pi^-}{4}} (x + a\sqrt{2}\sin x)$$

$$= \frac{\pi}{4} + a$$

$$\text{RHL} = \lim_{x \to \frac{\pi^+}{4}} f(x)$$

$$= \lim_{x \to \frac{\pi^+}{4}} (2x\cot x + b) = \frac{\pi}{2} + b$$

and $f\left(\dfrac{\pi}{4}\right) = 2 \times \dfrac{\pi}{4}\cot\left(\dfrac{\pi}{4}\right) + b$

$$= \frac{\pi}{2} + b$$

Since, $f(x)$ is continuous.

$\therefore \quad \text{LHL} = \text{RHL} = f\left(\dfrac{\pi}{4}\right)$

$$\Rightarrow \frac{\pi}{4} + a = \frac{\pi}{2} + b \Rightarrow a - b = \frac{\pi}{4}$$

At $x = \dfrac{\pi}{2}$, LHL $= \lim\limits_{x \to \frac{\pi^-}{2}} f(x)$

$$= \lim_{x \to \frac{\pi^-}{2}} (2x\cot x + b) = b$$

$$\text{RHL} = \lim_{x \to \frac{\pi^+}{2}} f(x)$$

$$= \lim_{x \to \frac{\pi^+}{2}} (a\cos 2x - b\sin x) = -a - b$$

and $$f\left(\frac{\pi}{2}\right) = 2\frac{\pi}{2}\cot\frac{\pi}{2} + b = b$$

$\because f(x)$ is continuous.

$\therefore$ $\text{LHL} = \text{RHL} = f\left(\frac{\pi}{2}\right)$

$\Rightarrow$ $b = -a - b$

$\Rightarrow$ $a + 2b = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$a = \frac{\pi}{6},\ b = -\frac{\pi}{12}$$

64. *(d)* $\therefore (\cos^2\theta - \sin^2\theta)^3 + 3(\cos^2\theta - \sin^2\theta)$

$= \cos^6\theta - \sin^6\theta - 3\cos^4\theta\sin^2\theta$

$+ 3\cos^2\theta\sin^4\theta + 3\cos^2\theta - 3\sin^2\theta$

$= \cos^6\theta - \sin^6\theta - 3\cos^4\theta(1 - \cos^2\theta)$

$+ 3(1 - \sin^2\theta)\sin^4\theta + 3\cos^2\theta - 3\sin^2\theta$

$= \cos^6\theta - \sin^6\theta + 3\cos^6\theta + 3\sin^4\theta$

$- 3\cos^4\theta - 3\sin^6\theta + 3\cos^2\theta - 3\sin^2\theta$

$= 4(\cos^6\theta - \sin^6\theta) + 3\cos^2\theta(1 - \cos^2\theta)$

$- 3\sin^2\theta(1 - \sin^2\theta)$

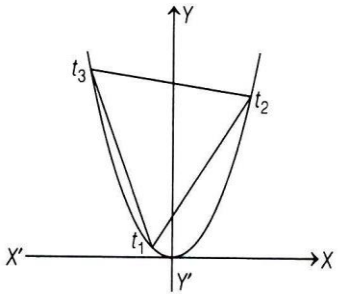
$= 4(\cos^6\theta - \sin^6\theta)$

Hence, $k = 4$

65. *(a)* $m_1 = 2 = \frac{t_2^2 - t_1^2}{t_2 - t_1} = t_1 + t_2 = 2$ ...(i)

Now, $\frac{2 - m_2}{1 + 2m_2} = \sqrt{3}$

[$\because$ given triangle is an equilateral triangle]



$\therefore$ $2 - m_2 = \sqrt{3} + 2\sqrt{3}m_2$

$\therefore$ $m_2(2\sqrt{3} + 1) = 2 - \sqrt{3}$

$\therefore$ $t_1 + t_3 = m_2 = \frac{2 - \sqrt{3}}{1 + 2\sqrt{3}}$ ...(ii)

