2024	3	22	JEE-T2	SEAT NUMBER				

IIT INSPIRE
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XI & XII Science (CBSE/state)
IIT- JEE (Mains + Advance)
NEET, MHT-CET , NDA
Mo. No. 9595445177/9021445177

Branches : Chhatrapati Sq., Mangalmurti Sq.

Day - 2

JEE TEST

Time : 3 Hrs. | (4 Pages) | Max. Marks: 300

Instructions

1. There are **three** parts in the questions paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
2. **Section A** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and – **1 mark** for wrong answer.
3. **Section B** This section contains 10 questions. In Section B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and – **1 mark** for wrong answer. For Section B, the answer should be rounded off to the nearest integer.

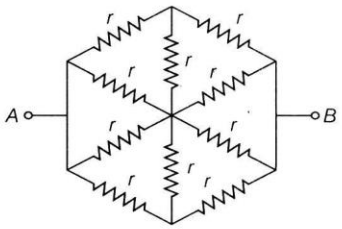
PHYSICS

Section A : Objective Type Questions

1. A series L-C-R circuit with $R = 120\,\Omega$ has resonance angular frequency 4×10^5 rad s^{-1}. At resonance, voltage across resistor and inductor are 60 V and 40 V, respectively. If angular frequency at which the circuit current lags behind the voltage by a phase of $\frac{\pi}{4}$ rad is $k \times 10^5$ rad / s, then the value of k is

(a) 4 (b) 2
(c) 8 (d) 12

2. The resistance of hexagon circuit between A and B represented in figure is
(Take, resistance of each resistor 1Ω)



(a) 1Ω (b) 0.5Ω
(c) 2Ω (d) 3Ω

3. The ratio of maximum acceleration to maximum velocity in a simple harmonic motion is $20\,s^{-1}$. At $t = 0$, the displacement is 10 m. What is the maximum acceleration? The initial phase is $\frac{\pi}{4}$.

(a) $4000\sqrt{2}\,ms^{-2}$ (b) $400\sqrt{2}\,ms^{-2}$
(c) $4\sqrt{2}\,ms^{-2}$ (d) $10\sqrt{2}\,ms^{-2}$

4. A proton of mass 1.67×10^{-27} kg and charge 1.6×10^{-19} C is projected with a speed of 2×10^6 m/s at an angle of 60° to the X-axis. If a uniform magnetic field of 0.104 T is applied along Y-axis, the path of proton is

(a) a circle of radius = 0.2 m and time period $\pi \times 10^{-7}$ s
(b) a circle of radius = 0.1 m and time period $2\pi \times 10^{-7}$ s
(c) a helix of radius = 0.1 m and time period $2\pi \times 10^{-7}$ s
(d) a helix of radius = 0.2 m and time period $4\pi \times 10^{-7}$ s

5. The velocity of the particle of mass m moving along a circle of radius 50 cm depends on the distance covered s as $v = ks^2$, then the acceleration of the particle when it completes one complete circle is

(a) $400\,k^2\,(4\pi^2 + 1)^{1/2}$ (b) $20\,\pi^3\,k^2(4\pi^2 + 1)^{1/2}$
(c) $40\,k^2\,(4\pi^2 + 1)^{1/2}$ (d) $2\pi^3\,k^2(\pi^2 + 1)^{1/2}$

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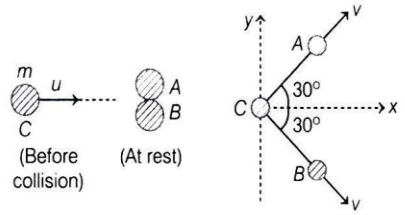
6. A billiard ball C of mass m moving with velocity u collides two identical balls A and B in contact and at rest. After the collision, ball C is stopped dead and balls A and B move along directions shown in figure with the same speed v. Then,



(After collision)

(a) $v = \frac{u}{\sqrt{3}}$ (b) $v = \frac{u}{\sqrt{2}}$

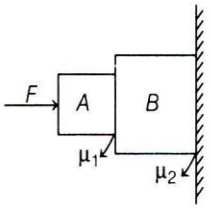
(c) loss in KE $= \frac{1}{3}mu^2$ (d) loss in KE $= \frac{1}{8}mu^2$

7. **Assertion** When a charge Q is taken from centre of the charged spherical shell to its surface, then the external work done is zero.

Reason The electric potential within the charged spherical shell remains constant.

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

8. Two blocks A and B of masses 1 kg and 2 kg are being pressed against a wall by a force F as shown in the figure below. If the coefficient of friction between the blocks is 0.15 and the wall is 0.2, then the frictional force applied by the wall on the block B is



(a) 20 N (b) 30 N (c) 50 N (d) 40 N

9. Two point masses A and B having masses in the ratio of 4 : 3 are separated by a distance of 1 m. When another point mass C of mass M is placed in between A and B, the force between A and C is $\frac{1}{3}$ rd of the force between B and C. Then, distance of C from A is

(a) $\frac{2}{3}$ m (b) $\frac{1}{3}$ m

(c) $\frac{1}{4}$ m (d) $\frac{2}{7}$ m

10. I. Stress and Young's modulus have same dimensions.

II. The dimensions of thermal conductivity of a substance is $[MLT^{-3}K]$.

III. Stress and strains are dimensionless physical quantities.

Which of the following statements is/are correct?

(a) I and II (b) II and III
(c) Only I (d) All of these

11. A gasoline engine takes in 5 mol of air at 20°C and 1 atm and compresses it adiabatically to (1/10) th of the original volume. Assume, air to be diatomic. The work done and change in internal energy is

(a) 46 kJ, – 46 kJ (b) 36 kJ, – 36 kJ
(c) –46 kJ, 46 kJ (d) 36 kJ, – 46 kJ

12. The maximum and minimum distance of a comet from the sun are 2×10^{12} m and 8×10^{10} m, respectively. If the speed of the comet at the nearest point in 8×10^{4} ms^{-1}, the speed at the farthest point is.

(a) 3.2×10^{5} ms^{-1} (b) 3.2×10^{3} ms^{-1}
(c) 1.6×10^{5} ms^{-1} (d) 4.2×10^{3} ms^{-1}

13. **Statement I** The root mean square speed of a given gas molecule is more than the average speed of that gas molecule.

Statement II In each degree of freedom, energy of one molecule of an ideal gas is $\frac{3}{2}RT$.

Choose the correct option.

(a) Statement I is true, Statement II is false.
(b) Statement I is false, Statement II is true.
(c) Both the Statements are true.
(d) Both the Statements are false.

14. A plane electromagnetic wave of frequency 60 MHz travels in vacuum along the positive x-direction. The electric field **E** at a particular space point x and an instant of time t is 9.6 $\hat{\mathbf{j}}$ Vm^{-1}.

The magnetic field **B** at time t is

(a) $3.5\times10^{-8}\hat{\mathbf{k}}$T (b) $3.0\times10^{-8}\hat{\mathbf{k}}$T
(c) $2.8\times10^{-8}\hat{\mathbf{k}}$T (d) $3.2\times10^{-8}\hat{\mathbf{k}}$T

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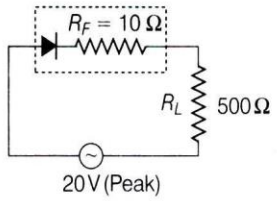
15. A particle is moving n times as fast as an electron. The ratio of de-Broglie wavelength of the particle to that of electron is x, if mass of the electron is y, then the mass of the particle is

(a) $\frac{y}{nx}$ (b) $\frac{x}{ny}$ (c) $\frac{n}{xy}$ (d) $\frac{xy}{n}$

16. In the circuit shown the barrier voltage of diode is 0.7 V. Match the physical quantities given in Column I to the results given in Column II and select the correct answer from the codes.



	Column I		Column II
A.	Peak current (in mA) in diode	1.	37.8
B.	Peak voltage (in V) across the ends of load	2.	40.0
C.	Peak current (in mA), if diode is ideal	3.	20.0
D.	Peak voltage (in V) across the ends of load, if diode is ideal	4.	18.9

Codes

	A	B	C	D		A	B	C	D
(a)	1	4	3	2	(b)	4	3	1	2
(c)	1	4	2	3	(d)	3	1	2	4

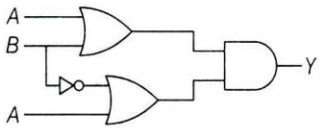
17. In an experiment for measurement of Young's modulus of Searle's apparatus, following readings are taken :
Load = 4.00 kg, length = 3.220 m, diameter = 0.045 cm and extension = 0.87 cm. If the error in each of load, length, diameter and extension is 0.01 unit, then the percentage error in measurement of Y is

(a) 50% (b) 45.2%
(c) 46.15% (d) 48%

18. In Bohr model of hydrogen atom, the force on the electron depends on the principle quantum number as

(a) $F \propto \frac{1}{n^3}$ (b) $F \propto \frac{1}{n^4}$
(c) $F \propto \frac{1}{n^5}$ (d) Does not depend on n

19. Find out the value of Y.



(a) A (b) B (c) $A+B$ (d) $A \cdot B$

20. A parallel plate capacitor of capacitance 120 pF is connected to a battery of emf 30 V. If a dielectric material of dielectric constant $K = \frac{3}{2}$ is inserted between the plates, then magnitude of induced charge will be

(a) 1.5 nC (b) 1.6 nC (c) 1.8 nC (d) 2.4 nC

Section B : Numerical Value Type Questions

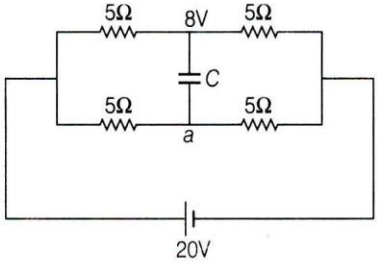
21. When a solid sphere is rolling without slipping along a level surface, then the percentage of its translational kinetic energy is x %. The value of x is ..... .

22. A non-isotropic solid metal cube has coefficient of linear expansion as $4 \times 10^{-4}/°C$ along X-axis and $5 \times 10^{-5}/°C$ along Y-axis. If coefficient of volumetric expansion of the solid is $57 \times 10^{-5}/°C$, then the coefficient of linear expansion along Z-axis is $n \times 10^{-5}/°C$. The value of n is ..... .

23. The focal length of a convex lens is 20 cm ($\mu_{glass} = 1.5$). Now, the lens in submerged in water ($\mu_{water} = \frac{4}{3}$). Find the ratio of focal length in medium to focal length in air.

24. In YDSE, a monochromatic light of wavelength λ is used. The intensity of light at a point on the screen, where the path difference is 2λ is c units. The intensity of light at a point where the path difference is $\frac{\lambda}{4}$ is given by $\frac{nc}{10}$, then the value of n is ..... .

25. An ideal cell of emf 20 V is connected in a circuit shown in figure below. Each resistance is 5 Ω. The potential difference (in V) across capacitor when it is fully charged will be ..... .



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26. A sonometer wire has a total length of 2 m between the fixed ends. There are two bridges placed below the wire, so that the three segments of the wire have their fundamental frequencies in the ratio of 1:2:3.

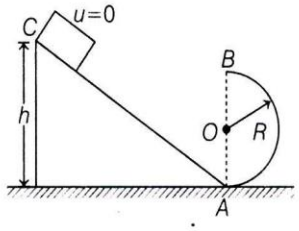
The distance of first bridge from first end is $\frac{x}{11}$ m, then the value of x is ..... .

27. A body is projected from the bottom of a smooth inclined plane with a velocity of 20 m/s. It is just sufficient to carry it to the top in 4 s, then the height of the plane is ..... m. (Take $g = 10\,ms^{-2}$)

28. Light emitted by hydrogen gas corresponding to transition from second excited state to ground state incident on a metal plate. The electron emitted from metal plate with maximum kinetic energy enters a magnetic field 8×10^{-4} T perpendicularly. If the radius of path of electron is 10 mm, then the work function of the metal is .....eV.

29. A closely wounded circular coil of radius 10 cm produces a magnetic field of 75.36×10^{-4} T at its centre. The current through the coil in ..... A. (Number of turns in the coil is 100 and $\pi = 3.14$)

30. A block of mass 6 kg is released from a height h on a smooth track.The radius of circular portion of the track is 10 m. What is the minimum value of h (in m), so that the block completes the vertical circle ?



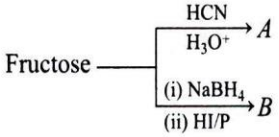
CHEMISTRY

Section A : Objective Type Questions

31. $Na_2B_4O_7 \cdot 10H_2O$ is correctly represented as

(a) $Na_2[B_4O_5(OH)_4] \cdot 8\,H_2O$
(b) $2\,NaBO_2 \cdot Na_2B_2O_3 \cdot 10\,H_2O$
(c) $Na_2[B_4(H_2O)_4O_7] \cdot 6\,H_2O$
(d) $2Na[B_4(H_2O)_5O_7] \cdot 5H_2O$

32. The formulas of A and B for the following reaction sequence are



(a) $A = C_7H_{14}O_8$, $B = C_6H_{14}$
(b) $A = C_7H_{13}O_7$, $B = C_7H_{14}O$
(c) $A = C_7H_{12}O_8$, $B = C_6H_{14}$
(d) $A = C_7H_{14}O_8$, $B = C_6H_{14}O_6$

33. Λ°_m for NaCl, HCl and NaA are 126.4, 425.9 and 100.5 S cm^2 mol^{-1}, respectively. If the conductivity of 0.001 M HA is 6×10^{-5} S cm^{-1}, degree of dissociation of HA is

(a) 0.75 (b) 0.25 (c) 0.150 (d) 0.50

34. Phenol on reaction with a mixture of conc. HNO_3 and conc. H_2SO_4 produces a compound. The degree of unsaturation and the nature of compound is

(a) 5, acidic (b) 7, basic (c) 6, neutral (d) 7, acidic

35. When C_2H_2, CH_4 and C_2H_4 passes through a test tube which have ammoniacal Cu_2Cl_2 find out which gas comes out unaffected from test tube?

(a) C_2H_2 and CH_4 (b) C_2H_2 and C_2H_4
(c) C_2H_4 and CH_4 (d) Only C_2H_2

36. Which of the following will not produce the benzyl amine?

(a) Benzamide $\xrightarrow{LiAlH_4}$
(b) Phenyl nitrile $\xrightarrow{LiAlH_4}$
(c) 2-phenylethyl isonitrile $\xrightarrow{LiAlH_4}$
(d) 2-phenyl ethanamide $\xrightarrow{Br_2 + KOH}$

37. "Ruhemann's purple" is a corroborative test for the presence of

(a) cupric particle
(b) protein
(c) starch
(d) reducing sugar

38. Given below are two statements.

Statement I The condition for any solution to be ideal is that it should obey Raoult's law over entire range of concentration.

Statement II A solution with $\Delta_{mix}S < 0$, $\Delta_{mix}V = 0$, $\Delta_{mix}H = 0$, $\Delta_{mix}G = 0$ is said to be an ideal solution.

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In the light of the above statements, choose the correct answer from the options given below.

(a) Both Statement I and Statement II are correct.

