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JEE-T1

SEAT NUMBER

# IIT INSPIRE ACADEMY OF SCIENCE

A Place for Excellence & Perfection

XI & XII Science (CBSE/state)

IIT- JEE (Mains + Advance)

NEET, MH-CET , NDA

Mo. No. 9595445177/9021445177

Branches : Chhatrapati Sq., Mangalmurti Sq.

Day - 1

# JEE TEST

Time : 3 Hrs. (4 Pages) Max. Marks : 300

**Instructions**

1. There are **three** parts in the questions paper consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
2. **Section A** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and – **1 mark** for wrong answer.
3. **Section B** This section contains 10 questions. In Section B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and – **1 mark** for wrong answer. For Section B, the answer should be rounded off to the nearest integer.

## PHYSICS

**Section A : Objective Type Questions**

**1.** A stone is thrown vertically up from the top of a tower which reaches the ground in time $t_1$ and another stone is thrown vertically down with the same speed which reaches ground in time $t_2$. A third stone is released from rest from the same location reaches the ground in time $t_3$, then

(a) $t_3 = \frac{t_1 - t_2}{2}$ (b) $t_3 = \sqrt{t_1 t_2}$

(c) $\frac{1}{t_3} = \frac{1}{t_2 - t_1}$ (d) $t_3^2 = t_1^2 - t_2^2$

**2.** Two massive bodies of masses $m_1$ and $m_2$ are at rest and separated by infinite distance. Now, they move towards each other due to their mutual attraction. The velocity of approach, when distance between $m_1$ and $m_2$ was $r$ , is

(a) $\sqrt{\frac{2G}{r}(m_1 + m_2)}$ (b) $\sqrt{\frac{2G}{r}(m_1 - m_2)}$

(c) $\sqrt{\frac{2Gm_1 m_2}{r(m_1 + m_2)}}$ (d) $\sqrt{\frac{2G}{r} m_1 m_2}$

**3. Assertion** The net dipole moment of a given volume containing large number of polar molecules is zero.

**Reason** The dielectric constant of polar molecules is independent of temperature.

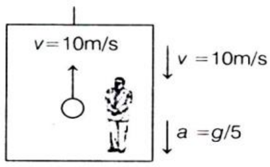
(a) Both Assertion and Reason are correct and Reason is correct explanation of Assertion.

(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

(c) Assertion is correct but Reason is incorrect.

(d) Reason is correct but Assertion is incorrect.

**4.** A boy is in a lift which is moving downwards with an acceleration $g/5$. At an instant when the velocity of the lift is 10 m/s, then boy throws a ball upwards with a velocity 10 m/s. The time after which, ball will return into the hand of the boy is




(a) 3 s (b) 4 s

(c) 5 s (d) Not possible

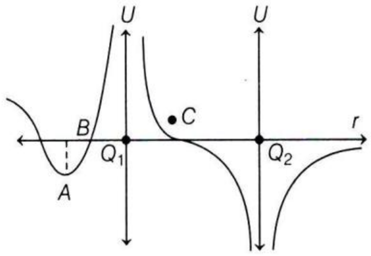
**5. Statement I** If the centre of mass of a system is at rest, the KE of the system must be zero.

**Statement II** The change in velocity depends on the external force.

(a) Statement I is true but Statement II is false.
(b) Both Statement I and Statement II are true.
(c) Statement I is false but Statement II is true.
(d) Both Statement I and Statement II are false.

**6.** Two resistances are expressed as $R_1 = (3 \pm 0.2)\,\Omega$ and $R_2 = (6 \pm 0.3)\,\Omega$. The equivalent resistance of their series and parallel combination are, respectively.
(a) $(9 \pm 0.1)\Omega, (2 \pm 0.12)\Omega$
(b) $(9 \pm 0.5)\Omega, (2 \pm 0.12)\Omega$
(c) $(9 \pm 0.8)\Omega, (2 \pm 0.1)\Omega$
(d) $(9 \pm 0.6)\Omega, (2 \pm 0.02)\Omega$

**7.** The curve represents the distribution of potential energy along the straight line joining two charges $Q_1$ and $Q_2$ separated by distance $r$, then which of the following statements are correct ?




I. Magnitude of charge $Q_1$ is less than that of charge $Q_2$
II. $Q_1$ is positive charge and $Q_2$ is negative charge
III. $A$ is a point of stable equilibrium
IV. $B$ and $C$ are points of unstable equilibrium.
(a) I and II
(b) I, II, III and IV
(c) I, II and III
(d) I and III

**8.** An electron with kinetic energy 2.0 eV is incident on a H-atom which is in excited state $n = 2$, then the collision
(a) must be elastic
(b) must be inelastic
(c) can be either elastic or inelastic
(d) can be either elastic or inelastic or super-elastic

**9.** The length of an open organ pipe is 60 cm and that of a closed organ pipe is 40 cm, then the ratio of the third overtone of closed organ pipe and the second overtone of open organ pipe is
(a) 4 : 3
(b) 3 : 4
(c) 7 : 4
(d) 4 : 7

**10.** A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electric power to electromagnetic waves and consume 100W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 5 m from the lamp will be
(a) $2.68\,\text{Vm}^{-1}$
(b) $3.15\,\text{Vm}^{-1}$
(c) $2.01\,\text{Vm}^{-1}$
(d) 0

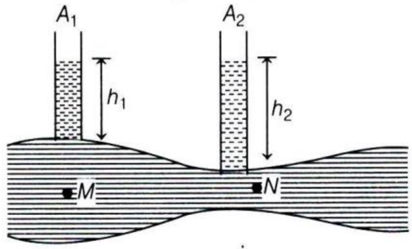
**11.** The specific heat at constant pressure is $C_p$ and specific heat at constant volume is $C_V$, then the ratio of specific heats $\left(\frac{C_p}{C_V}\right)$ in terms of degree of freedom ($f$) is given by
(a) $\left(1+\frac{3}{f}\right)$ (b) $\left(1+\frac{2}{f}\right)$ (c) $\left(1+\frac{f}{3}\right)$ (d) $\left(1+\frac{1}{f}\right)$

**12.** A pure germanium plate of area $3.5 \times 10^{-4}\ \text{m}^2$ and of thickness $1.5 \times 10^{-3}$ m is connected across a battery of potential 5 V. Find the amount of heat generated in the plate in 120 s. Given that, the concentration of carriers in germanium at room temperature is $1.6 \times 10^6$ per cubic metre. The mobilities of electrons and holes are $0.4\ \text{m}^2\,\text{V}^{-1}\ \text{s}^{-1}$ and $0.2\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$, respectively.
(a) $10.74 \times 10^{-11}$
(b) $9 \times 10^{-6}$
(c) $10.48 \times 10^{-9}$
(d) $6.26 \times 10^{-10}$

**13.** In an experiment of coefficient of viscosity, two drops of water of same size are falling through air with terminal velocity of 10 cm/s. If the two drops combine to form a single drop, then the terminal velocity is
(a) 15.9 cm/s
(b) 22.8 cm/s
(c) 12.8 cm/s
(d) 18.4 cm/s

**14.** For a projectile thrown into space with a speed $v$, the horizontal range is $\frac{\sqrt{3}v^2}{2g}$. The vertical range is $\frac{v^2}{8g}$. The angle which the projectile makes with the horizontal initially is
(a) 15° (b) 30° (c) 45° (d) 60°

**15.** A liquid is in steady flow inside a tube of non-uniform variation. Two capillaries are joined with the tube at two places, then

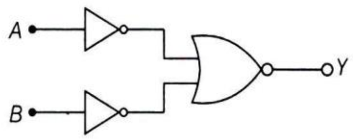



(a) $h_1 = h_2$
(b) $h_1$ must be greater than $h_2$
(c) $h_2$ may be greater than $h_1$
(d) $h_2$ must be greater than $h_1$

**16.** In changing the state of a gas adiabatically from an equilibrium state $A$ to another equilibrium state $B$, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state $A$ to $B$ *via* a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case ?

(a) 15.6 J (b) 11.2 J (c) 14.9 J (d) 16.9 J

**17.** Which of the following logic state is represented by the combination of logic gates ?




(a) NAND gate (b) NOR gate
(c) AND gate (d) OR gate

**18.** Match the Column I with Column II and select the correct option from the codes given below.

| | Column I | | Column II |
|---|---|---|---|
| A. | $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I$ | 1. | Gauss' law |
| B. | $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0\left(I + \varepsilon_0 \dfrac{d\phi_E}{dt}\right)$ | 2. | Faraday's laws of electromagnetic induction |
| C. | $\oint \mathbf{E}\cdot d\mathbf{l} = -\dfrac{d}{dt}\oint_s \mathbf{B}\cdot d\mathbf{S}$ | 3. | Ampere-Maxwell's law |
| D. | $\oint \mathbf{E}\cdot d\mathbf{S} = \dfrac{1}{\varepsilon_0}\oint_s \rho dS$ | 4. | Ampere's circuital law |

**Codes**

| | A | B | C | D | | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| (a) | 1 | 2 | 3 | 4 | (b) | 4 | 3 | 2 | 1 |
| (c) | 4 | 2 | 1 | 3 | (d) | 3 | 1 | 2 | 4 |

**19.** Two waves are travelling in the opposite direction, on superposition, produces standing wave.

The transverse displacement is given by

$$y(x,t) = 2\sin\left(\frac{8\pi}{9}x\right)\cos(400\pi t)$$

Speed of the travelling wave moving in the positive $x$-direction is ($x$ and $t$ are in metre and second, respectively)

(a) 160 m/s (b) 450 m/s
(c) 180 m/s (d) 220 m/s

**20.** A load of 4.0 kg is suspended from a ceiling through a steel wire of length 2.0 m and radius 2.0 mm. It is found that, the length of the wire increases by 0.031 mm as equilibrium is achieved.