Again, $\frac{m_3 - 2}{1 + 2m_3} = \sqrt{3}$

$\therefore$ $m_3 - 2 = \sqrt{3} + 2\sqrt{3}m_3$

$\therefore$ $m_3(1 - 2\sqrt{3}) = 2 + \sqrt{3}$

$\Rightarrow$ $m_3 = \frac{2 + \sqrt{3}}{1 - 2\sqrt{3}} = t_2 + t_3$

$\Rightarrow 2(t_1 + t_2 + t_3) = 2 + \frac{2 - \sqrt{3}}{1 + 2\sqrt{3}} + \frac{2 + \sqrt{3}}{1 - 2\sqrt{3}}$

$= 2 + \frac{(2 - \sqrt{3})(2\sqrt{3} - 1) - (2 + \sqrt{3})(2\sqrt{3} + 1)}{11}$

$= 2 + \frac{(5\sqrt{3} - 8) - (5\sqrt{3} + 8)}{11} = 2 - \frac{16}{11} = \frac{6}{11}$

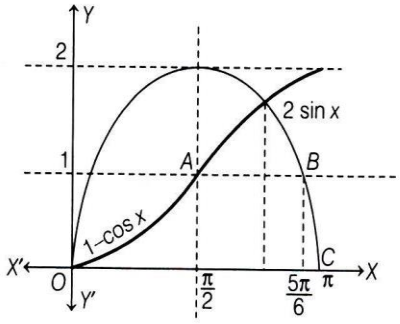
Hence, $t_1 + t_2 + t_3 = \frac{3}{11}$

Study Tactics

Using formula, $\frac{m_1 - m_2}{1 + m_1 m_2}$

$\because m_1$ is given, we get m_2 and m_3 in the form of t, then add m_1, m_2 and m_3, we get the required result.

66. *(d)* $f(x) = \text{Minimum}\{2\sin x, 1 - \cos x, 1\}$



$\int_0^{\pi} f(x)\,dx = \int_{OA} f(x)\,dx + \int_{AB} f(x)\,dx + \int_{BC} f(x)\,dx$

$= \int_0^{\pi/2} (1 - \cos x)\,dx + \int_{\pi/2}^{5\pi/6} dx + \int_{5\pi/6}^{\pi} 2\sin x\,dx$

$= [x - \sin x]_0^{\pi/2} + [x]_{\pi/2}^{5\pi/6} - 2[\cos x]_{5\pi/6}^{\pi}$

$= \frac{\pi}{2} - 1 + \frac{5\pi}{6} - \frac{\pi}{2} - 2\left(-1 + \frac{\sqrt{3}}{2}\right)$

$= 1 - \sqrt{3} + \frac{5\pi}{6}$

67. *(b)* Here, $T_n = \cot^{-1}\left(n^2 + \frac{3}{4}\right)$

$= \tan^{-1}\left(\frac{4}{4n^2 + 3}\right)$

$$= \tan^{-1}\left[\frac{1}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right]$$

$$= \tan^{-1}\left[\frac{\left(n+\frac{1}{2}\right)-\left(n-\frac{1}{2}\right)}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right]$$

$$= \tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(n-\frac{1}{2}\right)$$

$\therefore \quad S_\infty = \tan^{-1}\left(\infty+\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{2}\right)$

$$= \frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$$

$\Rightarrow \quad S_\infty = \cot^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \quad S_\infty = \tan^{-1} 2$

68. *(c)* Since, circles touches coordinate axes.

Let radius = r, then centre (r, r)

$\therefore$ Equation of circle will be

$$(x-r)^2+(y-r)^2 = r^2$$

$\Rightarrow \quad x^2+y^2-2rx-2ry+r^2 = 0$

This represents two circles having radius r_1 and r_2, then equation of circles is

$$x^2+y^2-2r_1x-2r_1y+r_1^2 = 0 \qquad \ldots(i)$$

and $\quad x^2+y^2-2r_2x-2r_2y+r_2^2 = 0 \qquad \ldots(ii)$

Circles Eqs. (i) and (ii) cuts each other orthogonally.

$\therefore \quad 2r_1r_2+2r_1r_2 = r_1^2+r_2^2$

$\Rightarrow \quad r_1^2+r_2^2 = 4r_1r_2$

$\Rightarrow \quad (r_1+r_2)^2 = 6r_1r_2 \qquad \ldots(iii)$

Since, circles passes through $P(\alpha, \beta)$.