(b) Both Statement I and Statement II all incorrect.

(c) Statement I is correct but Statement II is incorrect.

(d) Statement I is incorrect but Statement II is correct.

39. $CH_3CH{=}CH_2 \xrightarrow{A} CH_3{-}CH_2{-}CH_2OH$

The reagent *A* is

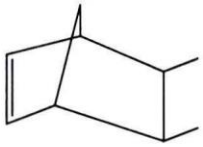
(a) H_2O/H_2SO_4

(b) $Hg(OAc)_2$ / H_2O followed by $NaBH_4$

(c) B_2H_6 followed by H_2O_2

(d) CH_3CO_2H/H_2SO_4

40. IUPAC name and degree of unsaturation of the following compound respectively is



(a) 2, 3- dimethyl bicyclo [2,2,1] hept-5 ene,2

(b) 1, 2- dimethyl bicyclo [2,2,1] hept-4 ene,3

(c) 5, 6- dimethyl bicyclo [2,2,1] hept-2 ene,3

(d) 4, 5- dimethyl bicyclo [2,2,1] hept-1 ene,2

41. Match List-I with List-II.

List-I	List-II
A. BF_4^-	I. Square planar
B. BrF_4^-	II. Bent shape
C. SF_4	III. Tetrahedral
D. BrF_2^+	IV. *See-saw*

Choose the correct answer from the options given below.

(a) A-II, B-III, C-I, D-IV

(b) A-III, B-IV, C-II, D-I

(c) A-III, B-I, C-IV, D-II

(d) A-IV, B-III, C-II, D-I

42. Which of the following statement(s) is/are true about product obtained by treatment of CCl_3CHO with chlorobenzene in presence of H_2SO_4?

A. It has one chiral centre.

B. It is used as an insecticide.

C. It is not easily metabolised by animals.

D. Its name is 1, 1, 1-trichloro-2, 2-*bis*-(*p*-chlorophenyl) ethane

Choose the correct answer from the options given below.

(a) A and B (b) B and C

(c) B, C and D (d) Only D

43. Match List-I with List-II.

	List-I		List-II
A.	$[Cr(NH_3)_4Cl_2]Cl$	I.	Paramagnetic exhibits ionisation isomerism
B.	$[Ti(H_2O)_5Cl](NO_3)_2$	II.	Diamagnetic and exhibits *cis-trans* isomerism
C.	$[Pt(en)(NH_3)Cl]NO_3$	III.	Paramagnetic and exhibits *cis-trans* isomerism
D.	$[Co(NH_3)_4(NO_3)_2]NO_3$	IV.	Diamagnetic and exhibits ionisation isomerism

Choose the correct answer from the options given below.

(a) A-IV, B-II, C-III, D-I

(b) A-III, B-I, C-IV, D-II

(c) A-II, B-I, C-III, D-IV

(d) A-I, B-III, C-IV, D-II

44. $[Co(NH_3)_4(NO_2)_2]$ Cl exhibits

(a) linkage, geometrical and ionisation isomerism

(b) linkage, ionisation and optical isomerism

(c) linkage, geometrical and optical isomerism

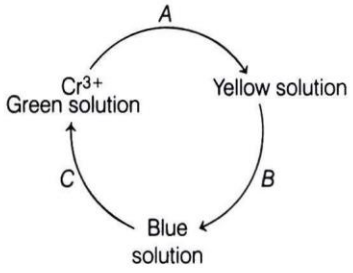
(d) ionisation, geometrical and optical isomerism

45. Which of the following contains a coordinate covalent bond?

(a) $N_2H_5^+$ (b) $BaCl_2$

(c) HCl (d) H_2O

46. Identify *A*, *B* and *C* in the following sequence.



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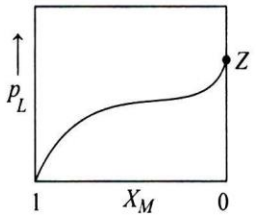
(a) Alkaline H_2O_2, acidified H_2O_2, on standing
(b) Alkaline O_3, acidified O_3, Zn/HCl
(c) Acidified H_2O_2, alkaline H_2O_2, heat
(d) Alkaline O_3, heat, NH_4OH

47. Consider the following compound.

A : Tetracyanomethane B : Carbon dioxide
C : Benzene D : 1, 3-butadiene

Ratio of σ-and π-bonds is in order.
(a) A = B < C < D (b) A = B < D < C
(c) A = B = C = D (d) C < D < A < B

48. For a solution formed by mixing liquids *L* and *M*, the vapour pressure of *L* plotted against the mole fraction of *M* in solution is shown in the following figure. Here, X_L and X_M represent mole fraction of *L* and *M*, respectively in the solution. The correct statement applicable to this system is



(a) The point *Z* represents vapour pressure of pure liquid *M* and Raoult's law is obeyed from $X_L = 0$ to $X_L = 1$.
(b) The point *Z* represents vapour pressure of pure liquid *L* and Raoult's law is not obeyed from $X_L \rightarrow 1$.
(c) The point *Z* represents vapour pressure of pure liquid *M* and Raoult's law is obeyed when $X_L \rightarrow 0$.
(d) Attractive intermolecular interaction between *L*—*L* in pure liquid *L* and *M*—*M* in pure liquid *M* are stronger than those between *L*—*M* when mixed in solution.

49. Which of the following statement is incorrect for glucose?
(a) Glucose exists in two crystalline from α and β.
(b) It reacts with hydroxylamine to give oxime.
(c) On prolonged heating with HI, it forms gluconic acid.
(d) Acetylation of glucose with acetic anhydride gives glucose penta acetate.

50. Given below are two statements, one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) $[Ti(H_2O)_6]^{3+}$ is coloured while $[Sc(H_2O)_6]^{3+}$ is colourless.

Reason (R) *d* - *d* transition is not possible in $[Sc(H_2O)_6]^{3+}$.

In the light of the above statements, choose the correct answer from the options given below.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

Section B : Numerical Value Type Questions

51. The chlorate ion can disproportionate in basic solution according to the reaction,

$$2ClO_3^- \rightleftharpoons ClO_2^- + ClO_4^-$$

The equilibrium concentration of perchlorate ions for a solution initially at 0.1M in chlorate ions at 298 K is ..... $\times 10^{-2}$. (Nearest integer)

[Given: $E^\circ_{ClO_4^-/ClO_3^-} = 0.39$ V and $E^\circ_{ClO_3^-/ClO_2^-} = 0.36$ V at 298 K.]

52. One atom of an element, *y* weighs 6.64×10^{-23} g. Then, the number of moles of atom in 20 kg is ..... .

53. 23.2 g of an organic compound having molecular formula, C_nH_{2n+2} is burnt in excess of $O_2(g)$ initially taken in a 44.82 L steel vessel. Before reaction, the gaseous mixture (*w*) is present at a temperature of 273 K with pressure of 2 atm. After complete combustion and loss of considerable amount of heat, the mixture of product and excess of O_2 had a temperature of 546 K and 4.6 atm pressure. The formula of compound is C_xH_y. The value of sum of *x* and *y* is ..... .

54. The pK_a of a weak acid (H*A*) is 4.1. The pOH of an aqueous buffered solution of H*A* in which 50% of the acid ionised is ..... . [Nearest integer]

55. $KMnO_4$ reacts with ferrous sulphate as follows,

$$MnO_4^- + 5Fe^{2+} + 8H^+ \longrightarrow Mn^{2+} + 5Fe^{3+} + 4H_2O$$

Here, 10 mL of 0.1 M $KMnO_4$ is equivalent to ..... of 0.1 M $FeSO_4$.

56. For a complex reaction,

$$P \xrightarrow{k} \text{Products}$$

$E_{a_1} = 200$ kJ/mol, $E_{a_2} = 90$ kJ/mol,
$E_{a_3} = 80$ kJ/mol and $A = 1$.
Overall rate constant, *k* is related to individual rate constant by the equation, $k = \left(\frac{k_1 k_2}{k_3}\right)^{2/3}$.

Activation energy (kJ/mol) for the overall reaction is ..... .

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57. The conductance of a 0.0015 M aqueous solution of a weak monobasic acid was determined by using platinised Pt electrodes. The area of cross-section and distance between the electrodes are 1 cm^2 and 120 cm respectively. The conductance of this solution was found to be 5×10^{-7} S. The pH of the solution is 4. The value of limiting molar conductivity (Λ°_m) of this weak monobasic acid in aqueous solution is $Z\times10^2$ S cm^2 mol^{-1}. The value of Z is .....

58. The number of correct statement(s) about the given reaction.

$$I^- + ClO_3^- + H_2SO_4 \longrightarrow Cl^- + HSO_4^- + I_2$$

A. Stoichiometric coefficient of HSO_4^- is 6.

B. Iodide is oxidised.

C. Sulphur is reduced.

D. H_2O is one of the products.

59. Ethylene glycol is used as an antifreeze in cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at $-6°C$ will be .....g.

(K_f for water = 1.86 K kg mol^{-1} and molar mass of ethylene glycol = 62 g mol^{-1})

60. An element undergoes a reaction as follows,

$X + 2e^- \longrightarrow X^{2-}$ and energy released

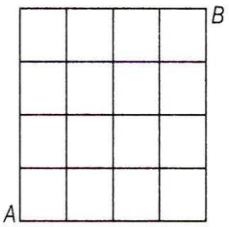
$= -30.86$ eV/atom. If the energy released is used to dissociate 4 g of H_2 molecule and equally into H^+ and H*, where H* is excited state of H-atoms where the electron travels in orbit whose circumference is equal to four times of its de-Broglie's wavelength. The least moles of X that would be required is ..... .

[Given: Ionisation energy of H = 13.6 eV/atom, binding energy of H_2 = 4.52 eV/molecule.]

MATHEMATICS

Section A : Objective Type Questions

61. In given figure, person can move towards right or in upward direction only, how many ways are there to reach the point B from point A ?



(a) 70 (b) 90 (c) 8 (d) 16

62. Solution of system of equation $2a^4 = b^4 + c^4$, $abc = 27$ knowing that $\log_b a$, $\log_c b$ and $\log_a c$ form a geometric progression, is

(a) $a = b = c = 1$ (b) $a = b = c = 3$

(c) $a = 3$ and $b = c = 2$ (d) $a = b = 2$ and $c = 3$

63. If $f(x, y) = c$ represents the circle having radius r which satisfies the differential equation $\frac{dy}{dx} = \frac{ax+5}{4y+b}$, then

(a) $a = -4, r = \frac{1}{4}\sqrt{b^2 + 4c + 25}$

(b) $a = 4, r = \frac{1}{4}\sqrt{b^2 + 8c + 100}$

(c) $r = 3, a = 3$

(d) $a = -4, r = \frac{1}{4}\sqrt{b^2 + 8c + 25}$

64. For complex number z and ω, $|z|^2\omega - |\omega|^2 z = z - \omega$, if and only if

(a) $z = \bar{\omega}$ or $\bar{z}\omega = 1$ (b) $z = \omega$ or $z\omega = 1$

(c) $z = \bar{\omega}$ or $z\bar{\omega} = \frac{1}{2}$ (d) $z = \omega$ or $z\bar{\omega} = 1$

65. If $\int \frac{e^{x-1}}{(x^2 - 5x + 4)} 2x\,dx = AF(x-1) + BF(x-4) + C$ and $F(x) = \int \frac{e^x}{x}\,dx$, then (A, B) is equal to

(a) $\left(\frac{-2}{3}, \frac{8}{3}\right)$ (b) $\left(\frac{-2}{3}, \frac{8e^3}{3}\right)$ (c) $\left(\frac{8}{3}, \frac{2}{3}\right)$ (d) $\left(\frac{-2}{3}, \frac{8e^{-3}}{3}\right)$

66. If $\left(\frac{1}{a}\right)^b = \lim_{x\to 0} \frac{8}{x^8}\left(1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)$, where a is a prime number, then ab

(a) is greater than $2b$

(b) is less than $2b$

(c) is equal to $2b$

(d) is equal to $5b$

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67. Let $p, q, r \in R$, such that $pqr = 5$, if

$$A = \begin{bmatrix} 3p & q & r \\ q & 3r & p \\ r & p & 3q \end{bmatrix} \text{ and } AA^T = 4I \text{ and } |A| \text{ is a}$$

positive real number, then $p^3 + q^3 + r^3$ is equal to

(a) 125 (b) 137 (c) 64 (d) 16

68. If $f(x) = \begin{cases} \dfrac{(3^x - 1)^2}{\sin\left(\dfrac{x}{c}\right)\log\left(1 + \dfrac{x}{3}\right)}, & x \neq 0 \\ 10(\log_e 3)^2, & x = 0 \end{cases}$ is continuous

at $x = 0$, then c is equal to

(a) $\frac{10}{3}$ (b) $\frac{3}{10}$ (c) $\frac{10}{4}$ (d) $\frac{3}{5}$

69. The coordiantes of the focus of the parabola $4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$ are

(a) $\left(\frac{4}{5}, \frac{3}{5}\right)$ (b) $\left(\frac{3}{5}, \frac{4}{5}\right)$

(c) $\left(\frac{4}{5}, -\frac{3}{5}\right)$ (d) $\left(-\frac{3}{5}, -\frac{4}{5}\right)$

70. Let f be a real valued function defined on the interval $[-10, 10]$ by

$$f(x) = \begin{cases} x - [x], & \text{if } [x] \text{ is odd} \\ 1 + [x] - x, & \text{if } [x] \text{ is even} \end{cases}$$

(where $[\cdot]$ is greatest integer function) and if $\int_{-10}^{10} f(x)\cos \pi x \, dx = \frac{20}{\pi^2} k$, then k is equal to

(a) 1 (b) 2
(c) 3 (d) 4

71. If $f : A \to B$ is an onto function such that $f(x) = \sqrt{|x| - x} + \frac{1}{\sqrt{|x| - x}}$, then A and B are respectively

(a) $(-\infty, 0), (0, \infty)$ (b) $(-\infty, 0), [2, \infty)$
(c) $(0, \infty), (2, \infty)$ (d) $(-\infty, 0], (0, \infty)$

72. A hyperbola having the transverse axis of length $2\sin\theta$ is confocal with the ellipse $4x^2 + 6y^2 = 12$. Then, its equation is

(a) $\frac{x^2}{\sin^2\theta} - \frac{y^2}{3\cos^2\theta} = 1$ (b) $\frac{x^2}{\text{cosec}^2\theta} - \frac{y^2}{\sec^2\theta} = 1$

(c) $\frac{x^2}{\sin^2\theta} - \frac{3y^2}{1 + 3\cos^2\theta} = 1$ (d) $\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$

73. A bag contains x blue balls and y red balls ($x \leq 13$). If two balls chosen simultaneously probability of (one blue, one red) is 0.5. Consider the following statements

I. The possible minimum number of blue balls is 3.

II. The possible maximum number of blue balls is 12.

(a) Statement I is true and Statement II is false
(b) Statement I is false and Statement II is true
(c) Statement I and Statement II both are false
(d) Statement I and Statement II both are true

74. A mirror and a source of light are situated at the origin O and at a point on OX, respectively. A ray of light from the source strikes the mirror and is reflected. If the direction ratios of the normal to the plane are proportional to 1, −1, 1, then direction cosines of the reflected rays are

(a) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (b) $-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

(c) $-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$ (d) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

75. $\mathbf{a}$ and $\mathbf{c}$ are unit collinear vectors and $|\mathbf{b}| = 6$, then $\mathbf{b} - 3\mathbf{c} = \lambda\mathbf{a}$, if λ is

(a) −9, 3 (b) 9, 3
(c) 3, −3 (d) None of these

76. The value of λ for which the lines

$$\frac{X-2}{1} = \frac{Y-9}{2} = \frac{Z-13}{3} \text{ and } \frac{X-\lambda}{-1} = \frac{Y-7}{2} = \frac{Z+2}{-3}$$

intersect, is

(a) −2 (b) −5
(c) −3 (d) 5

77. The line passing through the points (5, 1, a) and (3, b, 1) crosses the line $X = 0$ at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$, then

(a) $a = 2, b = 8$ (b) $a = 4, b = 6$
(c) $a = 6, b = 4$ (d) $a = 8, b = 2$

78. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

(a) 144 (b) 132 (c) −144 (d) −132

79. If $\Delta = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$, then Δ is independent of

(a) θ (b) ϕ
(c) θ and ϕ (d) None of these

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80. The area (in sq units) enclosed by the curve $y = \ln(x + e)$, $x = \ln(1 / y)$ and X-axis is

(a) 4 (b) 2 (c) – 3 (d) – 6

Section B : Numerical Value Type Questions

81. The number of three-digit numbers, whose middle digit is greater than the digit at unit place and hundredth place is .....