Taking, $g = 3.1\pi$ ms$^{-2}$, the Young's modulus of steel is

(a) $2.0\times10^{8}$ Nm$^{-2}$
(b) $2.0\times10^{9}$ Nm$^{-2}$
(c) $2.0\times10^{11}$ Nm$^{-2}$
(d) $2.0\times10^{13}$ Nm$^{-2}$

## Section B : Numerical Value Type Questions

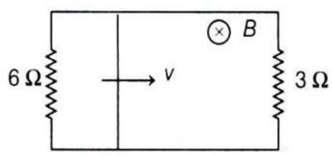
**21.** A mixture of gas consists of 3 moles of $O_2$ and 5 moles of Ar at temperature $T$. Considering only rotational and translation modes. The total internal energy of the system is found to be $xRT$. The value of $x$ is ……… .

**22.** The surface of some metal is radiated by waves of $\lambda_1 = 3.5\times10^{-7}$ m and $\lambda_2 = 6.2\times10^{-7}$ m, respectively. The ratio of the stopping potential in two cases is 2 : 1. The work function of the material is ………eV.

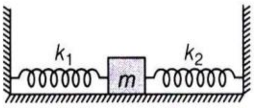
**23.** A transparent cube contains a small air bubble. Its apparent distance is 2 cm, when seen through one face and 5 cm, when seen through other face. If the refractive index of the material of the cube is 1.5, real length of the edge of the cube must be …… cm.

**24.** The resistance of a 10 m long wire is 10 Ω. Its length is increased by 25% by stretching the wire uniformly. The resistance of wire will change to (approximately) ………Ω.

**25.** A rectangular loop with a sliding conductor of length $l = 1.0$ m is situated in a uniform magnetic field $B = 2$T, perpendicular to the plane of loop. Resistance of conductor is $R = 2\Omega$. Two resistances of $6\Omega$ and $3\Omega$ are connected as shown in figure. The external force required to keep the conductor moving with a constant velocity $v = 2$ ms$^{-1}$ is ……… N.



**26.** Two springs with negligible masses and force constant $k_1 = 200$ N/m and $k_2 = 160$ N/m are attached to the block of mass $m = 10$ kg as shown in figure. Initially the block is at rest, at the equilibrium position in which both springs are neither stretched nor compressed. At time $t = 0$, sharp impulse of 50 N-s is given to the block with a hammer along the spring. Then, the amplitude of spring will be ...... m.

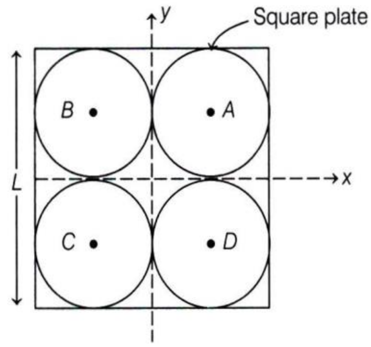



**27.** 10 g of ice at $-20°C$ is dropped into a calorimeter containing 10 g of water of 10°C and the specific heat of water is twice that of ice. When equilibrium is reached, the difference of masses of ice and water in calorimeter is $x$ gram, then the value of $x$ is ......... .

**28.** A $10 \mu F$ capacitor is fully charged by a 220 V supply. Then, it is disconnected from the supply and connected in series to another uncharged $5 \mu F$ capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$ J, then the value of $X$ to the nearest integer is ...... .

**29.** A proton with kinetic energy of 2 MeV moves from south to north direction. It gets an acceleration of $10^{12}$ m/s$^2$ by an applied magnetic field (west to east). The value of magnetic field is ......mT.

**30.** A thin square plate has mass $m$ and side $L$. Four holes, each of radius $R$, are cut out as shown in following figure.The ratio of moment of inertia of the remaining portion to the moment of inertia of the complete place is ......



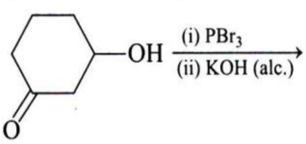

# CHEMISTRY

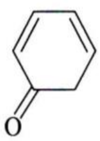
## Section A : Objective Type Questions

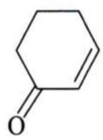
**31.** Bromine and fluorine react to form more than one compound. When an atom of bromine in its second excited state, reacts with fluorine to form a binary compound. The formula and shape of the compound is

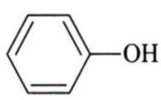
(a) BrF and linear
(b) $BrF_3$ and pyramidal
(c) $BrF_3$ and T-shaped
(d) $BrF_5$ and square pyramidal

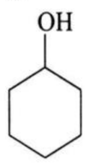
**32.** The major product of the given reaction is




(a) 


(b) 


(c) 

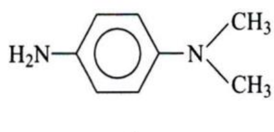

(d) 


**33.** Arrange the following in correct order of increasing dipole moment.

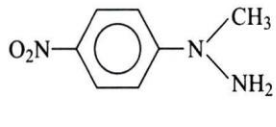
I. 1, 2, 3-trichlorobenzene
II. 1, 4-dichlorobenzene
III. 1, 2, 4-trichlorobenzene
IV. 1, 3, 5-trichlorobenzene

(a) II < IV < III < I
(b) II < IV < I < III
(c) II < III < IV < I
(d) I < III< II < IV

**34.** An organic compound having molecular formula $C_6H_4NO_2F$ is a cyclic compound which when treated with DMF and $(CH_3)_2NH$ produces $A$ and on further treatment with $H_2$/Pt produces $B$. The possible molecular structure of $B$ is

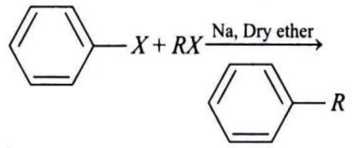
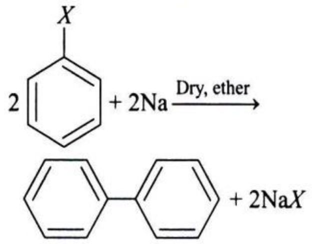
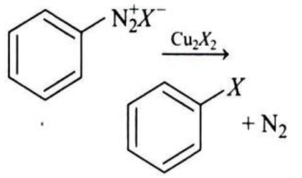
(a) 


(b) 


(c) 


(d) 

**35.** Match List-I with List-II.

| List-I | List-II |
|---|---|
| A.  | I. Fittig reaction |
| B.  | II. Wurtz-Fittig reaction |
| C.  | III. Hunsdiecker reaction |
| D. $C_6H_5COOAg \xrightarrow{Br_2} C_6H_5Br + CO_2 + Ag$ | IV. Sandmeyer reaction |

Choose the correct answer from the options given below.

(a) A-III, B-II, C-I, D-IV (b) A-IV, B-III, C-II, D-I
(c) A-II, B-I, C-IV, D-III (d) A-I, B-III, C-II, D-IV

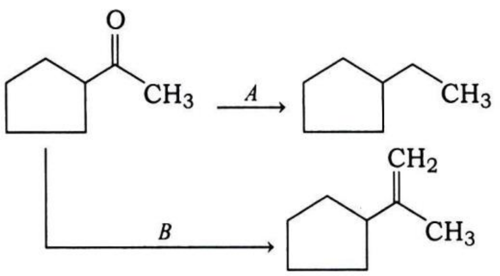
**36.** Match List -I with List-II.

| List -I | List-II |
|---|---|
| A. Myosin | I. Ascorbic acid |
| B. Vitamin | II. Fibrous protein |
| C. Haemoglobin | III. Steroids |
| D. Androgens | IV. Globular protein |

Choose the correct answer from the options given below.

(a) A-II, B-III, C-IV, D-I (b) A-II, B-I, C-IV, D-III
(c) A-I, B-IV, C-II, D-III (d) A-I, B-IV, C-III, D-II

**37.** On the basis of following reaction, choose the correct option for $A$ and $B$, respectively.



(a) $PPh_3 = CH_2$, Zn - Hg/HCl
(b) $PPh_3 = CH_2$, $LiAlH_4$
(c) $NaBH_4$, Zn - Hg / HCl
(d) Zn - Hg/HCl, $PPh_3 = CH_2$

**38.** A solution containing sulphate and sulphite ion can be easily distinguished by

(a) $FeSO_4$ (b) $Na_2[Fe(CN)_5NO]$
(c) $BaCl_2$ (d) $Na_3[Co(NO_2)_6]$

**39.** Formaldehyde + $NH_3 \longrightarrow X$
Here, '$X$' is

(a) nitro compound used as sedative.
(b) acid used as an antipyretic.
(c) tertiary amine used as an antiseptic.
(d) heterocyclic compound used as an antiseptic.