$\therefore \quad r^2-2(\alpha+\beta)r+\alpha^2+\beta^2 = 0$

Sum and product of roots

$$r_1+r_2 = 2(\alpha+\beta)$$

$$r_1r_2 = \alpha^2+\beta^2$$

From Eq. (iii),

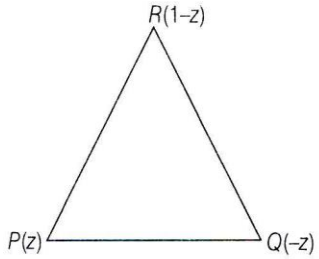
$$4(\alpha+\beta)^2 = 6(\alpha^2+\beta^2)$$

$\Rightarrow \quad \alpha^2-4\alpha\beta+\beta^2 = 0$

69. *(d)* $|PQ| = |z+z| = 2|z|$

$|QR| = |1-z+z| = 1$

$|PR| = |1-z-z| = |1-2z|$



Since, triangle is equilateral.

$|PQ| = |QR| = |PR|$

$\Rightarrow \quad 2|z| = 1 = |1-2z|$

$\Rightarrow \quad |z| = \frac{1}{2}$ and $|1-2z| = 1$

$\because \quad |1-2z| = 1$

$\Rightarrow \quad |1-2z|^2 = 1$

$\Rightarrow \quad (1-2z)(1-2\bar{z}) = 1$

$\Rightarrow \quad 1-2\bar{z}-2z+4z\bar{z} = 1$

$\Rightarrow \quad 1+4|z|^2-2(z+\bar{z}) = 1$

$\Rightarrow \quad 1+4\times\frac{1}{4}-2(z+\bar{z}) = 1$

$\Rightarrow \quad z+\bar{z} = \frac{1}{2}$

$\because \quad z+\bar{z} = 2\,\mathrm{Re}(z) = \frac{1}{2}$

$\Rightarrow \quad \mathrm{Re}(z) = \frac{1}{4}$

70. *(b)* Let $(1+x)^{10} = \sum_{i=0}^{10} {}^{10}C_i x^i$

On differentiating both sides w.r.t. x, we get

$$10(1+x)^9 = \sum_{i=1}^{10} i\,{}^{10}C_i x^{i-1} \qquad \ldots(i)$$

Again, on differentiating both sides w.r.t. x, we get

$$10\times 9(1+x)^8 = \sum_{i=1}^{10} i(i-1)\,{}^{10}C_i x^{i-2} \qquad \ldots(ii)$$

On substituting $x = 1$ in Eqs. (i) and (ii), we get

$$10\times 2^9 = \sum_{i=1}^{10} i\,{}^{10}C_i = f_2 \qquad \ldots(iii)$$

and $\quad 90\times 2^8 = \sum_{i=1}^{10} i(i-1)\,{}^{10}C_i = f_1 \qquad \ldots(iv)$

On adding Eqs. (iii) and (iv), we get

$$10\times 2^9+90\times 2^8 = \sum_{i=1}^{10} [i+i(i-1)]\,{}^{10}C_i$$

$$10\times 2^8(2+9) = \sum_{i=1}^{10} i^2\,{}^{10}C_i = f_3$$

$\Rightarrow \quad f_3 = 55 \times 2^9$

Thus, Statement I is true but Statement II is false.

Study Tactics

Differentiate the given expression twice with respect to x, then put $x = 1$ in both the equation and add both equation.

71. *(a)* $f(x) = \dfrac{x-3}{x+1}$

$$f^2(x) = f_o f(x) = \frac{\frac{x-3}{x+1} - 3}{\frac{x-3}{x+1} + 1}$$

$$= \frac{x-3-3x-3}{x-3+x+1} = \frac{-2x-6}{2x-2}$$

$$= -\frac{(x+3)}{x-1}$$

$$f^3(x) = f_o f_o f(x) = -\frac{\left(\frac{x-3}{x+1} + 3\right)}{\frac{x-3}{x+1} - 1} = x$$

$\therefore \; f^{3n}(x) = x, n \in N$

$\therefore \; f^{2100}(x) = x$

$\Rightarrow f^{2100}(2100) = 2100$

72. *(b)* Let r, l and h be the radius, slant height and height of cone respectively, at any time t.