82. If $\tan^{-1}(x+1) + \tan^{-1} x + \tan^{-1}(x-1) = \tan^{-1} 3$, then the value of x is equal to .....

83. $10\left(\frac{1}{2\cdot 4} + \frac{1\cdot 3}{2\cdot 4\cdot 6} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8} + \ldots \infty\right)$ is equal to .....

84. The number of points in $(-\infty, \infty)$, for which $x^2 - x\sin x - \cos x = 0$, is .....

85. If the mean deviation about the median of the numbers $k, 2k, 3k, \ldots 50k$ is 100, then $|k|$ is equal to ..... .

86. If a, b, c and $A, B, C \in R - \{0\}$ such that $\alpha A + \beta B + \gamma C + \sqrt{(\alpha^2 + \beta^2 + \gamma^2)(A^2 + B^2 + C^2)} = 0$, then value of $\frac{\alpha B}{\beta A} + \frac{\beta C}{\gamma B} + \frac{\gamma A}{\alpha C}$ is equal to .....

87. If $(2+\sqrt{3})^{x^2-2x+1} + (2-\sqrt{3})^{x^2-2x-1} = \frac{2}{2-\sqrt{3}}$, then $x = \alpha$ and β, then $2000\alpha + 2000\beta$ equals .....

88. A circle has the same centre as an ellipse and passing through the foci F_1 and F_2 of the ellipse, such that the curves intersect in 4 points. Let P be any one of their points of intersection, if the major axis of the ellipse is 17 and the area of the ΔPF_1F_2 is 30, then the distance between the foci is ..... .

89. Let $f : (0, \infty) \to (1, \infty)$ be a function such that $g(x) = \int_0^x f(t)\,dt$, if $g(x^2) = x^2(1+x)$ and $h(x) = f^{-1}(x)$, then $h(10)$ equals to .....

90. Let λ and α be real. The set of all value of λ, for which the system of linear equations.

$$\begin{aligned} \lambda x + (\sin\alpha)\, y + (\cos\alpha)\, z &= 0 \\ x + (\cos\alpha)\, y + (\sin\alpha)\, z &= 0 \\ -x + (\sin\alpha)\, y - (\cos\alpha)\, z &= 0 \end{aligned}$$

has a non-trivial solution is $[-u, v]$, then $2021\left(\frac{u^2}{v^2}\right)$ is equal to .....

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a

JEE PAPER-2

PHYSICS (Section A: Objective Type Questions)										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans	c	b	a	c	d	a	a	b	a	c
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans	c	b	a	d	a	c	c	b	a	c
	(Section B: Numerical Value Type Questions)									
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans	71.42	12	4	5	4	12	20	6.48	12	25
CHEMISTRY (Section A: Objective Type Questions)										
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans	a	a	c	d	c	c	b	c	c	c
Que.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
Ans	c	c	b	a	a	a	a	d	c	a
	(Section B: Numerical Value Type Questions)									
Que.	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans	2	500	14	10	50	140	6	3	800	2
MATHEMATICS (Section A: Objective Type Questions)										
Que.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
Ans	a	b	d	d	b	c	b	a	a	b
Que.	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
Ans	b	d	c	d	a	c	c	c	b	b
	(Section B: Numerical Value Type Questions)									
Que.	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
Ans	240	-1	5	2	8	3	4000	13	36	2021

DETAILED SOLUTIONS

Physics

1. ***(c)*** At resonance condition of series *L-C-R* circuit,

$$I = \frac{V}{R} = \frac{60}{120} = 0.5\,\text{A}$$

Thus, $V_L = IX_L = I\omega_r L$

$$\Rightarrow \quad L = \frac{V_L}{I\omega_r} = \frac{40}{0.5\times 4\times 10^5} = 2\times 10^{-4}\ \text{H}$$

Resonance angular frequency, $\omega_r = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \quad C = \frac{1}{L\omega_r^2} = \frac{1}{2\times 10^{-4}\times (4\times 10^5)^2} = \frac{1}{32}\times 10^{-6}\ \text{F}$$

$$\Rightarrow \quad C = \frac{10^{-6}}{32}\ \text{F}$$

Since, current lags behind the voltage by $\frac{\pi}{4}$ rad, hence

$$\tan\phi = \frac{X_L - X_C}{R} \Rightarrow \tan 45^\circ = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\Rightarrow \quad \omega L - \frac{1}{\omega C} - R = 0$$

$$\Rightarrow \quad \omega\times 2\times 10^{-4} - \frac{1}{\omega\times \frac{1}{32}\times 10^{-6}} - 120 = 0$$

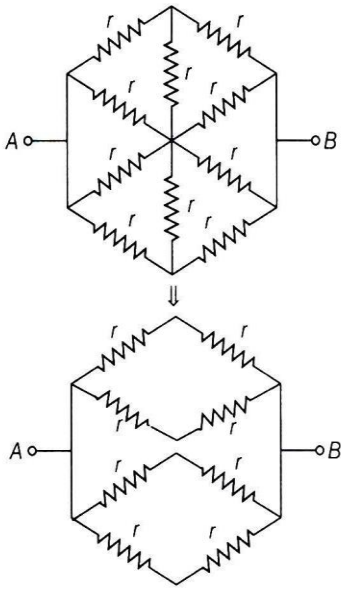
$$\Rightarrow \quad \omega^2 - 6\times 10^5\omega - 16\times 10^{10} = 0$$

$$\Rightarrow \quad \omega = \frac{6\times 10^5 \pm \sqrt{(6\times 10^5)^2 + (64\times 10^{10})}}{2}$$

$$= 8\times 10^5\ \text{rad/s} = k\times 10^5\ \text{rad/s} \qquad \text{(given)}$$

$$\therefore \quad k = 8$$

2. ***(b)*** The given hexagon circuit is symmetric about the line *AB*, so the modified circuit can be drawn as below.



The equivalent resistance between points *A* and *B* is given by

$$R_{eq} = (2r)||(2r)||(2r)||(2r)$$

$$\therefore \quad \frac{1}{R_{eq}} = \frac{1}{2r} + \frac{1}{2r} + \frac{1}{2r} + \frac{1}{2r} = \frac{4}{2r}$$

$$\therefore \quad R_{eq} = \frac{r}{2}$$

Given, $r = 1\,\Omega$

$$\therefore \quad R_{eq} = \frac{1}{2} = 0.5\,\Omega$$

3. ***(a)*** For simple harmonic motion,

$$\frac{\text{maximum acceleration}}{\text{maximum velocity}} = 20$$

$$\Rightarrow \quad \frac{\omega^2 a}{\omega a} = 20 \ \text{ or } \ \omega = 20\ \text{rad/s}$$

At $t = 0$, displacement, $x = 10$ m

$$x = a\sin(\omega t + \phi)$$

$$10 = a\sin\left(0 + \frac{\pi}{4}\right)$$

or, $$10 = a\sin\frac{\pi}{4}$$

$$10 = a\times\frac{1}{\sqrt{2}}$$

$$a = 10\sqrt{2}\,\text{m}$$

Maximum acceleration $= \omega^2 a$

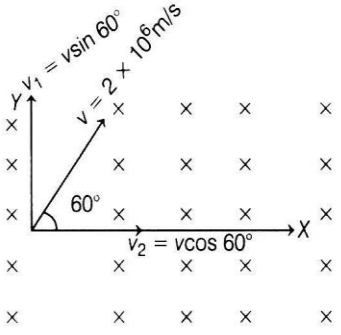
$$= (20)^2\times 10\sqrt{2} = 400\times 10\sqrt{2} = 4000\sqrt{2}\,\text{ms}^{-2}$$

4. ***(c)*** Given, $m = 1.67\times 10^{-27}$ kg,

$q = 1.6\times 10^{-19}$ C

$v = 2\times 10^6$ m/s,

$\theta = 60^\circ$, $B = 0.104$ T



As velocity of proton is inclined to the direction of magnetic field, so it will move on a circular path while moving in forward direction. So, the path of proton is helical path.

$\therefore$ Radius of the circular path,

$$r = \frac{mv_2}{Bq} = \frac{mv\cos 60°}{Bq}$$

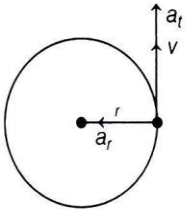
$$\therefore \quad r = \frac{1.67\times10^{-27}\times2\times10^{6}}{0.104\times1.6\times10^{-19}\times2} = 0.1\,\text{m}$$

Time period of revolution,

$$T = \frac{2\pi r}{v_2} = \frac{2\pi m}{Bq}$$

$$\therefore \quad T = \frac{2\pi\times1.67\times10^{-27}}{0.104\times1.6\times10^{-19}} = 2\pi\times10^{-7}\ \text{s}$$

5. ***(d)*** Radius of the circle, $r = 50\,\text{cm} = \frac{1}{2}\,\text{m}$, $s = 2\pi r$



$v = ks^2$

$\therefore$ Tangential acceleration,

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(ks^2) = 2ks.v = 2ks(ks^2)$$

$$\therefore \quad a_t = 2k^2s^3$$

The centripetal or radial acceleration,

$$a_r = \frac{v^2}{r} = \frac{k^2s^4}{r}$$

$\therefore$ Net acceleration, $a_{\text{net}} = \sqrt{a_t^2 + a_r^2}$

$$\therefore \quad a_{\text{net}} = \sqrt{(2k^2s^3)^2 + \left(\frac{k^2s^4}{r}\right)^2}$$

$$= k^2s^3\left(4 + \frac{s^2}{r^2}\right)^{1/2}$$

$$= k^2\times8\pi^3r^3\left(4 + \frac{4\pi^2r^2}{r^2}\right)^{1/2}$$

$$= k^2\times8\pi^3\times\frac{1}{8}\times2(\pi^2+1)^{1/2}$$

$$= 2\pi^3k^2(\pi^2+1)^{1/2}$$

6. ***(a)*** As net external force is zero.

$\therefore$ Applying conservation of momentum,

$$p_1 = p_2$$

$$\Rightarrow mu + 0 = mv\cos 30° + mv\cos 30°$$

$$\therefore \quad v = \frac{u}{\sqrt{3}} \qquad \ldots(\text{i})$$

Now, initial KE, $K_1 = \frac{1}{2}mu^2$

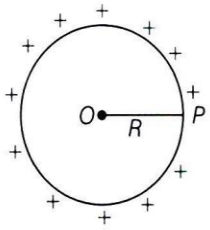
Final KE, $K_2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

$$= mv^2 = m\left(\frac{u}{\sqrt{3}}\right)^2 = \frac{1}{3}mu^2 \text{ [using Eq. (i)]}$$

$\therefore$ Loss in KE $= K_1 - K_2$

$$= \frac{1}{2}mu^2 - \frac{1}{3}mu^2 = \frac{1}{6}mu^2$$

7. ***(a)***



As electric field inside a charged spherical shell is zero.

$$\therefore \quad E = 0$$

$$\therefore \quad dV = -Edr = 0$$

$$\therefore \quad V_0 = V_p = \text{constant} = \frac{Q}{4\pi\varepsilon_0 R}$$

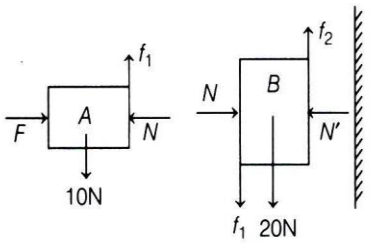
$\therefore$ The external work done,

$$W = \Delta U = q(V_p - V_0) = 0$$

8. ***(b)*** Given, $M_A = 1\,\text{kg}$, $M_B = 2\,\text{kg}$, $\mu_1 = 0.15$, $\mu_2 = 0.2$

As the blocks are at rest, so static friction acts in the system of blocks.

The FBDs of the blocks A and B are shown below.



For equilibrium of body A,

$$f_1 = 10\,\text{N}$$

For equilibrium of body B,

$$f_2 = f_1 + 20 = 10 + 20 = 30\,\text{N}$$

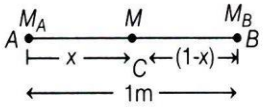
$\therefore$ Frictional force applied by the wall on the block,

$$f_2 = 30\,\text{N}$$

9. ***(a)*** Let a point mass at C is placed at a distance of x from the point mass at A as shown in figure.

Here, $\frac{M_A}{M_B} = \frac{4}{3}$, Force between A and C is

$$F_{AC} = \frac{GMM_A}{x^2}$$



Force between B and C is $F_{BC} = \frac{GMM_B}{(1-x)^2}$

According to question, $F_{AC} = \frac{1}{3}F_{BC}$

$\therefore \quad \frac{GM_AM}{x^2} = \frac{1}{3}\left(\frac{GM_BM}{(1-x)^2}\right)$

$\Rightarrow \quad \frac{M_A}{M_B} = \frac{x^2}{3(1-x)^2} \Rightarrow \frac{4}{3} = \frac{x^2}{3(1-x)^2}$

$\Rightarrow \quad 4 = \frac{x^2}{(1-x)^2} \Rightarrow 2 = \frac{x}{(1-x)}$

By solving, we get $x = \frac{2}{3}$ m

10. *(c)* [Stress] = [Young modulus] [Strain]

As, $[\text{Strain}] = \frac{[\text{Change in length}]}{[\text{Original length}]} = \frac{[M^0LT^0]}{[M^0LT^0]}$

$= [M^0L^0T^0] =$ Dimensionless

$\therefore$ [Stress] = [Young modulus]

Thermal conductivity, $[K]$

$$= \frac{[\text{Energy/Time}][\text{length}]}{[\text{Area}]\,[\text{Temperature difference}]}$$

$$= \frac{[ML^2T^{-3}][L]}{[L^2][K]} = [MLT^{-3}K^{-1}]$$

Strain is a dimensionless physical quantity, whereas stress is not a dimensionless physical quantity.