**40.** Phosphorus on reaction with NaOH produces a colourless gas with rotten fish smell. The gas gives a vortex ring. The colourless gas is

(a) $PH_3$ (b) $P_2O_3$
(c) $P_2O_5$ (d) $P_2S_5$

**41.** Elements $A$ and $D$ react to form more than one type of compounds. The solubility product values of three such compounds are as follows

| Compound | $K_{sp}$ |
|---|---|
| $AD$ | $5 \times 10^{-8}$ |
| $AD_2$ | $3 \times 10^{-14}$ |
| $AD_3$ | $2.8 \times 10^{-15}$ |

I. Solubility of compound $AD$ is more than that of compound $AD_3$.
II. Solubility of compound $AD_2$ is more than that of compound $AD_3$.
III. Solubility of compound $AD_3$ is more than that of compound $AD$ and less than compound $AD_2$.
IV. Solubility of compound $AD_2$ is less than $AD$ and $AD_3$.

Choose the most appropriate answer from the options given below.

(a) I, II and IV are correct.
(b) II, III and IV are correct.
(c) I, III and IV are correct.
(d) I and IV are correct.

**42.** Which of the following complex/ion has 3 geometrical isomers?

(a) $[Co(NH_3)_4Cl_2]^+$
(b) $[Co(H_2O)_2(NH_3)_2(en)]^{3+}$
(c) $[Pt(NH_3)_2Cl_2]$
(d) $[Cr(H_2O)_2(NH_3)_4]^{3+}$

**43.** A sulphate of a metal $(A)$ on heating gives two gases $(B)$ and $(C)$ and an oxide $(D)$, gas $(B)$ turns $K_2Cr_2O_7$ paper green, while gas $(C)$ forms a trimer in which there is no S—S bond. Compound $(D)$ with conc. HCl forms a Lewis acid $(E)$ which exists as a dimer. Identify compounds $(A)$, $(B)$, $(C)$, $(D)$ and $(E)$ respectively.

(a) $FeSO_4$, $SO_2$, $SO_3$, FeO, $Fe_2(PO_4)_3$
(b) $FeSO_4$, $SO_2$, $SO_3$, $Fe_2O_3$, $FeCl_3$
(c) $Al_2(SO_4)_3$, $SO_2$, $SO_3$, $Al_2O_3$, $FeCl_2$
(d) $FeSO_4$, $SO_3$, $SO_2$, FeO, $FeCl_2$

**44.** Two solutions of glucose have osmotic pressure 1.5 atm and 2.5 atm respectively. 1 L of first solution is mixed with 2 L of second solution at the same temperature. Then, the osmotic pressure of resultant solution will be

(a) 2.16 atm (b) 3.5 atm
(c) 5.12 atm (d) 7.2 atm

**45.** The bond order of $N_2^+$ and $N_2^-$ and $N_2$ is

(a) 2.5, 3 and 3 (b) 3, 2.5 and 3
(c) 2.5, 2.5 and 3 (d) 3, 3 and 2.5

**46.** Resistance of 0.2 M solution of an electrolyte is 60 Ω. The specific conductance of the solution is $1.5\ Sm^{-1}$. The resistance of 0.5 M solution of the same electrolyte is 280 Ω. The molar conductivity of 0.5 M solution of the electrolyte in $Scm^2mol^{-1}$ is

(a) 6.420 (b) 6.235 (c) 5.381 (d) 5.833

**47.** Identify the hormone that increase the glucose level in the blood.

(a) Oxytocin (b) Glucagon
(c) Insulin (d) Thyroxine

**48.** Fenton's reagent is a mixture of

(a) $FeCl_2$ and HCl (b) $Fe(OH)_3$ and $H_2O_2$
(c) $FeSO_4$ and $H_2O_2$ (d) $Fe_2(SO_4)_3$ and $HNO_3$

**49.** The incorrect statement about the 'Hinsberg reagent' is

(a) It is known as *p*-toluenesulphonyl chloride.
(b) On treatment with secondary, it leads to a product, that is soluble in alkali.
(c) It doesn't react with tertiary amines.
(d) It is used to distinguish primary and secondary amines.

**50.** Given below are two statements, one is labelled as Assertion (A) and the other is labelled as Reason (R).

**Assertion** (A) 1-chloro-3, 3-di-methyl-1-butene do not undergo nucleophilic substitution reaction easily.

**Reason** (R) Even though the intermediate carbocation is stabilised by loosely held π-electrons, the cleavage is difficult because of strong bonding.

In the light of the above statements, choose the correct answer from the options given below.

(a) Both (A) and (R) are true and (R) is the true explanation of (A).
(b) Both (A) and (R) are true but (R) is not the true explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

## Section B : Numerical Value Type Questions

**51.** Number of electrons will have magnetic quantum number $(m=0)$ in $Fe^{3+}$ ion is .......

**52.** The electrolysis of acetate solution produces ethane according to reaction,

$$2CH_3COO^- \longrightarrow C_2H_6(g) + 2CO_2(g) + 2e^-$$

The current efficiency of the process is 90%. If the current of 0.5 A is passed through the solution for 96.45 min at 27°C and 760 torr, then the volume of gas would be.......

**53.** 16 g of an ideal gas $SO_x$ occupies 5.6 L at STP. The value of $x$ for this gas is ...... .

**54.** A radioactive substance decays in such a way that its amount decreases from initial value of 80 dpm to 40 dpm in 40 minutes. The number of atoms of radioactive substance present initially in the sample are approximately ......... .

**55.** When a saturated solution of $PbCl_2$ is evaporated to dryness, the residue is found to weigh 4.5 g. The $K_{sp}$ for the solution is ......... $\times 10^{-7}$.

**56.** The azimuthal quantum number of the valence electrons of $Zn^+$ is ...... (Atomic number of $Zn = 30$)

**57.** If the standard molar enthalpy change for combustion of graphite powder is $-3.45 \times 10^2$ kJ $mol^{-1}$, the amount of heat generated on combustion of 2g of graphite powder is ......... kJ. (Nearest integer)

**58.** The sum of the ratio of magnetic moment of $Mn^{2+}$ and $Cr^{3+}$ is ......... . (Nearest integer)

**59.** The rise in boiling point of a solution containing 1.8 g glucose in 100 g of a solvent is 0.1°C. The molal elevation constant of the liquid is ......°C/m

**60.** How many of the given equation involve reduction of transition metal complex?

A. $Fe_2O_3 + 3CO \longrightarrow 2Fe + 3CO_2$
B. $TiO_2 + 2C + 2Cl_2 \longrightarrow TiCl_4 + 2CO$
C. $Cr_2O_3 + 2Al \longrightarrow 2Cr + Al_2O_3$
D. $TiCl_4 + 4Na \longrightarrow Ti + 4NaCl$

# MATHEMATICS

## Section A : Objective Type Questions

**61.** Let $A = \int_0^1 \frac{e^x}{x+1} dx$, then the value of $\int_0^1 \frac{xe^{x^2}}{x^2+1} dx$ is

(a) $A^2$ (b) $\frac{1}{2} A$

(c) $2A$ (d) $\frac{1}{2} A^2$

**62.** Let $z = 1 + ai$ be a complex number, $a > 0$, such that $z^3$ is a real number. Then, the sum of $1 + z + z^2 + \ldots\ldots + z^{11}$ is equal to

(a) $1215\sqrt{3}\, i$ (b) $-1215\sqrt{3}\, i$

(c) $-1365\sqrt{3}\, i$ (d) $1365\sqrt{3}\, i$

**63.** The function $g(x) = \lim_{n \to \infty} \frac{f(x) + x^n h(x)}{1 + x^n}$ shall be continuous everywhere, if

**Statement I** $f(1) = h(1)$

**Statement II** $f(-1) = h(-1)$

Then, which of the following option is correct?

(a) Statement I is true and Statement II is false
(b) Statement I is false and Statement II is true
(c) Statement I and Statement II both are true
(d) Statement I and Statement II both are false

**64.** The sum of $1 \cdot 2C_2 + 2 \cdot 3C_3 + \ldots + (n-1)nC_n$ is equal to

(a) $n(n-1)\, 2^{n-1}$ (b) $n(n-1)\, 2^{n-2}$

(c) $(n-1)\, 2^{n-2}$ (d) $n\, 2^{n-2}$

**65.** The area bounded by $y = x^3$, $3y + x = 4$ and $X$-axis is equal to

(a) 2 (b) 1.9

(c) 1.75 (d) 1.5

**66.** A rod of length $l$ moves such that its end $A$ and $B$ always lie on the lines $3x - y + 5 = 0$ and $y + 5 = 0$, then locus of the point $P$, which divide $AB$ internally in the ratio $2:1$, is

(a) $l^2 = \frac{1}{4}(3x + 3y - 5)^2 + (3y + 5)^2$

(b) $l^2 = \frac{1}{4}(3x - 3y + 5)^2 + (3y - 5)^2$

(c) $l^2 = \frac{1}{4}(3x - 3y - 5)^2 + (3y - 5)^2$

(d) None of the above

**67.** The distance of point of intersection of lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ from $(1, -4, 7)$, is