Then, $l^2 = r^2 + h^2$

$$\Rightarrow \quad 2l\frac{dl}{dt} = 2r\frac{dr}{dt} + 2h\frac{dh}{dt}$$

$$\Rightarrow \quad l\frac{dl}{dt} = r\frac{dr}{dt} + h\frac{dh}{dt}$$

$$= 7(3) + 24(-4)$$

$$= -75 \qquad \left[\because \frac{dh}{dt} = -4 \text{ and } \frac{dr}{dt} = 3\right]$$

Now, $l^2 = 7^2 + 24^2$

$\Rightarrow \quad l^2 = 625$

$\Rightarrow \quad l = 25$

$\therefore \quad \dfrac{dl}{dt} = -3$

$\therefore \quad \dfrac{ds}{dt} = \dfrac{d}{dt}(\pi rl)$

$$= \pi\left[l\frac{dr}{dt} + r\frac{dl}{dt}\right]$$

$$= \pi[25 \times 3 + 7 \times (-3)]$$

$$= \pi \times 54 = 54\,\pi \text{ cm}^2/\text{min}$$

73. *(c)* $\Delta = \begin{vmatrix} q^2+r^2 & pq & pr \\ pq & r^2+p^2 & qr \\ pr & qr & p^2+q^2 \end{vmatrix}$

$$= (q^2+r^2)[(r^2+p^2)(p^2+q^2) - q^2r^2]$$
$$- pq[pq(p^2+q^2) - pqr^2] + pr[pq^2r - (r^2+p^2)pr]$$
$$= (q^2+r^2)[r^2p^2 + p^4 + q^2r^2 + p^2q^2 - q^2r^2]$$
$$- pq[p^3q + pq^3 - pqr^2] + [p^2r^2q^2 - p^2r^2(r^2+p^2)]$$
$$= p^2q^2r^2 + p^4q^2 + p^2q^4 + r^4p^2 + r^2p^4 + p^2q^2r^2$$
$$- p^4q^2 - p^2q^4 + p^2q^2r^2 + p^2r^2q^2 - p^2r^4 - p^4r^2$$
$$= p^2q^2r^2(4)$$

$$\Rightarrow \quad \left(\frac{K}{2}\right)^2 = 4$$

$$\Rightarrow \quad \frac{K}{2} = \pm 2$$

$$\Rightarrow \quad K = \pm 4$$

74. *(c)* Given, a, b and c are in AP.

$$\Rightarrow \quad b - a = c - b = \lambda \text{ (let)}$$

$$\because \quad \frac{t-p}{ap} = \frac{t-q}{bq} = \frac{t-r}{cr}$$

$$\Rightarrow \quad \frac{\frac{t}{p}-1}{a} = \frac{\frac{t}{q}-1}{b} = \frac{\frac{t}{r}-1}{c}$$

$$\Rightarrow \quad \frac{\left(\frac{t}{p}-1\right) - \left(\frac{t}{q}-1\right)}{a-b} = \frac{\left(\frac{t}{q}-1\right) - \left(\frac{t}{r}-1\right)}{b-c}$$

$$\Rightarrow \quad t\left(\frac{1}{p} - \frac{1}{q}\right) = t\left(\frac{1}{q} - \frac{1}{r}\right)$$

$$\Rightarrow \quad \frac{1}{p} - \frac{1}{q} = \frac{1}{q} - \frac{1}{r}$$

$$\Rightarrow \quad \frac{2}{q} = \frac{1}{p} + \frac{1}{r}$$

Hence, p, q and r are in HP.

75. *(a)* Here, $p - 1 \geq 0$, $p \leq m + 1$

$$\Rightarrow \quad 1 \leq p \leq m+1$$

$$\Rightarrow \quad \frac{1}{m+1} \leq \frac{p}{m+1} \leq 1$$

Now, $\lambda^2 - 8 = \dfrac{{}^mC_{p-1}}{{}^{m+1}C_p}$

$$= \frac{m!}{(p-1)!(m-p+1)!} \times \frac{p!(m+1-p)!}{(m+1)!}$$

$$= \frac{p}{m+1}$$

$\therefore \quad \frac{1}{m+1} \le \lambda^2 - 8 \le 1$

$\Rightarrow \quad 8 < \frac{1}{m+1} + 8 \le \lambda^2 \le 9$

$\Rightarrow \quad 8 < \lambda^2 \le 9$

$\Rightarrow \quad -3 \le \lambda < -2\sqrt{2}$

or $\quad 2\sqrt{2} < \lambda \le 3$

Hence, $\lambda \in [-3, -2\sqrt{2})$

76. ***(b)*** Since, there are exactly two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose distance from the centre is the same, the points would either be the endpoints of the major axis or of the minor axis.