11. *(c)* Let $p_1 = 1$ atm, $n = 5$ mol, $T_1 = 293$ K

and $V_2 = \frac{V_1}{10}$

Using, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\Rightarrow \quad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 293(10)^{0.4} = 736$ K

[for diatomic gas, $\gamma = 1.4$]

Work done, $W = \frac{nR(T_1 - T_2)}{\gamma - 1}$

$$= \frac{5 \times 8.3 \times (293 - 736)}{0.4} = -46\text{ kJ}$$

$$\Delta U = \Delta Q - W = 0 - W = 46\text{ kJ}$$

12. *(b)* $r_{\max} = 2\times10^{12}$ m

$r_{\min} = 8\times10^{10}$ m

$v_{\max} = 8\times10^4$ ms^{-1}

By conservation of angular momentum,

$mvr =$ constant

$v_{\min} \times r_{\max} = v_{\max} \times r_{\min}$

$$v_{\min} = \frac{8\times10^4 \times 8\times10^{10}}{2\times10^{12}} = 4\times8\times10^{14}\times10^{-12}$$

$$= 32\times10^2 = 3.2\times10^3\text{ ms}^{-1}$$

13. *(a)* The rms speed of gas molecules is $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

The average speed of gas molecules is

$$v_{\text{av}} = \sqrt{\frac{8RT}{\pi M}}$$

$\therefore \quad v_{\text{rms}} : v_{\text{av}} = \sqrt{3} : \sqrt{\frac{8}{\pi}} = 1.732 : 1.595$

$\therefore \quad v_{\text{rms}} > v_{\text{av}}$

According to law of equipartition of energy, the energy of an ideal gas is distributed equally among its all degree of freedom. The energy associated with each degree of freedom per molecule is $\frac{1}{2}RT$.

14. *(d)* The magnitude of B is

$$B = \frac{E}{c} = \frac{9.6}{3\times10^8} = 3.2\times10^{-8}\text{ T}$$

In EM wave, **E** and **B** oscillates perpendicular to each other. As wave is propagating along $+x$-direction and **E** is along $+y$-direction. So, magnetic field **E** should be along $+z$-direction.

$\therefore \quad \mathbf{B} = 3.2\times10^{-8}\,\hat{\mathbf{k}}\text{ T}$

15. *(a)* Given, $v_p = nv_e$, $m_e = y$,

$$\frac{\lambda_p}{\lambda_e} = x$$

The de-Broglie wavelength of a particle is

$$\lambda = \frac{h}{mv}$$

$$\frac{\lambda_p}{\lambda_e} = \frac{m_ev_e}{m_pv_p} = x$$

$$\Rightarrow \frac{y}{m_p} \times \frac{v_e}{nv_e} = x$$

$\therefore \quad m_p = \frac{y}{nx}$

16. *(c)* Diode gets forward biased during positive half-cycle of alternating source, hence current flows. Let the barrier voltage of diode is V_B, its forward resistance be R_F and load resistance be R_L, then peak voltage of applied signal is

$$V_0 = V_B + i_0(R_F + R_L)$$

$\therefore \quad i_0 = \frac{V_0 - V_B}{R_F + R_L}$

$$= \frac{20 - 0.7}{10 + 500} = 37.8 \times 10^{-3} \text{ A}$$

$$= 37.8 \text{ mA}$$

Peak voltage across the ends of the load R_L,

$V_L = i_0 \times R_L = 37.8 \times 10^{-3} \times 500 = 18.9 \text{ V}$

For ideal diode, $V_B = 0$ and $R_F = 0$

$\therefore \quad i_0 = \frac{V_0}{R_L} = \frac{20}{500} = 0.04 \text{ A} = 40 \text{ mA}$

and $\quad V_L = i_0 R_L = 0.04 \times 500 = 20 \text{ V}$

17. *(c)* If Y = Young's modulus, m = mass of the wire, x = extension in the wire, A = area of cross-section of the wire and L = length of the wire, then Young's modulus by Searle's apparatus can be calculated as follows

$$Y = \frac{mgL}{Ax} = \frac{4mgL}{\pi d^2 x}$$

$$\therefore \quad \frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta L}{L} + 2 \times \frac{\Delta d}{d} + \frac{\Delta x}{x}$$

$$= \frac{0.01}{4.00} + \frac{0.01}{3.220} + 2 \times \frac{0.01}{0.045} + \frac{0.01}{0.87}$$

$$= 0.4615$$

$\therefore$ Percentage error in $Y = \left(\frac{\Delta Y}{Y} \times 100\right)\% = 46.15\%$

18. *(b)* According to Bohr's model,

$$F \propto \frac{v^2}{r}$$

Also, $\quad v \propto \frac{1}{n}$ $\quad (n$ = principal quantum number)

$$\Rightarrow \quad F \propto \frac{1}{n^4}$$

19. *(a)* $Y = (A + B) \cdot (\bar{B} + A)$

$= A \cdot \bar{B} + A \cdot A + B \cdot \bar{B} + B \cdot A$

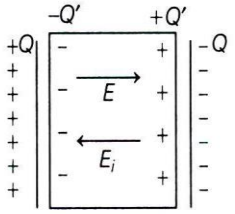
$= A \cdot \bar{B} + A + 0 + B \cdot A \quad (\because B \cdot \bar{B} = 0 \text{ and } A \cdot A = A)$

$= A(\bar{B} + 1 + B) = A \quad (\because \bar{B} + 1 + B = 1)$

20. *(c)* $C = 120 \text{pF} = 120 \times 0^{-12} \text{ F}$,

$V = 30 \text{V}, K = \frac{3}{2}$

Let induced charge on the dielectric be Q'



The charge on the capacitor with dielectric medium,

$$Q = C'V = KCV$$

$\therefore E$ = Electric field inside the plates of capacitor due to charge Q

$$= \frac{Q}{A\varepsilon_0}$$

E_i = Induced electric field $= \frac{Q'}{\varepsilon_0 A}$

$\therefore$ Resultant electric field between the plates of capacitor after insertion of dielectric material,

$$E' = E - E_i = \frac{Q - Q'}{\varepsilon_0 A} \quad \ldots(i)$$

Also, $\quad E' = \frac{E}{K} = \frac{Q}{\varepsilon_0 AK} \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$\frac{Q - Q'}{\varepsilon_0 A} = \frac{Q}{\varepsilon_0 AK}$$

$$\therefore \quad Q' = Q\left(1 - \frac{1}{K}\right) \quad [\because Q = KCV]$$

$$\therefore \quad Q' = KCV\left(1 - \frac{1}{K}\right) = (K - 1)CV$$

$$= \left(\frac{3}{2} - 1\right) \times 120 \times 10^{-12} \times 30 = 1.8 \text{ nC}$$

Study Tactics

Start from the calculation of electric field inside the plates of capacitor and then induced electric field. Equate both the electric field and finally, calculate the induced charge on dielectric.

21. (71.42) For a body rolling along a level surface,

$$(\text{KE})_{\text{Total}} = (\text{KE})_T + (\text{KE})_R$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow (\text{KE})_{\text{Total}} = \frac{1}{2}(mv^2 + I\omega^2) \quad \ldots(i)$$

Now, moment of inertia for the solid sphere,

$$I = \frac{2}{5}mr^2$$

Substituting this value of moment of inertia in Eq. (i), we get

$$(\text{KE})_{\text{Total}} = \frac{1}{2}\left(mv^2 + \frac{2}{5}mr^2\omega^2\right)$$

Angular velocity of a body is given as

$$\omega = \frac{v}{r} \Rightarrow v = \omega r$$

Substituting value of ωr as v in the above equation,

$$(\text{KE})_{\text{Total}} = \frac{1}{2}\left(mv^2 + \frac{2}{5}mv^2\right)$$

$$\Rightarrow (\text{KE})_{\text{Total}} = \frac{7}{10}mv^2$$

Now, the contribution of translational KE in total KE can be found on dividing the translational KE by total KE.

$$\frac{(KE)_{Trans}}{(KE)_{Total}} = \frac{\frac{1}{2}mv^2}{\frac{7}{10}mv^2}$$

$$= \frac{1}{2}\times\frac{10}{7}$$

$$= \frac{10}{14}$$

$$= 0.7142$$

$$\therefore \quad \%(KE)_{Trans} = 0.7142\times 100$$

$$= 71.42\%$$

22. (12) The coefficient of linear expansion,

$$\alpha = \frac{1}{T}\frac{\Delta l}{l}$$

The coefficient of volumetric expansion,

$$\gamma = \frac{1}{T}\cdot\frac{\Delta V}{V}$$

$\therefore$ For a cube, $V = xyz$

$$\therefore \quad \frac{\Delta V}{V} = \frac{\Delta x}{x}+\frac{\Delta y}{y}+\frac{\Delta z}{z}$$

$$\Rightarrow \quad \frac{1}{T}\cdot\frac{\Delta V}{V} = \frac{1}{T}\cdot\frac{\Delta x}{x}+\frac{1}{T}\cdot\frac{\Delta y}{y}+\frac{1}{T}\cdot\frac{\Delta z}{z}$$

$$\Rightarrow \quad \gamma = \alpha_x+\alpha_y+\alpha_z$$

$$\Rightarrow \quad \alpha_z = \gamma-(\alpha_x+\alpha_y)$$

$$= 57\times 10^{-5}-(40+5)\times 10^{-5}$$

$$\Rightarrow \quad \alpha_z = 12\times 10^{-5}/^\circ C = n\times 10^{-5}/^\circ C$$

$$\Rightarrow \quad n = 12$$

23. (4) $$\frac{1}{f_a} = \left(\frac{\mu_g}{\mu_a}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \ldots(i)$$

Similarly, $$\frac{1}{f_m} = \left(\frac{\mu_g}{\mu_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \ldots\ldots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{f_m}{f_a} = \frac{\left(\frac{\mu_g}{\mu_a}-1\right)}{\left(\frac{\mu_g}{\mu_m}-1\right)} = \frac{\left(\frac{1.5}{1}-1\right)}{\left(\frac{1.5}{\frac{4}{3}}-1\right)} = 4$$

$$\therefore \quad \frac{f_m}{f_a} = 4$$

24. (5) For path difference, $\Delta x = 2\lambda$

Phase difference, $\Delta\phi = \frac{2\pi}{\lambda}\times\Delta x = \frac{2\pi}{\lambda}\times 2\lambda = 4\pi$

Intensity, $I = I_1+I_2+2\sqrt{I_1I_2}\cos(\Delta\phi)$

As, light is monochromatic,

$$\therefore \quad I_1 = I_2 = I_0$$

At, $\Delta\phi = 4\pi$,

$$I = 4I_0 \qquad \text{(given)}$$

Also given, $I = c$

$$\therefore \quad I_0 = \frac{c}{4}$$

For path difference,

$$\Delta x = \frac{\lambda}{d}$$

Phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}\times\frac{\lambda}{4} = \frac{\pi}{2}$$

$$\therefore \quad I' = I_1+I_2+2\sqrt{I_1I_2}\cos(\Delta\phi)$$

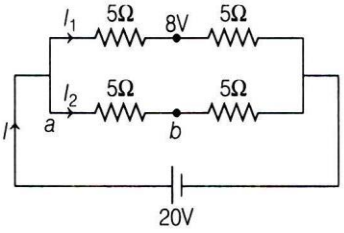
$$= \frac{c}{4}+\frac{c}{4}+2\times\frac{c}{4}\times 0$$

$$= \frac{c}{2}$$

According to question, $\frac{nc}{10} = \frac{c}{2}$

$$\Rightarrow \quad n = 5$$

25. (4) When capacitor is fully charged, then it behaves as open circuit.



$$\therefore \quad R_{net} = \frac{10\times 10}{10+10} = 5\Omega$$

$$I_1 = \frac{20-8}{5} = \frac{12}{5} = 2.4\text{ A}$$

$$I = \frac{E}{R_{net}} = \frac{20}{5} = 4\text{ A}$$

$$\therefore \quad I_2 = (4-2.4) = 1.6\text{ A}$$

$$\therefore \quad V_{ab} = I_2\times 5 = 1.6\times 5 = 8\text{ V}$$

$$V_a = 20\text{ V}$$

$$\therefore \quad V_{ab} = V_a - V_b$$

$$\Rightarrow \quad 8 = 20 - V_b$$

$$\therefore \quad V_b = 12\text{ V}$$

$\therefore$ Potential difference across the capacitor,

$$V = V_b - 8 = 12-8 = 4\text{ V}$$

26. (12) The fundamental frequency of the vibrating wire in sonometer is

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$\therefore \quad f \propto \frac{1}{l}$$

As, $f_1 : f_2 : f_3 = 1:2:3$

$$\therefore \quad l_1 : l_2 : l_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 6:3:2$$

$$l = l_1 + l_2 + l_3 = 2\,\text{m}$$

$$\therefore \quad l_1 = \left(\frac{6}{6+3+2}\right) \times 2 = \frac{12}{11}\,\text{m}$$

$\therefore$ The distance of first bridge form first end is

$$l_1 = \frac{12}{11}\,\text{m}$$

A compared with given equation in question.

$$\frac{x}{11} = \frac{12}{11} \Rightarrow x = 12$$

27. (20) Initial velocity,

$$u = 20\,\text{m/s}$$

Final velocity,

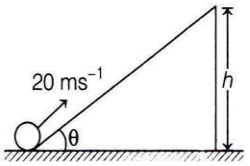
$$v = 0$$

Acceleration,

$$a = -g\sin\theta = -10\sin\theta$$

Displacement,

$$s = \frac{h}{\sin\theta}$$



Using $v = u + at$

$$0 = 20 + (-10\sin\theta)(4)$$

$$\Rightarrow \quad \sin\theta = \frac{20}{10\times 4} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30^\circ$$

From second equation of motion,

$$s = ut + \frac{1}{2}at^2$$

$$\frac{h}{\sin 30^\circ} = 20(4) + \frac{1}{2}(-10\sin 30^\circ)(4)^2$$

$$2h = 80 + \frac{1}{2}\times\left(\frac{-10}{2}\right)\times 16$$

$$h = 20\,\text{m}$$

28. (6.48) For transition of electron in hydrogen atom,

$n_1 = 3,\ n_2 = 1$

The energy of photon emitted by hydrogen atom,

$$E = -13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = -13.6\left[\frac{1}{3^2} - \frac{1}{1^2}\right] = 12.08\,\text{eV}$$