(a) $\sqrt{15}$ (b) $\sqrt{27}$

(c) $\sqrt{26}$ (d) $\sqrt{14}$

**68.** Let $h(x)$ be a function such that $h'(x) = h(x)$, $h(0) = 1$ and $g(x)$ be a function such that $h(x) + g(x) = 2x$. Then, $\int_0^2 h(x)\, g(x) dx$ is equal to

(a) $(e^2 - 1)(e^2 - 3)$ (b) $\frac{(e^2 + 1)(e^2 - 3)}{2}$

(c) $\frac{(e^2 + 1)(e^2 - 1)}{2}$ (d) $\frac{(e^2 + 1)(5 - e^2)}{2}$

**69.** If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2}$, then the value of $x^2 + y^2 + z^2 + 2xyz$ is equal to

(a) 8 (b) 1 (c) 0 (d) 3

**70.** Let $|\mathbf{a}| = |\mathbf{c}| = 1$ and $|\mathbf{b}| = 4$ with $\mathbf{a} \times \mathbf{b} = 2\mathbf{a} \times \mathbf{c}$. The angle between $\mathbf{a}$ and $\mathbf{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$. If $\mathbf{b} - 2\mathbf{c} = \lambda \mathbf{a}$ $(\lambda > 0)$, then $\lambda$ is equal to

(a) 1 (b) 3 (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

**71.** The total number of local maxima and local minima of the function $f(x) = \begin{cases} (3+x)^3, & -3 < x \le -1 \\ 3x^{\frac{2}{3}}, & -1 < x < 2 \end{cases}$ is

(a) 3 (b) 0 (c) 4 (d) 2

**72.** The number of positive integral solutions of the equation $abcde = 1050$ is $375\, k$, when $k \in N$. Then, $k$ is equal to

(a) 2 (b) 5 (c) 6 (d) 11

**73.** The equation of curve which satisfy the equation $\frac{dy}{dx} = \frac{y(x + y^3)}{x(y^3 - x)}$ and passing through the point (1, 1), is

(a) $y^3 + 2x + 3x^2 y = 0$
(b) $y^3 + 2x - 3x^2 y = 0$
(c) $y^3 + x + x^2 y = 0$
(d) $y^3 + 3x + 2x^2 y = 0$

**74.** Let $p$, $q$ and $r$ be three non-zero real numbers such that the equation $\sqrt{3}p\cos x + 2q\sin x = r$, $x$ is in first and fourth quadrant, has two distinct real roots $\alpha$ and $\beta$ with $\alpha + \beta = \frac{\pi}{3}$. Then, $q : p$ is equal to

(a) 2 : 1 (b) 3 : 2
(c) 1 : 2 (d) 2 : 3

**75.** The number of points with integral coordinates lying on or within the common region bounded by the curves $y^2 = 4|x|$ and the figure formed by joining the extremeties of latus rectum with straight lines is:

(a) 9 (b) 11
(c) 13 (d) 17

**76.** The range of the function $f(x) = [16^x + 2^{2x+1} + 1]$ is where, $[\cdot]$ denotes greatest integer function.

(a) Natural numbers (b) Whole numbers
(c) Even integers (d) Integers

**77.** If $x^3 + 2x^2 - x + \lambda = 0$ and $x^2 + 5x - 3\lambda = 0$ has a common root, then $\lambda$ can be

(a) 1 (b) 4 (c) –2 (d) 2

**78.** If $D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+1 & n^2+n \\ 2k-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\sum_{k=1}^{n} D_k = 56$, then $n$ equals to

(a) 7 (b) 6 (c) 5 (d) 4

**79.** If $C_i$ denotes ${}^4C_i$, then the value of $\sum_{i=1}^{4}\left(\frac{i \cdot C_i}{C_i + C_{4-i}}\right)^2$ is equal to

(a) $\frac{20}{3}$ (b) $\frac{15}{7}$
(c) $\frac{30}{7}$ (d) $\frac{15}{2}$

**80.** Let $p, q, r \in Z$ such that $p^2 + q^2 + r^2 = 2$, let $A = \begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix}$. Then, probability of $|A| \neq 0$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

## Section B : Numerical Value Type Questions

**81.** Let $g(x) = \sin hx$ and if $g[f(x)] = x$, then $\frac{1}{501} f\left(\frac{e^{1002}-1}{2e^{501}}\right)$ is .........

**82.** If $M = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ and $M^2 - \lambda M - I_2 = 0$, then $2^\lambda$ must be ........

**83.** A line is equally inclined by an angle $\alpha$ with each of $X$ and $Z$-axes. If $\beta$ is the angle which it makes with $Y$-axis, is such that $\sin^2\beta = 3\sin^2\alpha$, then $5\cos^2\alpha$ is equal to ...........

**84.** If ratio of second and third term in the expansion of $(x + y)^n$ is equal to the ratio of third and fourth term in the expansion of $(x + y)^{n+3}$, then $n$ is equal to ........ .

**85.** The value of three times of $\int_{-\pi/2}^{0} \sqrt{\cos x - \cos^3 x}\, dx$ is ......... .

**86.** Let $a_1 = \tan^{-1} 2$, and $\{a_n\}_{n=1}^{\infty}$ be decreasing sequence of positive real numbers satisfying the condition $\sin(a_{n+1} - a_n) + \frac{\sin(a_n)\sin(a_{n+1})}{2^{n+1}} = 0$ for $n \geq 1$. Then, the value of $\lim_{n\to\infty} \cot a_n$ is equal to .....

**87.** Let $z = -\frac{i}{2} + \frac{\sqrt{3}}{2}$. Then, the smallest positive integer $k$ such that $(z^{95} + i^{71})^{106} = z^k$, then the value of $\frac{k}{5}$ is equal to .........

**88.** An equilateral triangle is inscribed in an ellipse whose equation is $\frac{x^2}{9} + y^2 = 1$. If one vertex of triangle is (0, 1) and perimeter of triangle is $k$, then the value of $7\sqrt{3}\,k$ is .........

**89.** The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On review, it was found that an observation 10 was incorrect. Then, correct standard deviation if the wrong item is omitted, is .........

**90.** Four books $A$, $B$, $C$ and $D$ are to be arranged in a row such that $B$ does not follow $A$ and $C$ does not follow $B$ and $D$ does not follow $C$. Then, the number of ways of seating is .......

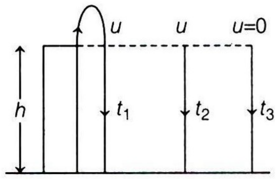
# JEE PAPER-1

| **PHYSICS (Section A: Objective Type Questions)** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Que.** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| **Ans** | b | a | c | c | c | b | a | d | c | a |
| **Que.** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| **Ans** | b | a | a | b | c | d | c | b | b | c |
| | **(Section B: Numerical Value Type Questions)** | | | | | | | | | |
| **Que.** | **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** |
| **Ans** | 15 | 0.5 | 10.5 | 15.6 | 2 | 0.83 | 10 | 8 | 0.5 | 0.26 |
| | | | | | | | | | | |
| **CHEMISTRY (Section A: Objective Type Questions)** | | | | | | | | | | |
| **Que.** | **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** |
| **Ans** | d | b | a | a | c | b | d | c | d | a |
| **Que.** | **41** | **42** | **43** | **44** | **45** | **46** | **47** | **48** | **49** | **50** |
| **Ans** | d | b | b | a | c | a | b | c | b | c |
| | **(Section B: Numerical Value Type Questions)** | | | | | | | | | |
| **Que.** | **51** | **52** | **53** | **54** | **55** | **56** | **57** | **58** | **59** | **60** |
| **Ans** | 11 | 1 | 2 | 4624 | 169 | 0 | 29 | 3 | 1 | 3 |
| | | | | | | | | | | |
| **MATHEMATICS (Section A: Objective Type Questions)** | | | | | | | | | | |
| **Que.** | **61** | **62** | **63** | **64** | **65** | **66** | **67** | **68** | **69** | **70** |
| **Ans** | b | c | c | b | c | d | c | d | b | b |
| **Que.** | **71** | **72** | **73** | **74** | **75** | **76** | **77** | **78** | **79** | **80** |
| **Ans** | d | b | b | c | b | a | c | a | d | b |
| | **(Section B: Numerical Value Type Questions)** | | | | | | | | | |
| **Que.** | **81** | **82** | **83** | **84** | **85** | **86** | **87** | **88** | **89** | **90** |
| **Ans** | 1 | 16 | 3 | 5 | 2 | 1 | 2 | 81 | 2 | 1 |

# DETAILED SOLUTIONS

## Physics

**1.** *(b)*




For stone 1,

$$h = -ut_1 + \frac{1}{2}gt_1^2 \quad \text{...(i)}$$

For stone 2,

$$h = ut_2 + \frac{1}{2}gt_2^2 \quad \text{...(ii)}$$

For stone 3,

$$h = \frac{1}{2}gt_3^2 \quad \text{...(iii)}$$

Multiplying Eq. (i) by $t_2$ and Eq (ii) by $t_1$ and adding, we get

$$h(t_1 + t_2) = -ut_1t_2 + \frac{1}{2}gt_1^2t_2 + ut_2t_1 + \frac{1}{2}gt_2^2t_1$$

$$h(t_1 + t_2) = \frac{1}{2}gt_1t_2(t_1 + t_2)$$

$$h = \frac{1}{2}gt_1t_2 \quad \text{...(iv)}$$

From Eqs. (iii) and (iv), we get

$$t_3^2 = t_1t_2$$

$$t_3 = \sqrt{t_1t_2}$$

**2.** *(a)* Consider $v_1$ and $v_2$ are velocities of approach at a distance *r*.

Apply conservation of momentum,

$$m_1v_1 - m_2v_2 = 0$$

$$m_1v_1 = m_2v_2 \quad \text{... (i)}$$

Now, apply conservation of energy,

Change in PE = Change in KE

$$\frac{Gm_1m_2}{r} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\frac{m_1^2v_1^2}{m_1} + \frac{m_2^2v_2^2}{m_2} = \frac{2Gm_1m_2}{r} \quad \text{... (ii)}$$