But $\sqrt{\frac{(2a^2 + 3b^2)}{\sqrt{3}}} > b.$

So, the points are the vertices of the major axis.

Hence, $a = \sqrt{\frac{2a^2 + 3b^2}{\sqrt{3}}}$

On squaring both sides, we get

$$a^2 = \frac{2a^2 + 3b^2}{3}$$

$\Rightarrow 3a^2 - 2a^2 = 3b^2$

$\Rightarrow \quad b^2 = \frac{1}{3}a^2$

$\because \quad e = \sqrt{1 - \frac{b^2}{a^2}}$

$\therefore \quad e = \sqrt{1 - \frac{a^2}{3a^2}} = \sqrt{1 - \frac{1}{3}} = \sqrt{\frac{2}{3}}$

77. ***(b)*** We have,

$$al + bm + cn = 0 \quad ...(i)$$

$$fmn + gnl + hlm = 0 \quad ...(ii)$$

From Eq. (i),

$$n = -\left(\frac{al + bm}{c}\right)$$

Put the value of n in Eq. (ii), we get

$$fm\left(-\frac{(al + bm)}{c}\right) + gl\left(-\frac{(al + bm)}{c}\right) + hlm = 0$$

or $\quad ag\frac{l^2}{m^2} + \frac{l}{m}(af + bg - ch) + bf = 0 \quad ...(ii)$

Let $X = \frac{l}{m}$

$$agX^2 + (af + bg - ch)X + bf = 0$$

Let two values of $X\left(= \frac{l}{m}\right)$ be $\frac{l_1}{m_1}$ and $\frac{l_2}{m_2}$.

Product of roots $= \frac{l_1 l_2}{m_1 m_2} = \frac{bf}{ag}$

$\therefore \quad \frac{l_1 l_2}{\frac{f}{a}} = \frac{m_1 m_2}{\frac{g}{b}} = \frac{n_1 n_2}{\frac{h}{c}}$

Now, lines are perpendicular, if

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$$

$\Rightarrow \quad k\left(\frac{f}{a} + \frac{g}{b} + \frac{h}{c}\right) = 0$

$\Rightarrow \quad \frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$

Hence, $\left(\frac{f}{a} + \frac{g}{b} + \frac{h}{c}\right)! + 2 = 0! + 2 = 3 \quad [\because 0! = 1]$

78. ***(d)*** $\because \ |\mathbf{r} - \mathbf{a}| = |\mathbf{r} - \mathbf{b}|$

$\Rightarrow \ |\mathbf{r} - \mathbf{a}|^2 = |\mathbf{r} - \mathbf{b}|^2$

$\Rightarrow \ (\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{a}) = (\mathbf{r} - \mathbf{b}) \cdot (\mathbf{r} - \mathbf{b})$

$\Rightarrow \ |\mathbf{r}|^2 + |\mathbf{a}|^2 - 2\mathbf{r} \cdot \mathbf{a}$

$$= |\mathbf{r}|^2 + |\mathbf{b}|^2 - 2\mathbf{r} \cdot \mathbf{b}$$

$\Rightarrow \quad 2\mathbf{r} \cdot (\mathbf{a} - \mathbf{b}) + |\mathbf{a}|^2 - |\mathbf{b}|^2 = 0$

$\Rightarrow \ 2\mathbf{r} \cdot (\mathbf{a} - \mathbf{b}) - (\mathbf{a} - \mathbf{b}) \cdot (\mathbf{b} + \mathbf{a}) = 0$

$\Rightarrow \quad \left(\mathbf{r} - \frac{\mathbf{a} + \mathbf{b}}{2}\right) \cdot (\mathbf{a} - \mathbf{b}) = 0$