Now, $B = 8\times 10^{-4}\,\text{T}$, $r = 10\,\text{mm} = 10\times 10^{-3}$ m

$\therefore$ Radius of circular path of electron in magnetic field is

$$r = \frac{mv}{Bq} = \frac{\sqrt{2mK}}{Bq}$$

$$\Rightarrow \text{Kinetic energy, } K = \frac{r^2B^2q^2}{2m}$$

$$\therefore \quad K = \frac{(10\times 10^{-3})^2 \times (8\times 10^{-4})^2 \times (1.6\times 10^{-19})^2}{2\times 9.1\times 10^{-31}}$$

$$= 9\times 10^{-19}\,\text{J} = 5.6\,\text{eV}$$

From Einstein photoelectric equation,

$$\text{KE} = E - \phi$$

$$\therefore \quad 5.6 = 12.08 - \phi$$

$$\therefore \quad \phi = 6.48\,\text{eV}$$

29. (12) $B = \dfrac{\mu_0 NI}{2r}$

$$I = \frac{2Br}{\mu_0 N}$$

$$I = \frac{2\times 75.36\times 10^{-4}\times 10\times 10^{-2}}{4\pi\times 10^{-7}\times 100} = 12\,\text{A}$$

30. (25) Given, $m = 6\,\text{kg}$, $R = 10$ m

For the block to complete vertical circle,

$$N_B \geq 0$$

$\therefore$ Applying conservation of energy at A and B,

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mg(2R)$$

$$\therefore \quad v_A^2 = v_B^2 + 4gR \qquad \ldots(\text{i})$$

Also, at point B,

$$N_B + mg = \frac{mv_B^2}{R}$$

$$\Rightarrow \quad v_B^2 = gR \qquad \ldots(\text{ii})$$

From Eqs. (i) and (ii),we get

$$v_A = \sqrt{5gR} \qquad \ldots(\text{iii})$$

Minimum velocity at A, $v_A = \sqrt{5gR}$

Now, applying conservation of energy at points A and C,

$$0 + mgh_{\min} = \frac{1}{2}mv_A^2$$

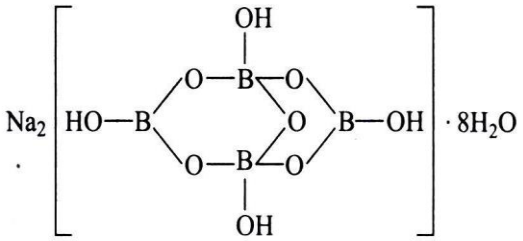
$$mgh_{\min} = \frac{1}{2}m(5gR) \qquad [\text{using Eq. (iii)}]$$

$$\Rightarrow \quad h_{\min} = \frac{5}{2}R = \frac{5}{2}\times 10 = 25\,\text{m}$$

$$\therefore \quad h_{\min} = 25\,\text{m}$$

Chemistry

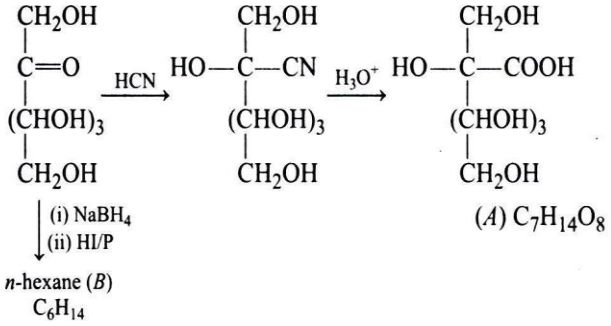
31. (*a*) Borax molecule is made up of two tetrahedral and two triangular units joined as follows



So, correct representation of borax is,

$$Na_2[B_4O_5(OH)_4]\cdot 8H_2O$$

32. *(a)* The reaction is represented as



33. *(c)* Given, $\Lambda^\circ_m NaCl = 126.4\ S\ cm^2 mol^{-1}$

$\Lambda^\circ_m HCl = 425.9\ S\ cm^2 mol^{-1}$

$\Lambda^\circ_m NaA = 100.5\ S\ cm^2 mol^{-1}$

$\Lambda^\circ_m HA = \Lambda^\circ_m HCl + \Lambda^\circ_m NaA - \Lambda^\circ_m NaCl$

$= 425.9 + 100.5 - 126.4$

$= 400\ S\ cm^2 mol^{-1}$

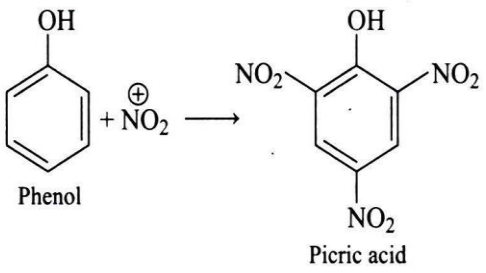
$$\Lambda_m = \frac{1000\kappa}{C} = 6\times10^{-5}\times\frac{1000}{0.001}$$

$= 60\ S\ cm^2 mol^{-1}$

$$\Lambda_m = \frac{\Lambda_m}{\Lambda^\circ_m} = \frac{60}{400} = 0.150$$

34. (*d*) Phenol undergoes electrophilic substitution reaction on reaction with a mixture of conc. HNO_3 and conc. H_2SO_4 (known as nitrating mixture). This reaction produces nitro compound and on successive nitration it produces trinitrophenol.

Conc. HNO_3 + Conc. $H_2SO_4 \longrightarrow \overset{+}{N}O_2$ (electrophile)

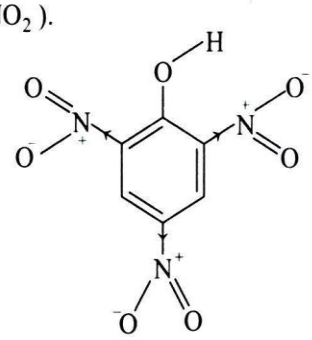


Molecular formula = $C_6H_3N_3O_7$

Degree of unsaturation $= (C+1) - \frac{H}{2} - \frac{X}{2} + \frac{N}{2}$

$$= (6+1) - \frac{3}{2} + \frac{3}{2} = 7 - 0 = 7$$

Nature of compound The compound is acidic in nature due to presence of three strong electron withdrawing groups (NO_2).



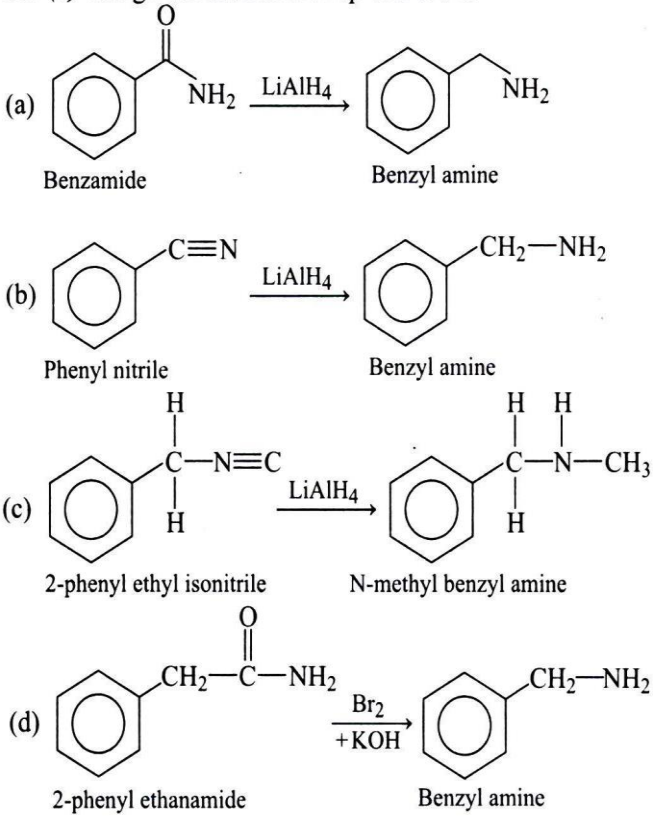
35. *(c)* Copper and silver alkylides are obtained by passing alkynes in the ammoniacal solution of cuprous chloride or silver nitrate respectively. The reaction are used for detecting the presence of acetylenic hydrogen atom is

$$HC\equiv CH + Cu_2Cl_2 + 2NH_4OH \longrightarrow$$

$$\underset{\text{Copper acetylide (Red ppt.)}}{Cu\cdot C\equiv C\cdot Cu\downarrow} + 2NH_4Cl + 2H_2O$$

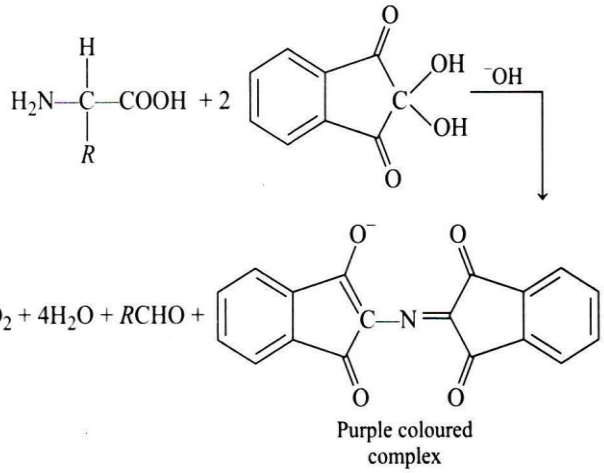
While, C_2H_4 and CH_4 comes out unaffected from test tube.

36. *(c)* The given reaction are represented as



Last process is a good example of Hofmann bromamide reaction in which carboxylamide are converted into amine having one carbon atom less than carboxyl amide.

37. *(b)* All α-amino acids on treatment with ninhydrin (2, 2-dihydroxyindane -1, 3-dione) give purple colouration. The purple coloured complex is Ruhemann's purple. This test is called ninhydrin test. Hence, it is a corroborative test for the presence of protein. The reaction is as follows



$CO_2 + 4H_2O + RCHO +$

Purple coloured complex

38. *(c)* Statement I is correct but statement II is incorrect.

For a solution to be ideal it should obey Raoult's law over entire range of concentration.

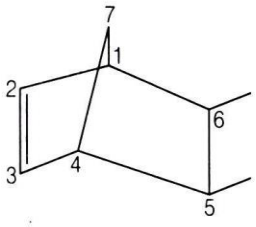
In case of ideal solutions, other conditions are

$$\Delta_{mix}S > 0, \Delta_{mix}V = 0, \Delta_{mix}H = 0, \Delta_{mix}G < 0$$

39. (*c*) $CH_3\underset{\text{Propene}}{-CH=CH_2} + B_2H_6 \xrightarrow{H_2O_2} \underset{\text{Propan-1-ol}}{CH_3-CH_2-CH_2OH}$

When propene reacts with diborane followed by H_2O_2, it gives primary alcohols which is in accordance with anti-Markownikoff's rule.

40. *(c)*



Total carbon atom forming the bicyclic ring = 7 (hept)

Functional group present ⇒ double bond (ene)

Position of double bond ⇒ 2-ene

Substituents ⇒ 2-methyl groups ⇒ dimethyl

Position of substituents = 5, 6 ⟶ 5, 6-dimethyl

Number of cyclic chain = 2 ⟶ Bicyclo

3-bridges are 2 carbons, 2 carbons and one carbon.

IUPAC name = 5, 6-dimethylbicyclo [2, 2, 1] hept- 2-ene

Molecular formula of compound is C_9H_{14}.

Degree of unsaturation can be calculated as

$$u = (C+1) - \frac{H}{2} - \frac{X}{2} + \frac{N}{2}$$

where, u = degree of unsaturation

C = number of carbons

H = number of hydrogens

X = number of halogens

N = number of nitrogens.

Hence, for a compound having molecular formula C_9H_{14} the degree of unsaturation may be calculated as

$$u = (9+1) - \frac{14}{2}$$
$$= 10 - 7$$
$$= 3$$

Study Tactics

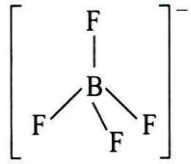
The problem can be solved by identifying the parent chain, functional group, position of functional group, substituent and their position one by one and then write the name of compound according to IUPAC names and finally calculate degree of unsaturation by using formula

$$u = (C + 1) - \frac{H}{2} - \frac{X}{2} + \frac{N}{2}$$

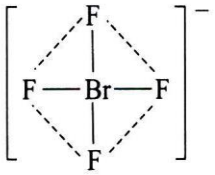
41. *(c)* The correct match is A-III, B-I, C-IV, D-II.

The molecules with their shapes are given below.

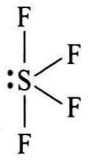
(A) BF_4^- – Tetrahedral



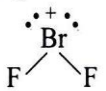
(B) BrF_4^- – Square planar



(C) SF_4 – *See-saw*



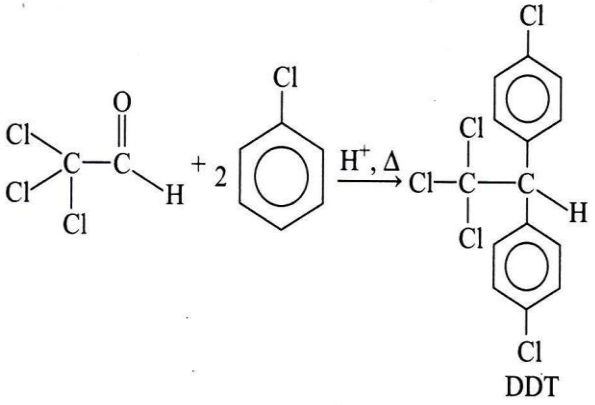
(D) BrF_2^+ – Bent-shape



42. (*c*) Among the given statements B, C and D are correct while statement A is incorrect.

Preparation of DDT

DDT is *p*, *p*-Dichlorodiphenyltrichloroethane prepared by reaction of chlorobenzene with CCl_3—CHO in presence of H_2SO_4 through electrophilic substitution reaction.



DDT

Characteristics of DDT

- It is used as an insecticide to kill mosquito so that people are prevented by malaria.
- It is highly toxic towards fish.
- It is toxic for animals because it is not easily metabolised by animal.
- It has no chiral centre.

43. (*b*) The correct match is A-III, B-I, C-IV, D-II.

A. Cr^{3+} has $3d^3$ configuration, with 3 unpaired electrons. Hence, it shows paramagnetic behaviour. Complex of the type Ma_4b_2 shows *cis-trans* isomerism

B. Ti^{3+} has $3d^1$ configuration, hence shows paramagnetic behaviour. Complex gives Cl^- and NO_3^- ions in solution hence, shows ionisation isomerism.

C. Pt^{2+} has $5d^8$ configuration but ligands are strong field ligands hence, it forms square planar complex. thus, all electrons are paired and it also exhibits ionisation isomerism.

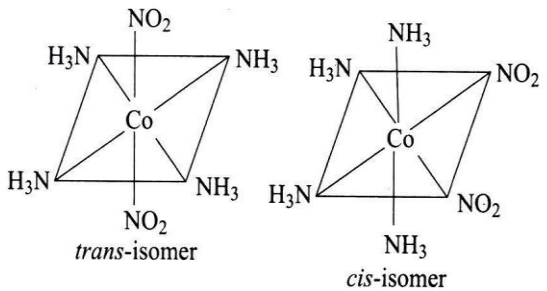
D. Co^{3+} has $3d^6$ configuration. But, ligands present are strong enough to cause electron pairing, hence, it shows diamagnetic behaviour and exhibits *cis-trans* isomerism as it is Ma_4b_2 type complex.