Substitute value of $m_1v_1$ from Eq. (i)

$$\frac{m_2^2v_2^2}{m_1} + \frac{m_2^2v_2^2}{m_2} = \frac{2Gm_1m_2}{r}$$

$$(m_2v_2)^2 = \frac{2G}{r}\frac{(m_1m_2)^2}{(m_1 + m_2)}$$

$$v_2 = \sqrt{\frac{2Gm_1^2}{r(m_1 + m_2)}}$$

Again from Eq. (i),

$$v_1 = \sqrt{\frac{2Gm_2^2}{r(m_1 + m_2)}}$$

Approach velocity $= v_1 - (-v_2) = v_1 + v_2$

$$= \sqrt{\frac{2G}{r}(m_1 + m_2)}$$

**3.** *(c)* For polar molecules, the net dipole moment of a given volume containing large number of molecules is zero, because of the dipole moments of molecules are oriented in different direction due to thermal agitation. The dielectric constant of polar molecules depends on its temperature. Hence, option (c) is correct.

**4.** *(c)* This question is based on one-dimensional motion. Here, one thing should be noticed that the final displacement of the boy and the ball is same.

| Boy | Ball |
|---|---|
| $u = +10$ m/s, | $u = -10$ m/s, |
| $a = +2\,\text{m/s}^2$ | $a = +10\,\text{m/s}^2$ |

$\therefore \quad s_{\text{Boy}} = s_{\text{Ball}}$

$$10t + \frac{1}{2}\times 2\times t^2 = -10t + \frac{1}{2}\times 10\times t^2$$

$$\Rightarrow \quad 20t = 5t^2 - t^2$$

$$\Rightarrow \quad 20t = 4t^2$$

$$\Rightarrow \quad t = 5\,\text{s}$$

**5.** *(c)* Statement I is false but the Statement II is true.

If a body at rest explodes into two parts due to internal force, then its centre of mass will be at rest but the KE of the system is non-zero.

The change in velocity, depends on the external force.

**6.** *(b)* In series combination,

$$R = R_1 + R_2$$

$\therefore \quad R_{eq} = 3 + 6 = 9\Omega$

and $\Delta R_{eq} = \Delta R_1 + \Delta R_2$

$$= 0.2 + 0.3 = 0.5$$

Thus $\quad R = (9 \pm 0.5)\,\Omega$

In parallel combination,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$R_{eq} = 2\Omega$$

Again $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{\Delta R}{R_{eq}^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\frac{\Delta R}{4} = \frac{0.2}{9} + \frac{0.3}{36} \Rightarrow \Delta R = 0.12\ \Omega$$

Thus, $R = (2 \pm 0.12)\Omega$

Hence, option (b) is correct.

**7.** ***(a)*** Clearly, $Q_1$ is +ve and $Q_2$ is −ve because potential energy due to + ve charge is + ve and due to −ve charge, is −ve.

Secondly, $|Q_2|$ is more than $|Q_1|$ as seen from graph. Also potential energy at point $A$ would be $U = qV$.

For stable equilibrium, $U$ should be minimum and $U$ would be minimum only when charge placed at $A$ is +ve.

Thus, stable equilibrium would depend on the sign of charge placed at $A$ and thus statement III is not always true.

Points $B$ and $C$ are not equilibrium points.

**8.** ***(d)*** Initially electron is in excited state $n = 2$,

Now, $E_3 - E_2 = \frac{-13.6}{9} + \frac{13.6}{4} = 1.9\text{eV}$

Incident electron energy > $(E_3 - E_2)$.

So, it is capable of exciting it to $n = 3$ state. In that case, some part of the KE of incident electron would be converted into KE of exciting electron.

Thus, collision would be inelastic [as KE is not conserved]. If the excited hydrogen atom de-excited to $n = 1$, then the electron does not lose its kinetic energy, instead gain some kinetic energy. It is the case of superelastic collision.

If H-atom were in ground state and incident electron energy were less than $(E_2 - E_1)$ which is equal to 10.2 eV. Then, collision would always be elastic.

**9.** ***(c)*** Given, length of open organ pipe,

$$l_1 = 60\,\text{cm} = 0.6\,\text{m}$$

length of closed organ pipe,

$$l_2 = 40\,\text{cm} = 0.4\,\text{m}$$

As we know, for 3rd overtone of closed organ pipe,

$$(\nu_3)_c = \frac{7v}{4l_2} = \frac{7v}{4(0.4)} \quad \text{... (i)}$$

Here, $v$ is velocity of sound.

and for 2nd overtone of open organ pipe,

$$(\nu_2)_o = \frac{3v}{2l_1} = \frac{3v}{2\times(0.6)} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{(\nu_3)_c}{(\nu_2)_o} = \frac{\frac{7v}{4\times 0.4}}{\frac{3v}{2\times 0.6}} = \frac{7v\times 2\times 0.6}{3v\times 4\times 0.4} = \frac{7}{4}$$

Therefore, the ratio of third overtone of closed organ pipe and second overtone of open organ pipe is 7 : 4.

**10.** ***(a)*** Average intensity $= \frac{P}{4\pi R^2} = \frac{1}{2}\varepsilon_0 E_0^2 c$

or $$E_0 = \sqrt{\frac{P}{2\pi R^2 \varepsilon_0 c}}$$

$$= \sqrt{\frac{3}{2\times 3.14\times 25\times 8.85\times 10^{-12}\times 3\times 10^8}}$$

$$= 2.68\,\text{Vm}^{-1}$$

**11.** ***(b)*** As we know that,

$$C_V = \frac{f}{2}R \text{ and } C_p = C_V + R$$

$$\therefore \quad C_p = \frac{f}{2}R + R = \left(1 + \frac{f}{2}\right)R$$

Now $$\frac{C_p}{C_V} = \frac{\left(1+\frac{f}{2}\right)R}{\frac{f}{2}(R)} = \frac{2+f}{f} = \left(1 + \frac{2}{f}\right)$$

Hence, option (b) is correct.

**12.** ***(a)*** Conductivity of semiconductor is given by

$$\sigma = n_e e\mu_e + n_h e\mu_h$$

Since semiconductor is intrinsic

$\Rightarrow \quad n_e = n_h = n_i$

or $\quad \sigma = n_i e(\mu_e + \mu_h)$

$$= 1.6\times 10^6 \times 1.6\times 10^{-19}\ (0.4 + 0.2)$$

$$= 1.536\times 10^{-13}\ \Omega^{-1}\text{m}^{-1}$$

Current flowing, $I = jA$ ...(i)

where, $j$ = current density $= \sigma E = \sigma\left(\frac{V}{d}\right)$

From Eq. (i), we get

$$I = 1.536\times 10^{-13}\left(\frac{5}{1.5\times 10^{-3}}\right)(3.5\times 10^{-4})$$

$$= 1.79\times 10^{-13}\ \text{A}$$

Heat produced, $H = VIt = 5\times 1.79\times 10^{-13}\times 120$

$$= 10.74\times 10^{-11}\ \text{J}$$

**13.** ***(a)*** Here, volume of new drop = volume of two small drops.

$$\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3 \Rightarrow R^3 = 2r^3 \quad \text{... (i)}$$

where, $R$ = radius of new drop

and $r$ = radius of each small drop.

Each drop acquires a velocity equal to its terminal velocity, when viscous force acting on it equals to its effective weight. Let $v_1$ and $v_2$ are the velocities of new drop and each small drop, respectively.

Then, $6\pi\eta R v_1 = \frac{4}{3}\pi R^3(\rho-\sigma)g$ ... (ii)

and $6\pi\eta r v_2 = \frac{4}{3}\pi r^3(\rho-\sigma)g$ ... (iii)

From Eqs. (ii) and (iii), we get

$$\frac{Rv_1}{rv_2} = \frac{R^3}{r^3} \Rightarrow \frac{Rv_1}{rv_2} = \left(\frac{R}{r}\right)^3$$

Using Eqs. (i),

$$(2)^{\frac{1}{3}}\frac{v_1}{v_2} = 2 \Rightarrow \frac{v_1}{v_2} = 2^{2/3}$$

$$v_1 = 2^{\frac{2}{3}} \cdot v_2 = 2^{\frac{2}{3}} \times 10 = 15.9\ \text{cm/s}$$

**14.** *(b)* As, $R = \frac{\sqrt{3}v^2}{2g} \Rightarrow \frac{v^2 \sin 2\theta}{g} = \frac{\sqrt{3}v^2}{2g}$

or $\sin 2\theta = \frac{\sqrt{3}}{2}$ or $2\theta = 60°$

$\therefore$ $\theta = 30°$

**15.** *(c)* $p_M > p_N$

[pressure at $M$ is greater than pressure at $N$]

So, velocity, $v_M < v_N$

From this observation, it seems that $h_1 > h_2$ but there is no relation given between $A_1$ and $A_2$. So, $h_2$ may be greater than $h_1$.