79. ***(a)*** Number of elements in $X = 5$

Number of subsets of $X = 2^5 = 32$

Total combination of A_1 and $A_2 = 32 \times 32 = 1024$

For disjoint combination there are three possibilities for $x \in X$

(i) $x \in A_1$, then $x \notin A_2$

(ii) $x \notin A_1$, then $x \in A_2$

(iii) $x \notin A_1$ and $x \notin A_2$

Number of disjoint combination $= 3^5 = 243$

Probability that A_1 and A_2 are not disjoint

$$= \frac{1024 - 243}{1024} = \frac{781}{1024}$$

80. ***(d)*** $\because \quad \left|\frac{z - 12}{z - 8i}\right| = \frac{5}{3}$

$\Rightarrow \quad 3|x + iy - 12| = 5|x + iy - 8i|$

$\Rightarrow 9[(x - 12)^2 + y^2] = 25[x^2 + (y - 8)^2]$

$\Rightarrow 2x^2 + 2y^2 + 27x - 50y + 38 = 0 \quad ...(i)$

and $\left|\frac{z - 4}{z - 8}\right| = 1 \Rightarrow x = 6 \quad ....(ii)$

On solving Eqs. (i) and (ii), we get

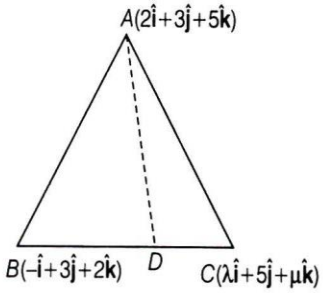
Point of intersection are (6, 8), (6, 17)

$\therefore \quad z_1 = 6+8i,\ z_2 = 6+17i$

$\therefore \ \Sigma|z_n| = |z_1| + |z_2|$

$= 10 + \sqrt{325} = 10 + 5\sqrt{13}$

81. *(4)* Given, A, B and C are vertices of ΔABC.



D is mid-point of BC, then position vector of D

$$= \left(\frac{\lambda-1}{2}\right)\hat{i} + 4\hat{j} + \left(\frac{\mu+2}{2}\right)\hat{k}$$

DR's of $AD = \frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}$

But it is given that DC's of AD is $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

$$\therefore \quad \frac{\lambda-5}{2} = 1 = \frac{\mu-8}{2}$$

$$\Rightarrow \quad \lambda = 7 \text{ and } \mu = 10$$

$$\therefore \quad 2\lambda - \mu = 4$$

82. *(1)* We have,

$$x\sin x\frac{dy}{dx} + (x + x\cos x + \sin x)y = \sin x$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{1}{\sin x} + \cot x + \frac{1}{x}\right)y = \frac{1}{x}$$

Here, $P = \frac{1}{\sin x} + \cot x + \frac{1}{x}$ and $Q = \frac{1}{x}$

$$\text{IF} = e^{\int\left(\frac{1}{\sin x}+\cot x+\frac{1}{x}\right)dx}$$

$$= e^{\log\left(x\tan\frac{x}{2}\sin x\right)}$$

$$= x\tan\frac{x}{2}\sin x$$

$$= x\tan\frac{x}{2}2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$= x(1-\cos x)$$

Now, solution is given by

$$y(\text{IF}) = \int Q(\text{IF})\,dx + C$$

$$xy(1-\cos x) = \int \frac{1}{x}\times x(1-\cos x)dx + C = x - \sin x + C$$

Now, $\quad y\left(\frac{\pi}{2}\right) = 1 - \frac{2}{\pi}$

$$\Rightarrow \quad C = 0$$

$$\therefore \quad y(x) = \frac{x-\sin x}{x(1-\cos x)}$$

$$= \frac{x-\left(x-\frac{x^3}{3!}+\ldots\right)}{x\left[1-\left(1-\frac{x^2}{2!}+\ldots\right)\right]}$$

Hence, $\lim_{x\to 0} y(x) = 3\lim_{x\to 0}\frac{x^3}{6}\times\frac{1}{x^3/2}$

$$= 3\times\frac{1}{3} = 1 \qquad [\text{as } x\to 0$$

Study Tactics

The given equation is in the form of linear differential equation. So, first we solve for integrating factor in which we use the property of logarithm, then we get the required solution of differential equation. At last, use expansion series of sin x and cos x.