44. (*a*)

(i) NO_2 is an ambidentate group. It can show linkage isomerism by linking through either N- or O-atom.

(ii) The given complex can show ionisation isomerism with the complex $[Co(NH_3)_4NO_2Cl]NO_2$.

(iii) The complex show geometrical isomerism.

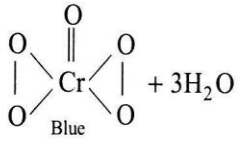


trans-isomer *cis*-isomer

45. (*a*) $N_2H_4 + H^+ \longrightarrow [N_2H_4 \longrightarrow H]^+$

A coordinate bond is present between N and H and a covalent bond with 4 hydrogens. Thus, it form coordinate covalent bond.

46. (*a*) (*A*) $\underset{\text{Green}}{2Cr^{3+}} + \underbrace{10OH^- + 3H_2O_2}_{A} \longrightarrow \underset{\text{Yellow}}{2CrO_4^{2-}} + 8H_2O$

(*B*) $CrO_4^{2-} + \underbrace{2H_2O_2 + 2H^+}_{B} \longrightarrow$  $+ 3H_2O$

Blue

In aqueous solution, CrO_5 is unstable and it further decomposes.

(*C*) $2\,CrO_5 \longrightarrow \underset{\text{(Amphoteric)}}{Cr_2O_3} + \frac{7}{2}O_2$

$Cr_2O_3 + 3\,H_2SO_4 \longrightarrow Cr_2(SO_4)_3 + 3\,H_2O$

47. (*a*) A. Tetracyanomethane : $C(CN)_4$ has 8 σ and 8 π-bond. So, ratio is 1.

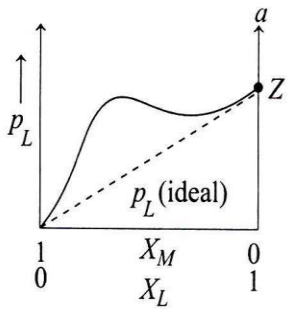
B. CO_2 has 2 σ and 2 π-bonds. So, ratio is 1.

C. Benzene has 6 σ and 3 π-bonds. So, the ratio is 2.

D. 1, 3-butadiene has 9 σ and 2 π-bonds. So, the ratio is $\frac{9}{2}$.

Hence, the correct order is A = B < C < D.

48. (*d*) .



The graph representing the deviation from Raoult's law therefore intermolecular forces of attraction between

$M—L < M—M$ or $L—L$

$p_L \geq p_L^\circ X_L$

but when $X_L = 1$, the mixture has almost pure liquid L so, $p_L = p_L^\circ$

49. (*c*) The incorrect statement for glucose is given in option (c). Its correct form is, on prolonged heating with HI, glucose forms *n*-hexane.

$$\begin{array}{l}CHO\\|\\(CHOH)_4\\|\\CH_2OH\end{array} \xrightarrow{HI,\Delta} CH_3—CH_2—CH_2—CH_2—CH_2—CH_3$$

(*n*– hexane)

50. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

$[Sc(H_2O)_6]^{3+}$ has no unpaired electron in its *d*-subshell and thus *d* - *d* transition is not possible. $[Ti(H_2O)_6]^{3+}$ has one unpaired electron in its *d*-subshell which gives rise to *d*-*d* transition.

51. *(2)* **At anode**

$$H_2O(l) + ClO_3^-(aq) \longrightarrow ClO_4^-(aq) + 2H^+(aq) + 2e^-$$

At cathode

$$2H^+(aq) + ClO_3^-(aq) + 2e^- \longrightarrow ClO_2^-(aq) + H_2O(l)$$

$$2ClO_3^-(aq) \longrightarrow ClO_2^-(aq) + ClO_4^-(aq)$$

$$E^\circ_{cell} = -0.39 + 0.36 = -0.03$$

$$E^\circ_{cell} = \frac{RT}{2F} \ln K$$

$$-0.03 = \frac{0.06}{2} \log K \text{ or } K = 0.1$$

$$\underset{0.1-2x}{2ClO_3^-} \rightleftharpoons \underset{x}{ClO_4^-} + \underset{x}{ClO_2^-}$$

$$\frac{x^2}{(0.1-2x)^2} = 0.1 = \frac{1}{10}$$

$$3.16x = 0.1 - 2x \Rightarrow 5.16x = 0.1$$

$$x = \frac{0.1}{5.16} = 0.0193 \approx 1.93 \times 10^{-2} \approx 2 \times 10^{-2}$$

52. (*500*) Atomic weight of element,

$$y = 6.64 \times 10^{-23} \times N_A \approx 40$$

Number of moles of $y = \frac{20 \times 1000}{40} = 500$

53. (*14*) Equation of combustion,

$$C_nH_{2n+2} + \left(\frac{3n+1}{2}\right) O_2 \longrightarrow n\,CO_2 + (n+1)\,H_2O$$

Initial pressure of $C_n H_{2n+2}$ is p (assumed).

Increase in pressure

$$= p\left[(2n+1) - 1 - \left(\frac{3n+1}{2}\right)\right] = \left(\frac{n-1}{2}\right) p \qquad \ldots(i)$$

$\because$ Molecular mass of organic compound $= 14 \times n + 2$

at 546 K and 4.6 atm or 273 K and 2 atm

Increase in pressure

$\Rightarrow 2.3 - 2 = 0.3$ atm

$$p = \frac{nRT}{V} = \frac{23.2}{M} \times \left(\frac{0.0821 \times 273}{44.82}\right) = \frac{11.6}{14n+2}$$

Substitute the value of p in Eq. (i)

$$\frac{(n-1)}{2} \times \frac{11.6}{14n+2} = 0.3$$

On solving, $n = 4$, so formula of compound is C_4H_{10}.

Thus, $x + y = 4 + 10 = 14$

Study Tactics

This problem can be solved by using concept of ideal gas equation and chemical equation involved in combustion reaction. Students are advised to follow the steps,

Write chemical equation involved in combustion of hydrocarbon.

Calculate increase in pressure by assuming initial pressure = p

Then, calculate number of moles using ideal gas equation.

54. *(10)* From the aqueous buffered solution of H*A*, 50% H*A* is ionised.

$$[HA] = [A^-]$$

Buffer solution of weak acid

$HA \longrightarrow$ acidic buffer

$$pH = pK_a + \log\frac{[A^-]}{[HA]}$$

$$pH = pK_a + \log\frac{0.5}{0.5} = pK_a + \log 1$$

$$pH = pK_a = 4.1$$

$$pOH = pK_w - pH$$

$$pOH = 14 - 4.1 = 9.9 \approx 10$$

55. (*50*) $KMnO_4$ reacts with ferrous sulphate according to the given reaction. So, reaction can be represented as

$$2KMnO_4 + 10FeSO_4 + 8H_2SO_4 \longrightarrow$$

$$K_2SO_4 + 2MnSO_4 + 5Fe_2(SO_4)_3 + 8H_2O$$

2 moles of $KMnO_4$ reacts with 10 moles of $FeSO_4$.

The number of moles of $KMnO_4$ in 10 ml of 0.1 M

$$= 0.1 \times 0.01 = 10^{-3} \text{ moles}$$

Therefore, 10^{-3} moles $KMnO_4$ reacts with 5×10^{-3} moles of $FeSO_4$.

Hence, 10 mL of 0.1 M $KMnO_4$ is equivalent to its 5 times, i.e. 50 mL of 0.1 M $FeSO_4$.

56. *(140)* According to Arrhenius equation,

$$k = Ae^{-E_a/RT}$$

So, $k_1 = Ae^{-\frac{E_{a_1}}{RT}}$,

$$k_2 = Ae^{-\frac{E_{a_2}}{RT}}$$

and $k_3 = Ae^{-\frac{E_{a_3}}{RT}}$

$$k = \left(\frac{k_1 k_2}{k_3}\right)^{2/3} \qquad \text{[Given]}$$

$$\therefore \quad k = \left(\frac{Ae^{-\frac{E_{a_1}}{RT}} \cdot Ae^{-\frac{E_{a_2}}{RT}}}{Ae^{-E_{a_3}/RT}}\right)^{2/3} = Ae^{-E_a/RT}$$

$$e^{\frac{1}{RT}\left[\frac{2}{3}(E_{a_3} - (E_{a_1} + E_{a_2}))\right]} = e^{-E_a/RT}$$

$$E_a = \frac{2}{3}[E_{a_1} + E_{a_2} - E_{a_3}]$$

$$E_a = \frac{2}{3}[200 + 90 - 80] = \frac{2}{3}[210]$$

$$= 140 \text{ kJ / mol}$$

57. (*6*) The formula for conductance is $G = \kappa \times \frac{a}{l}$

$$5\times10^{-7} = \kappa \times \frac{1}{120}$$

$$\kappa = 6\times10^{-5}\ \text{S cm}^{-1}$$

$$\Lambda_m^C = \frac{\kappa\times1000}{M} = \frac{6\times10^{-5}\times1000}{0.0015} = 40$$

$\because$ $\quad pH = 4$

$\therefore$ $\quad [H^+] = 10^{-4} = C\alpha = 0.0015\alpha$

$$\alpha = \frac{\Lambda_m^C}{\Lambda_m^\circ} = \frac{10^{-4}}{0.0015} = 0.06667$$

$$\Lambda_m^\circ = \frac{\Lambda_m^C}{\alpha}$$

$$\Lambda_m^\circ = \frac{40\,\text{Scm}^2/\text{mol}}{0.06667}$$

$$\Lambda_m^\circ = 6\times10^2\ \text{S cm}^2/\text{mol}$$

But

$$\Lambda_m^\circ = Z\times10^2\ \text{S cm}^2/\text{mol}$$

Hence, $\quad Z = 6$.

58. (*3*) Statements (A), (B) and (D) are correct.

Balanced chemical equation is

$$6\overset{-1}{I^-} + ClO_3^- + 6H_2SO_4 \longrightarrow Cl^- + 6HSO_4^- + 3\overset{0}{I_2} + 3H_2O$$

The stoichiometric coefficient of HSO_4^- is 6.

Iodide's oxidation state has changed from -1 to 0 oxidation state, i.e. iodide is oxidised.

H_2O is one of the products in the reaction.

Sulphur oxidation state remains the same after reactions, i.e. +6, hence sulphur is not reduced.

59. (*800*) ΔT_f = freezing point of H_2O – freezing point of ethylene glycol solution

$= 0 - (-6^\circ) = 6^\circ C$

$K_f = 1.86\ \text{K kg mol}^{-1}$

w_1 = Mass of ethylene glycol in grams

w_2 = Mass of solvent (H_2O) in grams = 4000 g

m_1 = Molar mass of ethylene glycol = 62 g mol^{-1}

i = van't Hoff factor = 1

($\because$ ethylene glycol is non-electrolyte)

$$\Delta T_f = iK_f m$$

From, $\quad \Delta T_f = \frac{1000\,K_f\,w_1(i)}{m_1\,w_2}$

$\therefore$ $\quad 6 = \frac{1000\times1.86\times w_1\times1}{62\times4000}$

$$w_1 = 800\ \text{g}$$

60. (*2*) $X + 2e^- \longrightarrow X^{2-}$;

energy released = – 30.86 eV/atom

Total energy released = number of moles of molecule × energy released by one mole of molecule

$$= y \times 30.86\,N_A\,\text{eV}$$

Number of moles of $H_2 = \frac{4}{2} = 2$

According to de-Broglie, $2\pi r = n\lambda$

Now, it is given that, $2\pi r = 4\lambda$

$\therefore$ $\quad n = 4$ [Energy level]

$$2H_2 \longrightarrow 2H^+ + 2H^*$$

Total energy required = total energy required to dissociate two moles of H_2 + total energy required in ionisation of two H_2 to two H^+ + total energy required in ionisation of two H to 4th excited energy level.

$$= 2\times4.52\times N_A + 2\times13.6\,N_A + 2\times13.6\times\left(1-\frac{1}{16}\right)\times N_A$$

$$= N_A\ (9.04 + 27.2 + 27.2\times0.94)$$

$$= N_A\ (61.80)\ \text{eV}$$

We know that, during formation of H^+ and H^* in above reaction,

Total energy required = Total energy released

$\therefore$ $\quad (61.80\times N_A)\ \text{eV} = (-30.86y)\,N_A\ \text{eV}$

$$y = \frac{61.80}{30.86} \simeq 2$$

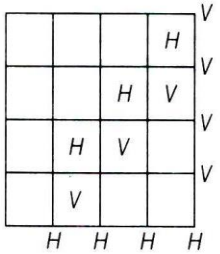
Hence, number of moles required = 2.

Study Tactics

This problem is based upon the conceptual mixing of de-Broglie equation and energy consideration during formation of molecule. So, students are advised to calculate the energy required and energy released during process. Before calculating energy required write the reaction of formation of H^+ and H^* by using concept of de-Broglie's equation.

Mathematics

61. (*a*) We have, shown two possible path in the following figure



If horizontal movement is written as H and vertical movement is written as V.

Then, these paths can be written as

$HHHHVVVV$ and $HV\ HV\ HV\ HV$

Hence, the possible path is basically arrangement of 4 H and 4 V.

$\therefore$ Total number of ways $= \frac{8!}{4!4!} = \frac{8\times7\times6\times5\times4!}{4\times3\times2\times1\times4!} = 70$

62. *(b)* $\log_b a, \log_c b$ and $\log_a c$ are in GP.

$\Rightarrow \quad (\log_c b)^2 = \log_b a \times \log_a c$

$\Rightarrow \quad (\log_c b)^2 = \log_b c$

$\Rightarrow \quad (\log_c b)^2 = \frac{1}{\log_c b}$

$\Rightarrow \quad (\log_c b)^3 = 1$

$\Rightarrow \quad \log_c b = 1$

$\Rightarrow \quad b = c\ (a, b, c > 0)$

Then, $2a^4 = b^4 + c^4$

$\Rightarrow \quad 2a^4 = 2b^4$

$\Rightarrow \quad a = b = c$

By $abc = 27 \Rightarrow a^3 = 27 \Rightarrow a = 3$

$\therefore\ a = b = c = 3$

63. *(d)* Given, $\frac{dy}{dx} = \frac{ax+5}{4y+b}$

$\Rightarrow (4y+b)\,dy = (ax+5)\,dx$

$\Rightarrow \quad 2y^2 + by = \frac{ax^2}{2} + 5x + C$

$\Rightarrow \quad 2y^2 - \frac{ax^2}{2} + by - 5x - C = 0$

represents a circle, then $2 = -\frac{a}{2} \Rightarrow a = -4$

$\therefore\ 2(x^2 + y^2) + by - 5x - C = 0$

$\Rightarrow x^2 + y^2 - \frac{5}{2}x + \frac{b}{2}y - \frac{C}{2} = 0$ is a circle of radius,

$$r = \sqrt{\left(\frac{-5}{4}\right)^2 + \left(\frac{b}{4}\right)^2 + \frac{C}{2}}$$

Hence, $r = \frac{1}{4}\sqrt{b^2 + 8C + 25}$

64. *(d)* $|z|^2\omega - z|\omega|^2 - z + \omega = 0$ ...(i)

$\Rightarrow \omega(|z|^2 + 1) - z(|\omega|^2 + 1) = 0$

$\Rightarrow \quad \frac{\omega}{z} = \frac{|\omega|^2 + 1}{|z|^2 + 1}$

$\Rightarrow \frac{\omega}{z}$ is a real number.