**16.** *(d)* Given, work done, $W = -22.3$ J

Work done is taken negative as work is done on the system.

In an adiabatic change, $\Delta Q = 0$

Using first law of thermodynamics,

$$\Delta U = \Delta Q - W = 0 - (-22.3) = 22.3\ \text{J}$$

For another process between state $A$ and state $B$,

Heat absorbed $(\Delta Q) = +9.35$ cal

$$= +(9.35 \times 4.19)\ \text{J} = +39.18\ \text{J}$$

Change in internal energy between two states *via* different paths are equal.

$\therefore$ $\Delta U = 22.3$ J

$\therefore$ From first law of thermodynamics,

$$\Delta U = \Delta Q - W$$

or $W = \Delta Q - \Delta U = 39.18 - 22.3$

$$= 16.88\ \text{J} \approx 16.9\ \text{J}$$

**17.** *(c)* From the figure, the truth table is given below :

| *A* | *B* | *Y* |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

i.e., AND gate

**Study Tactics**

$Y = \overline{\overline{A} + \overline{B}} = \overline{\overline{AB}} = AB$ (AND gate)

**18.** *(b)* According to Gauss's law

$$\oint \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\oint_s \rho\, d\mathbf{S}$$

[$\because Q = \oint_s \rho\, d\mathbf{S}, \rho =$ surface charge density]

According to Faraday's law of electromagnetic induction,

$$\oint \mathbf{E}\cdot d\mathbf{l} = \frac{-d\phi}{dt} = \frac{-d}{dt}\oint \mathbf{B}\cdot d\mathbf{S}$$

$\because$ $\phi = \oint \mathbf{B}\cdot d\mathbf{S}$

According to Ampere-Maxwell's law,

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0\left(I + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

According to Ampere's circuital law,

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I$$

Hence, option (b) is correct.

**19.** *(b)* Compare, $y(x,t) = 2\sin\left(\frac{8\pi x}{9}\right)\cos(400\pi t)$

with $y(x,t) = 2A\sin kx \cos\omega t$

$$k = \frac{8\pi}{9}, \omega = 400\pi$$

Now, speed of travelling wave, $v = \frac{\omega}{k}$

$$= \frac{400\pi}{\frac{8\pi}{9}} = 450\ \text{m/s}$$

**20.** *(c)* As we know that,

$$Y = \frac{MgL}{\pi r^2 \times l} = \frac{4 \times 3.1 \times \pi \times 2.0}{\pi \times (2\times10^{-3})^2 \times (0.031\times10^{-3})}$$

$$= \frac{4 \times 3.1 \times 2}{4\times10^{-6} \times 3.1 \times 10^{-5}}$$

$$= 2 \times 10^{11}\ \text{N/m}^2$$

**21.** (15) As we know that, internal energy of gas with $f$ degree of freedom,

$$U = \frac{nfRT}{2}$$

$\therefore$ $U_{O_2} = 3 \times \frac{5}{2}RT$ [$\because n = 3, f = 5$]

$U_{Ar} = 5 \times \frac{3}{2}RT$ [$\because n = 5, f = 3$]

Total internal energy,

$$U = U_{O_2} + U_{Ar} = 15RT = xRT$$

So, $x = 15$

**22.** (0.5) According to Einstein photoelectric equation,

$$(KE)_{max} = \frac{hc}{\lambda} - W$$

$\therefore$ Stopping potential,

$$V = \frac{1}{e}\left(\frac{hc}{\lambda} - W\right) \quad [\because (KE)_{max} = eV]$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{hc}{\lambda_1} - W}{\frac{hc}{\lambda_2} - W}$$

$$\Rightarrow \quad \frac{2}{1} = \frac{\frac{12400}{3500} - W}{\frac{12400}{6200} - W} \quad [\because hc = 12400\,\text{eV-Å}]$$

$$\Rightarrow \quad \frac{2}{1} = \frac{3.5 - W}{2 - W}$$

$$\Rightarrow \quad 4 - 2W = 3.5 - W \Rightarrow W = 0.5\,\text{eV}$$

**23.** (10.5) Now, $\mu = \dfrac{\text{real depth}}{\text{apparent depth}}$

or, real depth = $\mu \times$ apparent depth

Therefore, real length of the edge of the cube

= $\mu \times$ (apparent distance of the bubble from one face + apparent distance of the bubble from second face).

$$= 1.5 \times (2 + 5) = 1.5 \times 7 = 10.5\,\text{cm}$$

**24.** (15.6) Given, $l_2 = l + \dfrac{25}{100} l = \dfrac{5l}{4}$

$\therefore$ Volume remain unchanged.

$$A_1 l_1 = A_2 l_2$$

$$Al = A_2 \frac{5}{4} l$$

$$A_2 = \frac{4A}{5}$$

Now,

$$R_2 = \frac{\rho l_2}{A_2} = \frac{\rho\, 5l/4}{4A/5} = \frac{25\rho l}{16A}$$

$$= \frac{25}{16} \times R = \frac{25 \times 10}{16} \quad [\because R = 10\,\Omega]$$

$$= 15.6\,\Omega$$

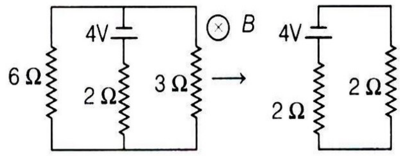
**25.** (2) Motion emf induced in the conductor,

$$e = Blv = 2(1)(2) = 4\text{ V}$$

This acts as a cell of emf 4V and internal resistance $2\Omega$, $6\Omega$ and $3\Omega$ resistors are in parallel.

$$\therefore \quad \frac{1}{R_p} = \frac{1}{6} + \frac{1}{3} = \frac{1+2}{6} = \frac{3}{6} = \frac{1}{2}$$

$$R_p = 2\Omega$$




$\therefore$ Current through the conductor ($i$)

$$= \frac{E}{R_p + r} = \frac{4}{2+2} = 1\text{A}$$

Magnetic force on the conductor

$$= Bil = 2(1)(1) = 2\text{N}$$

Therefore, to keep the conductor moving with a constant velocity, a force of 2 N to be applied to the right side.

**26.** (0.83) $T = 2\pi\sqrt{\dfrac{m}{k_1 + k_2}} = 2\pi\sqrt{\dfrac{10}{360}} = \dfrac{\pi}{3}\,\text{s}$

The maximum velocity is always at equilibrium position, since at any other point, there will be a restoring force.

$$\therefore \quad v_{max} = \frac{\text{impulse}}{\text{mass}} = \frac{50}{10} = 5\,\text{m/s}$$

$$\Rightarrow \quad \omega = \frac{2\pi}{T} = 6\,\text{rad/s}$$

$$\Rightarrow \quad A = \text{amplitude} = \frac{v_{max}}{\omega} = \frac{5}{6} = 0.83\,\text{m}$$

**27.** (10) Heat given by water to reach at 0°C

$$Q_1 = mc\Delta T = 10 \times 1 \times 10 = 100\,\text{cal}$$

Heat taken by ice to melt,

$$Q_2 = mc\Delta T + mL = 10 \times 0.5 \times [0 - (-20)] + 10 \times 80 = 900\,\text{cal}$$

As $Q_1 < Q_2$, so ice will not completely melt and final temperature = 0° C.

As heat given by water is cooling up to 0°C is only just sufficient to increase the temperature of ice from –20°C to 0°C, hence mixture in equilibrium will consist of 10 g ice and 10 g water at 0°C

### Study Tactics

When the temperature of a substance is increased or decreased by $\Delta T$, then heat transfer is calculated by $Q = mc\Delta T$, whereas during its phase change, it is $Q = mL$.