83. *(5)* $\because 0 < x < \frac{\pi}{2}$

$$\therefore \ 0 < \sin^2 x < 1$$

Then, $\sin^2 x + \sin^4 x + \sin^6 x + \ldots\infty = \frac{\sin^2 x}{1-\sin^2 x}$

$$= \tan^2 x$$

$$\therefore \exp\left[(\sin^2 x + \sin^4 x + \sin^6 x + \ldots\infty)\log_e 2\right]$$

$$= \exp\left[\tan^2 x \log_e 2\right]$$

$$= \exp\left[\log_e 2^{\tan^2 x}\right] = e^{\log_e 2^{\tan^2 x}}$$

$$= 2^{\tan^2 x}$$

Let $y = 2^{\tan^2 x}$

$\because$ y satisfies the quadratic equation $y^2 - 9y + 8 = 0$.

$$\Rightarrow \quad y = 1, 8$$

If $y = 1 = 2^{\tan^2 x}$

$$\therefore \quad 2^{\tan^2 x} = 2^0$$

$$\Rightarrow \quad \tan^2 x = 0$$

$$x = 0 \text{ (impossible)} \qquad [\because x > 0]$$

Now, if $\quad y = 8 = 2^{\tan^2 x}$

$$\therefore \quad 2^{\tan^2 x} = 2^3$$

$$\Rightarrow \quad \tan^2 x = 3$$

$$\Rightarrow \quad \tan x = \sqrt{3}$$

So, $\dfrac{\sin x-\cos x}{\sin x+\cos x}=\dfrac{\tan x-1}{\tan x+1}$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$=\frac{(\sqrt{3}-1)^2}{3-1}$$

$$=\frac{3+1-2\sqrt{3}}{2}$$

$$=2-\sqrt{3}$$

On comparing it with $a-\sqrt{b}$, we get

$$a=2 \text{ and } b=3$$

Hence, $a+b=5$

84. *(1)* Given, $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$$A^2=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta-\sin^2\theta & -\cos\theta\sin\theta-\sin\theta\cos\theta\\ \sin\theta\cos\theta+\sin\theta\cos\theta & -\sin^2\theta+\cos^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix}$$

Now, $A^n=\begin{bmatrix}\cos n\theta & -\sin n\theta\\ \sin n\theta & \cos n\theta\end{bmatrix}$

$$\therefore \quad B=A+A^6=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}+\begin{bmatrix}\cos 6\theta & -\sin 6\theta\\ \sin 6\theta & \cos 6\theta\end{bmatrix}$$

At $\theta=\dfrac{\pi}{7}$,

$$B=\begin{bmatrix}\cos\pi/7 & -\sin\pi/7\\ \sin\pi/7 & \cos\pi/7\end{bmatrix}+\begin{bmatrix}\cos 6\pi/7 & -\sin 6\pi/7\\ \sin 6\pi/7 & \cos 6\pi/7\end{bmatrix}$$

$$B=\begin{bmatrix}2\cos\pi/2\cos 5\pi/14 & -2\sin\pi/2\cos 5\pi/14\\ 2\sin\pi/2\cos 5\pi/14 & 2\cos\pi/2\cos 5\pi/14\end{bmatrix}$$

$$\Rightarrow |B|=\begin{vmatrix}0 & -2\cos\dfrac{5\pi}{14}\\ 2\cos\dfrac{5\pi}{14} & 0\end{vmatrix}$$

$$\Rightarrow |B|=4\cos^2\frac{5\pi}{14}$$

$$\Rightarrow |B|=4\times 0.18825$$

$$|B|=0.7530$$

$\therefore |B|$ lies in (0, 1).

$$\alpha=0,\beta=1$$

Hence, $\alpha+\beta=1$

85. *(5)* $\sum_{i=1}^{n}(x_i+1)^2=9n$ …(i)

$$\sum_{i=1}^{n}(x_i-1)^2=5n \quad \text{…(ii)}$$

On adding Eqs. (i) and (ii), we get

$$\sum_{i=1}^{n}(x_i+1)^2+(x_i-1)^2=14n$$

$$\Rightarrow \sum_{i=1}^{n}2(x_i^2+1)=14n$$

$$\Rightarrow \sum_{i=1}^{n}x_i^2+\sum_{i=1}^{n}1=\frac{14n}{2}$$

$$\Rightarrow \sum_{i=1}^{n}x_i^2=6n$$

$$\Rightarrow \frac{\Sigma x_i^2}{n}=6 \quad \text{…(iii)}$$

On subtracting Eqs. (i) and (ii), we get

$$\sum_{i=1}^{n}(x_i+1)^2-(x_i-1)^2=4n$$

$$\Rightarrow 4\sum_{i=1}^{n}x_i=4n$$

$$\Rightarrow \frac{\Sigma x_i}{n}=1$$

$$\therefore \text{ Variance}=\frac{\Sigma x_i^2}{n}-\left(\frac{\Sigma x_i}{n}\right)^2$$