$\therefore \quad \frac{\omega}{z} = \frac{\bar{\omega}}{\bar{z}} \Rightarrow \bar{\omega}z = \bar{z}\omega$ ...(i

From Eq. (i), we get

$z\bar{z}\omega - z\omega\bar{\omega} - z + \omega = 0$ [$\because z\bar{z} = |z|$

$\Rightarrow \quad z(z\bar{\omega} - 1) - \omega(\bar{\omega}z - 1) = 0$ [using Eq. (ii

$\Rightarrow \quad (z - \omega)(z\bar{\omega} - 1) = 0$

$\Rightarrow \quad z = \omega$ or $z\bar{\omega} = 1$

65. *(b)* Given, $\int \frac{e^{x-1}\,2x\,dx}{x^2 - 5x + 4} = AF(x-1) + BF(x-4) + C$

$$\frac{2x}{x^2 - 5x + 4} = \frac{2x}{(x-4)(x-1)} = \frac{A_1}{x-4} + \frac{A_2}{x-}$$

$\Rightarrow \quad 2x = A_1(x-1) + A_2(x-4)$

On solving, we get

$A_1 = \frac{8}{3}, A_2 = \frac{-2}{3}$

$$\therefore \int \frac{e^{x-1}\,2x\,dx}{x^2 - 5x + 4} = \int e^{x-1}\left(\frac{-\frac{2}{3}}{x-1} + \frac{\frac{8}{3}}{x-4}\right)dx$$

$$= \frac{-2}{3}\int \frac{e^{x-1}}{x-1}\,dx + \frac{8}{3}\int \frac{e^{x-1}}{x-4}\,dx$$

$$= \frac{-2}{3}F(x-1) + \frac{8}{3}e^3F(x-4) + C$$

$A = \frac{-2}{3}$ and $B = \frac{8}{3}e^3$

$(A, B) \equiv \left(\frac{-2}{3}, \frac{8}{3}e^3\right)$

Study Tactics

First, use partial fraction method of integration, then use $F(x) = \int \frac{e^x}{x}\,dx$ to get expression in the form of $C \cdot F(x)$ where, C is constant.

66. *(c)* $\left(\frac{1}{a}\right)^b = \lim_{x\to0}\frac{8}{x^8}\left(1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right.$

$$= \lim_{x\to0}\frac{8}{x^8}\left[\left(1 - \cos\frac{x^2}{2}\right)\left(1 - \cos\frac{x^2}{4}\right)\right]$$

$$= \lim_{x\to0}\frac{8}{x^8}\left[\left(2\sin^2\frac{x^2}{4}\right)\left(2\sin^2\frac{x^2}{8}\right)\right]$$

$$= \lim_{x\to0} 32\left(\frac{\sin\frac{x^2}{4}}{x^2}\right)^2\left(\frac{\sin\frac{x^2}{8}}{x^2}\right)^2$$

$$= \lim_{x\to 0} 32\left(\frac{\sin\frac{x^2}{4}}{\frac{4x^2}{4}}\right)^2\left(\frac{\sin\frac{x^2}{8}}{8\times\frac{x^2}{8}}\right)^2$$

$$= \frac{32}{4^2\times 8^2} = \frac{1}{4\times 8} = \frac{1}{2^5}$$

$\therefore \left(\frac{1}{a}\right)^b = \left(\frac{1}{2}\right)^5$

$a = 2$ and $b = 5$

$\therefore \quad a\cdot b = 10 = 2b$

67. ***(b)*** $\because$ Given, $pqr = 5,\ AA^T = 4I$

and $$A = \begin{bmatrix} 3p & q & r \\ q & 3r & p \\ r & p & 3q \end{bmatrix} = A^T$$

$\therefore \quad AA^T = 4I$

$\Rightarrow \quad AA = 4I$

$\Rightarrow \quad A^2 = 4I$

$\Rightarrow \quad |A|^2 = |4I| = 4^3\,|I|$

$\Rightarrow \quad |A| = \pm 8$

But $|A|$ is positive real number.

$\Rightarrow \quad |A| = 8$

Now, $|A| = 8$

$$\Rightarrow \quad \begin{vmatrix} 3p & q & r \\ q & 3r & p \\ r & p & 3q \end{vmatrix} = 8$$

$\Rightarrow \quad 29pqr - 3(p^3 + q^3 + r^3) = 8$

$\Rightarrow \quad 3(p^3 + q^3 + r^3) = 29\times 5 - 8 = 137$

68. ***(a)*** $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{(3^x - 1)^2}{\sin\left(\frac{x}{c}\right)\log\left(1+\frac{x}{3}\right)}$

$$= \lim_{x\to 0}\left(\frac{3^x - 1}{x}\right)^2 \times \frac{x^2}{x\sin\frac{x}{c}} \times \frac{1}{\frac{\log\left(1+\frac{x}{3}\right)}{\frac{x}{3}}\times\frac{x}{3}}$$

$$= \lim_{x\to 0}\left(\frac{3^x - 1}{x}\right)^2 \times \lim_{x\to 0}\frac{c}{\left(\sin\frac{x}{c}\right)} \times \lim_{x\to 0}\frac{3}{\log\left(1+\frac{x}{3}\right)}$$

$$= (\log_e 3)^2 \times c \times 3 = 3c(\log_e 3)^2$$

Also, $f(0) = 10(\log_e 3)^2$

$\because$ $f(x)$ is continuous at $x = 0$.

$\therefore \quad 3c(\log_e 3)^2 = 10(\log_e 3)^2$

$\Rightarrow \quad c = \frac{10}{3}$

69. ***(a)*** Given equation is $(2x - y)^2 = 8x + 6y - 5$

Since, the lines $2x - y = 0$ and $8x + 6y - 5 = 0$ are not at right angles, so adjusting the equations further, we rewrite it as

$4x^2 + y^2 + 1 - 4xy - 4x + 2y = 4x + 8y - 4$

$\Rightarrow \quad (2x - y - 1)^2 = 4(x + 2y - 1)$

$$\Rightarrow \quad \left(\frac{2x - y - 1}{\sqrt{2^2 + 1^2}}\right)^2 = \frac{4}{\sqrt{5}}\left(\frac{x + 2y - 1}{\sqrt{1^2 + 2^2}}\right),$$

which is of the form $Y^2 = 4aX$, since the straight lines $2x - y - 1 = 0$ and $x + 2y - 1 = 0$ are perpendicular to each other.

Focus is obtained from $X = a$ and $Y = 0$

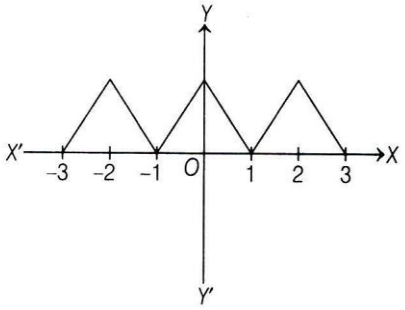
$\Rightarrow \frac{x + 2y - 1}{\sqrt{5}} = \frac{1}{\sqrt{5}}$ and $\frac{2x - y - 1}{\sqrt{5}} = 0$

$\Rightarrow x + 2y - 2 = 0$ and $2x - y - 1 = 0$

Solving them, the focus is $\left(\frac{4}{5}, \frac{3}{5}\right)$.

70. ***(b)*** $f(x) = \begin{cases} x - [x], & \text{if } [x] \text{ is odd} \\ 1 + [x] - x, & \text{if } [x] \text{ is even} \end{cases}$

$$= \begin{cases} \{x\}, & 2n - 1 \le x < 2n \\ 1 - \{x\}, & 2n \le x < 2n + 1 \end{cases}$$



$f(x)$ is periodic.

$$I = \int_{-10}^{10} f(x)\cos\pi x\,dx$$

$$= 2\int_0^{10} f(x)\cos\pi x\,dx$$

$$= 2\times 5\int_0^2 f(x)\cos\pi x\,dx$$

$$= 10\left[\int_0^1 (1 - x)\cos\pi x\,dx + \int_1^2 (x - 1)\cos\pi x\,dx\right]$$

$$= 10[I_1 + I_2]$$

$$I_1 = \int_0^1 (1-x)\cos \pi x \, dx$$

$$= -\int_0^1 x\cos \pi x \, dx$$

$$I_2 = \int_1^2 (x-1)\cos \pi x \, dx$$

On putting $x - 1 = t$

$$= -\int_0^1 t\cos \pi t \, dt$$

$$= -\int_0^1 x\cos \pi x \, dx$$

$$\therefore \quad I = 10\left(-2\int_0^1 x\cos \pi x \, dx\right)$$

$$= -20\left[\frac{x\sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right]_0^1$$

$$= -20\left(-\frac{1}{\pi^2} - \frac{1}{\pi^2}\right)$$

$$= \frac{40}{\pi^2}$$

$$\because \int_{-10}^{10} f(x)\cos \pi x \, dx = \frac{20}{\pi^2} k = \frac{40}{\pi^2}$$

$$= \frac{20}{\pi^2} \times 2$$

$$\Rightarrow \quad k = 2$$

Study Tactics

First, using property of greatest integer function, draw the graph. Then, by properties of definite integration get the required result.

71. *(b)* $f(x) = \sqrt{|x| - x} + \frac{1}{\sqrt{|x| - x}}$

If $x \geq 0$, then $|x| - x = 0$

If $x < 0$, then $|x| - x = -2x$

$\therefore f(x) = \sqrt{-2x} + \frac{1}{\sqrt{-2x}}$,

where $-2x > 0$

$\Rightarrow \quad A = (-\infty, 0)$

We know that AM $\geq$ GM

$$\Rightarrow \frac{\sqrt{-2x} + \frac{1}{\sqrt{-2x}}}{2} \geq \left(\sqrt{-2x} \times \frac{1}{\sqrt{-2x}}\right)^{1/2}$$

$$\Rightarrow \quad f(x) \geq 2$$

$$\Rightarrow \quad B = [2, \infty)$$

Study Tactics

Use property of modulus function, then apply AM $\geq$ GM.

72. *(d)* Given that the length of transverse axis is $2\sin\theta = 2a$

$\Rightarrow a = \sin\theta$

Also, for the ellipse

$$4x^2 + 6y^2 = 12$$

$$\Rightarrow \quad \frac{x^2}{3} + \frac{y^2}{2} = 1$$

Here, $a^2 = 3$ and $b^2 = 2$

$$\because \quad e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{2}{3}} = \frac{1}{\sqrt{3}}$$

Hence, the focus of ellipse is $(\pm 1, 0)$.

As the hyperbola is confocal with the ellipse, the focus of hyperbola is $(1, 0)$.

Now, $a'e' = 1$

$\Rightarrow \sin\theta \cdot e' = 1$

$\Rightarrow \quad (e')^2 = \text{cosec}^2\theta$

$\therefore \quad b'^2 = a'^2(e'^2 - 1) = \sin^2\theta(\text{cosec}^2\theta - 1)$

$\Rightarrow \quad b'^2 = 1 - \sin^2\theta = \cos^2\theta$

Therefore, the equation of hyperbola is

$$\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$$

73. *(c)* Total number of balls $= x + y$

$$\text{Required probability} = \frac{{}^xC_1 \times {}^yC_1}{{}^{x+y}C_2} = \frac{1}{2}$$

$$\Rightarrow \frac{2xy}{(x+y)(x+y-1)} = \frac{1}{2}$$

$$4xy = (x+y)(x+y-1)$$

$$4xy = x^2 + xy - x + xy + y^2 - y$$

$$y^2 - (2x+1)y + x^2 - x = 0$$

For real solution,

$$\Delta = (2x+1)^2 - 4(1)(x^2 - x)$$

$$= 4x^2 + 1 + 4x - 4x^2 + 4x = 8x + 1$$

must be a perfect square

$$\Rightarrow \quad x = 1, 3, 6, 10$$

$\therefore$ Statement I and Statement II both are false.

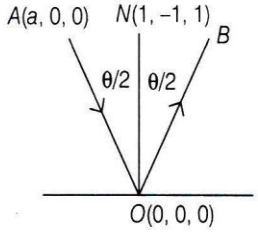
74. *(d)* Let the source of light be situated at $A(a, 0, 0)$, where $a \neq 0$. Let OA be the incident ray, OB be the reflected ray and ON be the normal to the mirror at O.

$$\therefore \quad \angle AON = \angle NOB = \frac{\theta}{2}$$

Direction ratios of OA are proportional to a, 0, 0 and s its direction cosines are 1, 0, 0.

Direction cosines of ON are $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$\therefore \cos\frac{\theta}{2} = \frac{1}{\sqrt{3}}$



Let l, m, n be the direction cosines of the reflected ray OB.

Thus, $\frac{l+1}{2\cos\frac{\theta}{2}} = \frac{1}{\sqrt{3}}, \frac{m+0}{2\cos\frac{\theta}{2}} = -\frac{1}{\sqrt{3}}$

and $\frac{n+0}{2\cos\frac{\theta}{2}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad l = \frac{2}{3} - 1, m = -\frac{2}{3}, n = \frac{2}{3}$

$\Rightarrow \quad l = -\frac{1}{3}, m = -\frac{2}{3}, n = \frac{2}{3}$

Hence, direction cosines of the reflected ray are $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$.

75. *(a)* We have, $\mathbf{b} - 3\mathbf{c} = \lambda\mathbf{a}$

On taking scalar product with $\mathbf{c}$, we get

$(\mathbf{b} - 3\mathbf{c})\cdot\mathbf{c} = \lambda(\mathbf{a}\cdot\mathbf{c})$

$\Rightarrow \quad \mathbf{b}\cdot\mathbf{c} - 3(\mathbf{c}\cdot\mathbf{c}) = \lambda(\mathbf{a}\cdot\mathbf{a})$

$\Rightarrow \quad \mathbf{b}\cdot\mathbf{c} - 3 = \lambda$

$\Rightarrow \quad \mathbf{b}\cdot\mathbf{c} = 3 + \lambda \quad \ldots(i)$

$[\because |\mathbf{a}| = |\mathbf{c}| = 1$ and $\mathbf{a}, \mathbf{c}$ are collinear vectors$]$

Again, $\mathbf{b} - 3\mathbf{c} = \lambda\mathbf{a}$

$\Rightarrow \quad |\mathbf{b} - 3\mathbf{c}| = |\lambda\mathbf{a}| \Rightarrow |\mathbf{b} - 3\mathbf{c}|^2 = \lambda^2|\mathbf{a}|^2$

$\Rightarrow |\mathbf{b}|^2 + 9|\mathbf{c}|^2 - 6(\mathbf{b}\cdot\mathbf{c}) = \lambda^2|\mathbf{a}|^2$

$\Rightarrow \quad 36 + 9 - 6(3 + \lambda) = \lambda^2$ [from Eq. (i)]

$\Rightarrow \quad 27 - 6\lambda = \lambda^2$

$\Rightarrow \quad \lambda^2 + 6\lambda - 27 = 0$

$\Rightarrow \quad \lambda = -9, 3$

Time Saver Tip

We have, $\bar{b} - 3\bar{c} = \lambda\bar{a}$

$\because$ Angle between any two collinear vectors is 0° or 180°.