**28.** (8) Initial energy of charge capacitor,

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times 10 \times 10^{-6} \times (220)^2 = 242 \times 10^{-3}\text{ J}$$

Common potential after redistribution,

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{C_1V_1}{C_1 + C_2} \quad [\because V_2 \approx 0]$$

$$= \frac{10 \times 10^{-6} \times 220}{15 \times 10^{-6}} = 220 \times \frac{2}{3}\,\text{V}$$

Now, energy of charged capacitors,

$$U_2 = \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2}(15\times10^{-6})\left(\frac{220\times2}{3}\right)^2 = 161\times10^{-3}\ \text{J}$$

Loss of energy,

$$\Delta U = U_1 - U_2$$

$$= 242\times10^{-3} - 161\times10^{-3}$$

$$= 81\times10^{-3} = 8.1\times10^{-2}$$

$$= \frac{8.1}{100}\ \text{J} \approx \frac{8}{100} = \frac{X}{100}\ \text{J}$$

Hence, $X = 8$

**29.** (0.5) As $v \perp B$, force on the charged particle,

$$F = Bqv$$

If $a$ is acceleration of particle, then

$$F = ma$$

so, $$ma = Bqv$$

$$B = \frac{ma}{qv} \quad \ldots\text{(i)}$$

If $K$ is kinetic energy of particle, then

$$K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2K}{m}} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$B = \frac{ma}{q\sqrt{\frac{2K}{m}}} = \frac{m^{3/2}a}{q(2K)^{1/2}} \quad \ldots\text{(iii)}$$

Here, $m = 1.6\times10^{-27}$ kg

$$a = 10^{12}\ \text{m/s}^2$$

$$q = 1.6\times10^{-19}\ \text{C}$$

$$K = 2\ \text{MeV}$$

$$= 2\times10^6\times1.6\times10^{-19}\ \text{J}$$

$$= 2\times1.6\times10^{-13}\ \text{J}$$

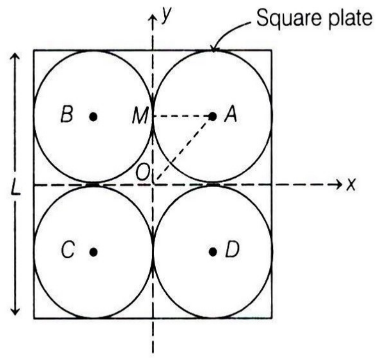
Substituting these values in Eq (iii), we get

$$B = \frac{(1.6\times10^{-27})^{3/2}\times10^{12}}{1.6\times10^{-19}\times(2\times2\times1.6\times10^{-13})^{1/2}}$$

$$\frac{(1.6)^{3/2}\times10^{\frac{-27\times3}{2}}\times10^{12}}{(1.6)^{3/2}\times10^{-19}\times2\times10^{-13/2}}$$

$$= \frac{1}{2}\times10^{-3} = 0.5\ \text{mT}$$

**30.** (0.26) According to the figure, mass per unit area of the plate, $\alpha = \frac{M}{L^2}$




But mass of each hole, $m = \alpha\times\pi R^2$

$$= \frac{M}{L^2}\times\pi R^2 = \frac{\pi MR^2}{L^2}$$

Since $L = 4R$, $m = \frac{\pi M}{16}$ ...(i)

Distance of the centre of a hole and centre $O$ of the plate

$$x = AO = \sqrt{2}R \quad [\because \text{In the right-angled } \Delta OAM$$

$$OA = \sqrt{(OM)^2 + (MA)^2}$$

$$= \sqrt{R^2+R^2} = \sqrt{2R^2} = \sqrt{2}R = x]$$

Using parallel axes theorem, the moment of inertia of a hole about the axis,

$$I = \frac{1}{2}mR^2 + mx^2 = m\left(\frac{R^2}{2} + 2R^2\right) \quad [\because x = \sqrt{2}R]$$

$$= \frac{\pi M}{16}\cdot\frac{5R^2}{2} = \frac{5\pi MR^2}{32} \quad \text{[using Eq. (i)]}$$

Moment of inertia of four holes about the $Z$-axis

$$= 4I = 4\times\frac{5\pi MR^2}{32} = \frac{5\pi MR^2}{8}$$

Moment of inertia of the complete square plate,

$$I_1 = \frac{ML^2}{6} = \frac{M}{6}(4R)^2 \quad [\because L = 4R]$$

$$= \frac{8}{3}MR^2$$

Therefore, moment of inertia of the remaining portion about the $Z$-axis,

$$I_2 = \frac{8}{3}MR^2 - \frac{5\pi}{8}MR^2 = \left(\frac{8}{3} - \frac{5\pi}{8}\right)MR^2$$

The required ratio $= \frac{I_2}{I_1}$

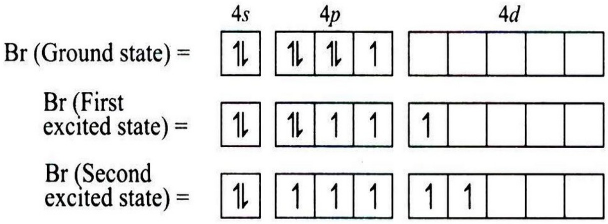
$$= \frac{\left(\frac{8}{3} - \frac{5\pi}{8}\right)MR^2}{\frac{8}{3}MR^2} = \left(\frac{8}{3} - \frac{5\pi}{8}\right)\times\frac{3}{8}$$

$$= \frac{64 - 15\pi}{3\times8}\times\frac{3}{8} = \frac{64-15\pi}{64}$$

$$= 1 - \frac{15\pi}{64} = 0.26$$

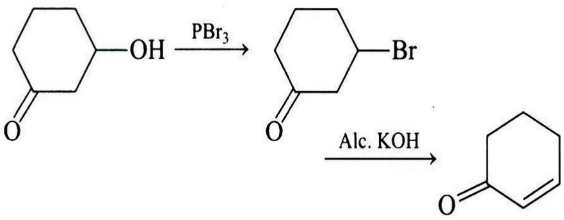
# Chemistry

**31.** *(d)* Bromine in its second excited state, reacts with fluorine to form $BrF_5$. Its shape is square pyramidal.

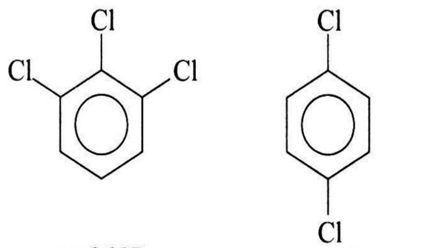



$\therefore$ Br in second excited state will react with F to form $BrF_5$ having square pyramidal shape.

**32.** *(b)* The reaction takes place as follows

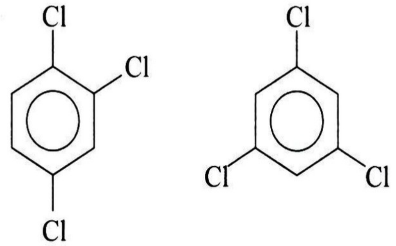



**33.** *(a)* Structure of given compounds are as follows




$\mu = 2.39D$
1,2,3-trichlorobenzene
(I)

$\mu = 0$
1,4-dichlorobenzene
(II)




$\mu = 1.26D$
1,2,4-trichlorobenzene
(III)

$\mu = 0.28D$
1,3,5-trichlorobenzene
(IV)

Thus, the increasing order of dipole moment is $II < IV < III < I$.

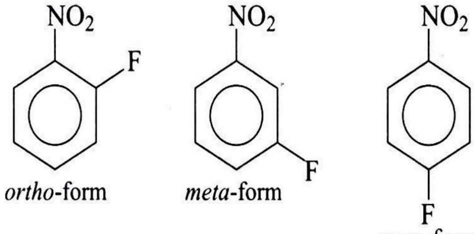
**34.** *(a)* **Determination of molecular structure** Structure of given compound may be determined as molecular formula

[Degree of unsaturation $(u)$]

$$= (C + 1) - \frac{H}{2} - \frac{X}{2} + \frac{N}{2}$$

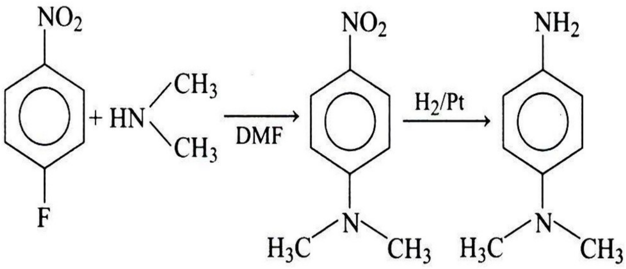
$$= (6 + 1) - \frac{4}{2} - \frac{1}{2} + \frac{1}{2} = 7 - 2 = 5$$

Since, the organic compound is cyclic, it must contain benzene ring as final product which is also aromatic in nature.



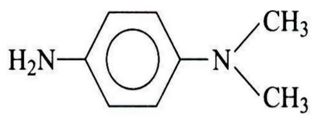

*ortho*-form  *meta*-form  *para*-form

From the given options, we will choose *para* form as a starting material and further sequence of chemical reaction will be as follows




*(A)*  *(B)*

Hence, the possible structure of compound *B* is




### Study Tactics

The problem includes conceptual mixing of determination of molecular structure of compound and nucleophilic substitution reaction on substituted benzene ring.

**35.** *(c)* The correct match is A-II, B-I, C-IV, D-III.

**36.** *(b)* The correct match is A-II, B-I, C-IV, D-III.

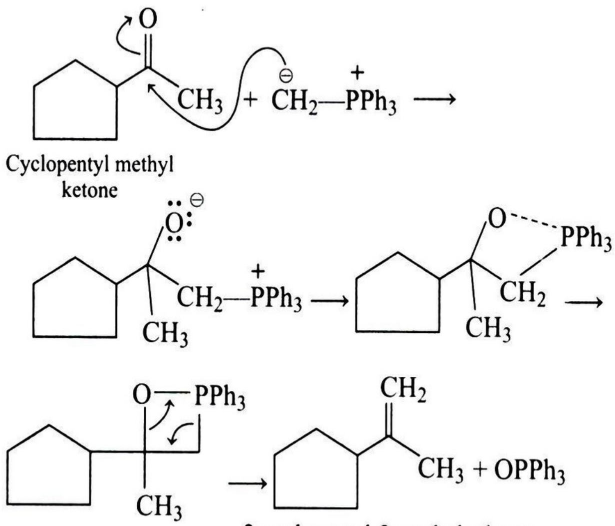
- **Myosin** is an example of fibrous protein.
- **Vitamin** Ascorbic acid is also known as vitamin-C.
- **Haemoglobin** It is an example of globular protein.
- **Androgens** is an example of steroids.