$$=6-1=5$$

86. *(15)* $\because 17^2=289=290-1$

Now, $17^{256}=(290-1)^{128}$

$$=(1-290)^{128}$$

$$=(290)^{128}-{}^{128}C_1(290)^{127}+{}^{128}C_2(290)^{126}-\ldots$$

$$\ldots-{}^{128}C_{125}(290)^3+{}^{128}C_{126}(290)^2-{}^{128}C_{127}(290)+1$$

$$=1000m+{}^{128}C_{126}(290)^2-{}^{128}C_{127}(290)+1$$

where, m is a positive integer.

$$=1000m+\frac{128\times 127}{2}\times(290)^2-128(290)+1$$

$$=1000m+683527680+1$$

Last three digits are 6, 8 and 1.

$$\therefore \quad x=6+8+1$$

$$=15$$

87. *(1)* $\log_4\left(\frac{2f(x)}{1-f(x)}\right)=x$

$\Rightarrow \quad \frac{2f(x)}{1-f(x)}=4^x$

$\Rightarrow \quad 2f(x)=4^x-4^x f(x)$

$\Rightarrow \quad (2+4^x)f(x)=4^x$

$\Rightarrow \quad f(x)=\frac{4^x}{2+4^x} \quad \ldots(i)$

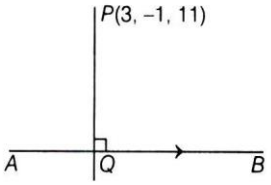
Now, $f(1-x)=\frac{4^{1-x}}{2+4^{1-x}} \quad \ldots(ii)$

On adding Eqs. (i) and (ii), we get

$f(1-x)+f(x)=1$

$\Rightarrow \quad f(20)+f(-19)=1$

88. *(53)* Let the coordinates of Q be $(2\lambda, 3\lambda+2, 4\lambda+3)$ which is any point on the straight line AB.



$\therefore$ DR's of PQ are $(2\lambda-3, 3\lambda+3, 4\lambda-8)$.

Also, perpendicular to straight line AB

$\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$ having DR's (2, 3, 4)

Thus, $2(2\lambda-3)+3(3\lambda+3)+4(4\lambda-8)=0$

$\Rightarrow \quad 4\lambda-6+9\lambda+9+16\lambda-32=0$

$\Rightarrow \quad 29\lambda-29=0$

$\Rightarrow \quad \lambda=1$

Hence, the coordinates of Q are (2, 5, 7).

$\therefore \quad |PQ|=\sqrt{(3-2)^2+(-1-5)^2+(11-7)^2}$

$=\sqrt{1+36+16}=\sqrt{53}=\sqrt{k}$

$k=53$

89. *(17)* $x+\frac{1}{x}=1 \Rightarrow x=-\omega,-\omega^2$

$\therefore \quad p=x^{2005}+\frac{1}{x^{2005}}$

$=-\omega-\omega^2=1$

$\therefore \quad p=1$

$\Rightarrow \quad 2^{2^n}-2=2^{4k}-2, k\in Z$

$=16^k-2$

Last digit of 16^k is 6.

$\therefore \quad q=6-2=4$

$p^2+q^2=1^2+4^2=17$

90. *(54)* Let n be the number of sides of the polygon.

$n150=(n-2)180$

$\Rightarrow \quad 150n-180n=-360$

$\Rightarrow \quad 30n=360$

$\Rightarrow \quad n=12$

Then, the number of diagonals $={}^{12}C_2-12$

$=66-12$

$=54$

Time Saver Tip

We know that

Number of sides $=\frac{360^\circ}{180^\circ-\text{Interior angle}}$

$=\frac{360^\circ}{30^\circ}=12$

Hence, the number of diagonals $={}^nC_2-n$

$=54$