So, $\bar{a}\cdot\bar{c} = \pm 1$

$\therefore \quad \bar{b} = \lambda\bar{a} + 3\bar{c} \Rightarrow |\bar{b}|^2 = |\lambda\bar{a} + 3\bar{c}|^2$

$\Rightarrow 36 = 9 + \lambda^2 + 6\lambda \Rightarrow \lambda^2 + 6\lambda - 27 = 0 \Rightarrow \lambda = -9, 3$

76. *(c)* $\because \frac{X-2}{1} = \frac{Y-9}{2} = \frac{Z-13}{3} = \mu$

$\Rightarrow \quad X = \mu + 2, Y = 2\mu + 9, Z = 3\mu + 13$

$\because \quad \frac{X-\lambda}{-1} = \frac{Y-7}{2} = \frac{Z+2}{-3} = \delta$

$\Rightarrow \quad X = -\delta + \lambda,$

$Y = 2\delta + 7$

and $Z = -3\delta - 2$

Now, $\mu + 2 = -\delta + \lambda$

$\mu + \delta = -2 + \lambda \quad \ldots(i)$

$\Rightarrow \quad 2\mu + 9 = 2\delta + 7$

$\Rightarrow \quad 2\mu - 2\delta = -2$

$\Rightarrow \quad \mu - \delta = -1 \quad \ldots(ii)$

$\Rightarrow \quad 3\mu + 13 = -3\delta - 2$

$\Rightarrow \quad 3\mu + 3\delta = -15$

$\Rightarrow \quad \mu + \delta = -5 \quad \ldots(iii)$

On solving Eqs. (i), (ii) and (iii), we get

$\lambda = -3,$

$\mu = -3,$

$\delta = -2$

77. *(c)* Equation of line passing through $(5, 1, a)$ and $(3, b, 1)$ is

$\frac{x-5}{5-3} = \frac{y-1}{1-b} = \frac{z-a}{a-1} = \lambda$

i.e. $x = 0$

$x = 2\lambda + 5 = 0$

$\Rightarrow \quad \lambda = -\frac{5}{2}$

Since, $y = \lambda(1-b) + 1 = \frac{17}{2}$

$= -\frac{5}{2}(1-b) + 1 = \frac{17}{2}$

$\Rightarrow \quad b = 4$

Also, $z = \lambda(a-1) + a = \frac{-13}{2}$

$= \frac{-5}{2}(a-1) + a = \frac{-13}{2}$

$\Rightarrow \quad a = 6$

78. *(c)* $(1 - x - x^2 + x^3)^6$

$\Rightarrow \quad [(1-x) - x^2(1-x)]^6 \Rightarrow [(1-x)^6(1-x^2)^6]$

$\Rightarrow (1 - {}^6C_1x + {}^6C_2x^2 - {}^6C_3x^3 + {}^6C_4x^4 - {}^6C_5x^5 + {}^6C_6x^6)$

$\cdot (1 - {}^6C_1x^2 + {}^6C_2x^4 - {}^6C_3x^6 + \ldots)$

$\Rightarrow$ Coefficient of x^7 is

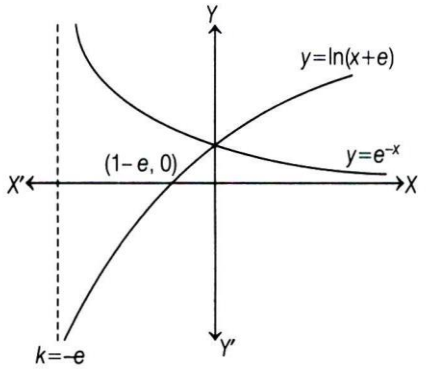
${}^6C_1{}^6C_3 - {}^6C_3{}^6C_2 + {}^6C_5{}^6C_1$

$\Rightarrow 6\cdot\frac{6\cdot5\cdot4}{3\cdot2} - \frac{6\cdot5\cdot4}{3\cdot2\cdot1}\cdot\frac{6\cdot5}{2} + 6\cdot6 = 120 - 300 + 36$

$= -144$

79. *(b)* $\therefore \Delta = \sin\theta\cos\phi(0+\sin^2\theta\cos\phi)$
$-\sin\theta\sin\phi(0-\sin^2\theta\sin\phi)$
$+\cos\theta(\cos^2\phi\sin\theta\cos\theta+\cos\theta\sin\theta\sin^2\phi)$
$=\sin^3\theta(\cos^2\phi+\sin^2\phi)+\sin\theta\cos^2\theta(\sin^2\phi+\cos^2\phi)$
$=\sin^3\theta+\sin\theta\cos^2\theta$
$=\sin\theta(\sin^2\theta+\cos^2\theta)$
$=\sin\theta$ $[\because \sin^2\theta+\cos^2\theta=1]$
which is independent of ϕ.

80. *(b)* The given curves are
$y=\ln(x+e)$
and $x=\ln(1/y)$
$\Rightarrow \frac{1}{y}=e^x$
$\Rightarrow y=e^{-x}$



$\therefore$ Required area $=\int_{1-e}^{0}\ln_e(x+e)dx+\int_0^\infty e^{-x}dx$
$=\int_1^e \ln t\,dt+\int_0^\infty e^{-x}\,dx$ $[\because x+e=t]$
$=[t\ln t-t]_1^e-(e^{-x})_0^\infty$
$=1+1$
$=2$ sq units

81. *(240)* To form any number, we have 10 digits i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 out of which middle digit can be x, (where $2\le x\le 9$), then the hundredth can be chosen in $(x-1)$ ways (as this place can not be zero) and units place can be chosen in x ways.

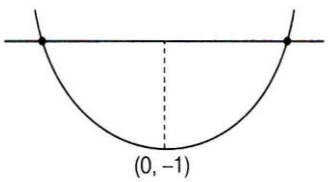
Thus, number of required numbers with middle digit is $(x-1)x$.

As $2\le x\le 9$, the number of required numbers is
$$\sum_{x=2}^{9}(x-1)x=\sum_{x=2}^{9}(x^2-x)$$
$=240$

82. *(–1)* $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}3-\tan^{-1}x$
$\Rightarrow \tan^{-1}\left(\frac{x+1+x-1}{1-(x+1)(x-1)}\right)=\tan^{-1}\left(\frac{3-x}{1+3x}\right)$,
when $-\sqrt{2}<x<\sqrt{2}$ and $3x<1$
$\Rightarrow \tan^{-1}\left(\frac{2x}{2-x^2}\right)=\tan^{-1}\left(\frac{3-x}{1+3x}\right)$,
when $-\sqrt{2}<x<\sqrt{2}$ and $x<\frac{1}{3}$
$\Rightarrow \frac{2x}{2-x^2}=\frac{3-x}{1+3x}$, when $-\sqrt{2}<x<\frac{1}{3}$
$\Rightarrow 2x(1+3x)=(2-x^2)(3-x)$, when $-\sqrt{2}<x<\frac{1}{3}$
$\Rightarrow 2x+6x^2=6-2x-3x^2+x^3$
$\Rightarrow x^3-9x^2+4x+6=0$
$\Rightarrow (x+1)(x^2-10x+6)=0$, when $-\sqrt{2}<x<\frac{1}{3}$
$\Rightarrow x=-1$ and neglecting $x^2-10x+6=0$ as its roots does not $\in\left(-\sqrt{2},\frac{1}{3}\right)$
Hence, $x=-1$

83. *(5)* Given series, $10\left(\frac{1}{2\cdot4}+\frac{1\cdot3}{2\cdot4\cdot6}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6\cdot8}+\ldots\infty\right)$
nth term of sequence,
$$T_n=\frac{1\cdot3\cdot5\ldots(2n-1)}{2\cdot4\cdot6\ldots(2n)(2n+2)}$$
$$=\frac{1\cdot3\cdot5\ldots(2n-1)}{2\cdot4\cdot6\ldots(2n)(2n+2)}[(2n+2)-(2n+1)]$$
$$=\frac{1\cdot3\cdot5\ldots(2n-1)}{2\cdot4\cdot6\ldots2n}-\frac{1\cdot3\cdot5\ldots(2n-1)(2n+1)}{2\cdot4\cdot6\ldots2n(2n+2)}$$
$$T_n=f(n)-f(n+1)$$
Sum of infinite term,
$S_n=\sum T_n$
$=f(1)-f(2)+f(2)-f(3)+\ldots\infty$
$=f(1)=\frac{1}{2}$
$10S_n=5$

84. *(2)* Let $f(x)=x^2-x\sin x-\cos x$



$\therefore f'(x)=2x-x\cos x-\sin x+\sin x$
$=2x-x\cos x$

$$f'(x)=0$$
$$\Rightarrow x(2-\cos x)=0$$
or $x=0$ $[\because 2-\cos x>0$ for all real $x]$

Also, $x=0$ is point of minima $f(0)=-1<0$

and $\lim_{x\to\infty} f(x)\to\infty$,

$\lim_{x\to-\infty} f(x)\to\infty$

Hence, it meets X-axis at two points and two solutions.

85. *(8)* Median $=\dfrac{25\text{th term}+26\text{th term}}{2}$

$$=\frac{25k+26k}{2}=25.5k$$

Mean deviation about median = 100

$$\Rightarrow \frac{\sum|x_i-25.5k|}{50}=100$$
$$\Rightarrow 24.5k+23.5k+\ldots+1.5k+0.5k+0.5k+1.5k+\ldots+23.5k+24.5k=5000$$
$$\Rightarrow k+3k+5k+\ldots+49k=5000$$
$$\Rightarrow k(1+3+5+\ldots+49)=5000$$
$$\Rightarrow k\cdot\frac{25}{2}(1+49)=5000$$
$$\Rightarrow k=8$$

86. *(3)* Let $r_1=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ and $r_2=A\hat{i}+B\hat{j}+C\hat{k}$

$$\Rightarrow r_1\cdot r_2=\alpha A+\beta B+\gamma C$$
$$\Rightarrow |r_1||r_2|=\sqrt{(\alpha^2+\beta^2+\gamma^2)(A^2+B^2+C^2)}$$
$$\therefore\quad r_1\cdot r_2=-|r_1||r_2|$$

$\Rightarrow r_1$ and r_2 are anti-parallel.

$$\Rightarrow\frac{\alpha}{A}=\frac{\beta}{B}=\frac{\gamma}{C}=k,\text{ where } k \text{ is any constant.}$$
$$\Rightarrow\frac{\alpha B}{\beta A}+\frac{\beta C}{\gamma B}+\frac{\gamma A}{\alpha C}=3.$$

87. *(4000)* $\because 2-\sqrt{3}=\dfrac{1}{2+\sqrt{3}}$

The given equation can be written as

$$\Rightarrow (2+\sqrt{3})^{x^2-2x+1}+\frac{1}{(2+\sqrt{3})^{x^2-2x+1}}=\frac{2}{2-\sqrt{3}}$$
$$\Rightarrow (2+\sqrt{3})^{x^2-2x+1}+\frac{1}{(2+\sqrt{3})^{x^2-2x+1}\cdot(2+\sqrt{3})^{-2}}=2(2+\sqrt{3}) \quad \ldots(i)$$

On putting $(2+\sqrt{3})^{x^2-2x+1}=t$,

From Eq. (i), we get

$$t+\frac{(2+\sqrt{3})^2}{t}=2(2+\sqrt{3})$$
$$\Rightarrow t^2-2(2+\sqrt{3})t+(2+\sqrt{3})^2=0$$
$$\Rightarrow [t-(2+\sqrt{3})]^2=0$$
$$\Rightarrow t=2+\sqrt{3}$$
$$\Rightarrow (2+\sqrt{3})^{x^2-2x+1}=2+\sqrt{3}$$
$$\Rightarrow x^2-2x+1=1$$
$$x(x-2)=0$$
$$x=0,2$$

i.e. $\alpha=0$ and $\beta=2$

So, $2000\alpha+2000\beta=4000$

88. *(13)* Let ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

and circle $x^2+y^2=a^2e^2$

Radius of circle = ae

Point of intersection of circle and ellipse is

$$\left[\frac{a}{e}\sqrt{2e^2-1},\frac{a}{e}(1-e^2)\right].$$

Now, area of ΔPF_1F_2

$$=\frac{1}{2}\begin{vmatrix}\frac{a}{e}\sqrt{2e^2-1} & \frac{a}{e}(1-e^2) & 1\\ ae & 0 & 1\\ -ae & 0 & 1\end{vmatrix}$$
$$=\frac{1}{2}\left|\frac{a}{e}(1-e^2)(2ae)\right|=30$$
$$\Rightarrow a^2(1-e^2)=30 \qquad [\text{given}]$$
$$\Rightarrow a^2e^2=a^2-30=\left(\frac{17}{2}\right)^2-30=\frac{169}{4}$$
$$\Rightarrow 2ae=13$$

89. *(36)* Given, $f:(0,\infty)\to(1,\infty)$

$$g(x)=\int_0^x f(t)dt$$
$$\Rightarrow g'(x)=f(x) \quad \ldots(i)$$

Now, $g(x^2)=x^2(1+x)$

$$g(x)=x(1+\sqrt{x})$$
$$g'(x)=1+\frac{3}{2}\sqrt{x} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$f(x) = 1 + \frac{3}{2}\sqrt{x} = y \qquad \text{[say]}$$

$$\sqrt{x} = \frac{2}{3}(y-1)$$

$$x = \left(\frac{2}{3}(y-1)\right)^2$$

$$f^{-1}(x) = 4/9(x-1)^2 = h(x) \qquad \text{[given]}$$

$$\therefore \quad h(10) = 4/9(10-1)^2 = \frac{4}{9} \times 9 \times 9$$

$$= 36$$

Study Tactics

Since, $f(x)$ is a derivative of $g(x)$, then we get inverse of $f(x)$ by replacing x in terms of y.

90. *(2021)* Given the system has a non-trivial solution.

$$\text{So,} \quad \begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$\Rightarrow \lambda(-\cos^2\alpha - \sin^2\alpha) - (-\sin\alpha\cos\alpha - \sin\alpha\cos\alpha) - (\sin^2\alpha - \cos^2\alpha) = 0$$

$$\Rightarrow \quad -\lambda + \sin 2\alpha + \cos 2\alpha = 0$$

$$\lambda = \sin 2\alpha + \cos 2\alpha$$

$$\lambda = \sqrt{2}\cos(2\alpha - \pi/4)$$

$$\because -1 \le \cos(2\alpha - \pi/4) \le 1, \forall\, \alpha \in R$$

So, $-\sqrt{2} \le \lambda \le \sqrt{2}$ i.e. $\lambda \in [-\sqrt{2}, \sqrt{2}]$

Hence, $2021\left(\frac{u^2}{v^2}\right) = 2021 \times \frac{2}{2} = 2021$