**37.** *(d)* **Wittig reaction** The conversion of cyclopentyl methylketone to 2-cyclopentyl-2-methyl ethene is done by the use of Wittig reagent, i.e. phosphorus ylide.

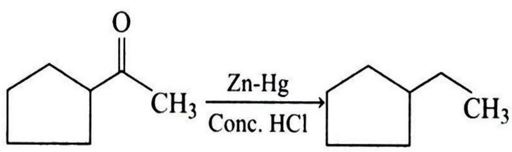
Therefore, the reagent *B* is

$$PPh_3{=}CH_2 \text{ or } \overset{\oplus}{P}Ph_3{-}\overset{\ominus}{C}H_2$$

Reaction involved by using is as follows




**Clemmensen reduction** When carbonyl compound is treated with amalgamated zinc in presence of concentrated HCl, it converts $\rangle C{=}O$ to $\rangle CH_2$.




This is a direct method for conversion of ketone to hydrocarbon. Thus, the reagent *A* used is Zn-Hg in conc. HCl.

**Study Tactics**

This problem includes conceptual mixing of Wittig reaction and Clemmensen reduction.

**38.** *(c)* $SO_3^{2-}(aq) + SO_4^{2-}(aq) \xrightarrow{BaCl_2} \underset{\text{White ppt.}}{BaSO_4} + 2Cl^- + SO_3^{2-}$

Therefore, using $BaCl_2$ sulphite and sulphate get separated easily.

**39.** *(d)* $\underset{\text{Formaldehyde}}{6HCHO} + 4NH_3 \longrightarrow \underset{\text{Hexamethylene - tetramine}}{(CH_2)_6N_4} + 6H_2O$

When formaldehyde reacts with ammonia, hexamethylene tetramine is formed which is used as urinary antiseptic (urotropine).

**40.** *(a)* The reaction is given as

$$\underset{\text{(Tetraphosphorus)}}{P_4} + 3NaOH + 3H_2O \longrightarrow \underset{\text{(Sodium hypophosphite)}}{3NaH_2PO_2} + \underset{\text{(Phosphine)}}{PH_3}$$

Thus, phosphorus $(P_4)$ on reaction with NaOH produces phosphine $(PH_3)$ gas.

**41.** *(d)* Solubility of salt *AD*

$$AD \rightleftharpoons \underset{(S)}{A^+} + \underset{(S)}{D^-}$$

$$K_{sp} = \underset{S}{[A^+]}\underset{S}{[D^-]}$$

$\therefore \quad K_{sp} = S^2$

or $\quad S = \sqrt{K_{sp}} = \sqrt{5 \times 10^{-8}} = 2.23 \times 10^{-4}$

Solubility of salt $AD_2$

$$AD_2 \rightleftharpoons \underset{S}{A^{+2}} + \underset{2S}{2D^-}$$

$$K_{sp} = [A^{2+}][D^-]^2 = S(2S)^2 = 4S^3$$

$$S = \sqrt[3]{K_{sp}/4} = \sqrt[3]{\frac{3 \times 10^{-14}}{4}} = 1.95 \times 10^{-5}$$

Solubility of salt $AD_3$

$$AD_3 \rightleftharpoons \underset{S}{A^{3+}} + \underset{3S}{3D^-}$$

$$K_{sp} = \underset{S}{[A^{3+}]}\underset{(3S)^3}{[D^-]^3} = S\,(3S)^3 = 27S^4$$

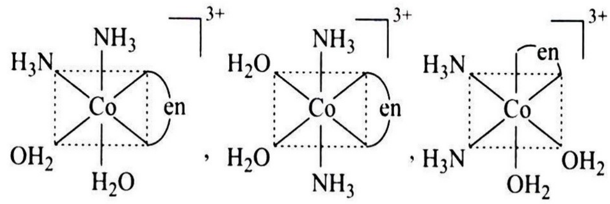
$$S = \sqrt[4]{K_{sp}/27} = \sqrt[4]{\frac{2.8 \times 10^{-15}}{27}} = 1.0 \times 10^{-4}$$

It means solubility order is $AD > AD_3 > AD_2$.

$\therefore$ Only statements (I) and (IV) are correct.

**42.** *(b)* Among the given complexes

$[Co(H_2O)_2(NH_3)_2(en)]^{3+}$ has 3 geometrical isomers.



while, all other have 2 geometrical isomers.

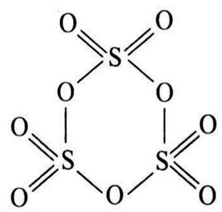
**43.** *(b)* $\underset{(A)}{2FeSO_4} \xrightarrow{\Delta} \underset{(D)}{Fe_2O_3} + \underset{(B)}{SO_2} + \underset{(C)}{SO_3}$

$$\underset{(D)}{Fe_2O_3} + \underset{\text{Conc.}}{6HCl} \longrightarrow \underset{(E)}{2FeCl_3} + 3H_2O$$

[Lewis acid and exists as a dimer]

$$K_2Cr_2O_7(aq) + H_2SO_4(aq) + \underset{(B)}{3SO_2(g)} \longrightarrow K_2SO_4(aq) + \underset{\text{(green)}}{Cr_2(SO_4)_3(aq)} + H_2O(l)$$

$SO_3$ forms a trimer in which no S—S bond is present.
(*C*)



**44.** *(a)* Given, osmotic pressure

($\pi_1$) for one solution = 1.5 atm

($\pi_2$) for other solution = 2.5 atm

$V_1 = 1\,L$

$V_2 = 2\,L$

The osmotic pressure of resultant solution is

$$\pi_R (V_1 + V_2) = \pi_1 V_1 + \pi_2 V_2$$

$$3\pi_R = 1.5 \times 1 + 2.5 \times 2$$

$$\pi_R = \frac{6.5}{3} = 2.16\,\text{atm}$$

**45.** *(c)*

$$N_2^+ = \frac{9-4}{2} = \frac{5}{2} = 2.5$$

$$N_2^- = \frac{10-5}{2} = \frac{5}{2} = 2.5$$

$$N_2 = \frac{10-4}{2} = 3$$

**Study Tactics**

$N_2$ has 14 electrons and its bond order is 3. Every electron added or subtracted to 14, reduces the bond order by 0.5.

**46.** *(a)* Given for 0.2 M solution,

$R = 60\,\Omega$

$\kappa = 1.5\,\text{Sm}^{-1} = 1.5 \times 10^{-2}\,\text{S cm}^{-1}$

Now,

$$R = \rho \frac{l}{A} = \frac{1}{\kappa} \times \frac{l}{A}$$

$$\frac{l}{A} = R \times \kappa = 60 \times 1.5 \times 10^{-2}\,\text{cm}^{-1}$$

For, 0.5 M solution

$R = 280\,\Omega$

$\kappa = ?$

As we know,

$$R = \rho \frac{l}{A} = \frac{1}{\kappa} \times \frac{l}{A}$$

$$\kappa = \frac{1}{R} \times \frac{l}{A} = \frac{1}{280} \times 60 \times 1.5 \times 10^{-2}\ \text{S cm}^{-1}$$

$$= 0.321 \times 10^{-2}\ \text{S cm}^{-1} = 3.21 \times 10^{-3}\ \text{S cm}^{-1}$$

Now, $\Lambda_m = \dfrac{\kappa \times 1000}{M}$

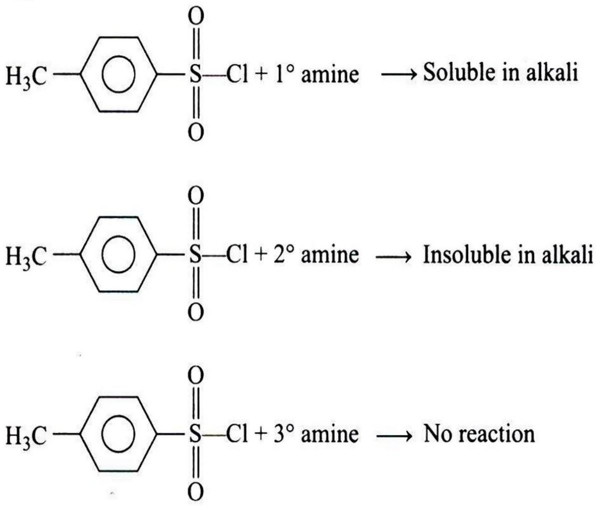
$$= \frac{3.21 \times 10^{-3}\,\text{Scm}^{-1} \times 1000\,\text{cm}^3\text{L}^{-1}}{0.5\,\text{mol L}^{-1}}$$

$$= 6.420\,\text{S cm}^2\,\text{mol}^{-1}$$

**47.** *(b)* Among the given hormones, glucagon increase the glucose level in the blood. While insulin keeping the blood glucose level within the narrow limit.

**48.** *(c)* Fenton's reagent is a mixture of ferrous sulphate and hydrogen peroxide ($FeSO_4$ and $H_2O_2$) and it can be used for the conversion of benzene to phenol.

**49.** *(b)* *p*-toluenesulphonyl chloride is the derivative of Benzene sulphonyl chloride also known as Hinsberg's reagent.




**50.** *(c)* (A) is true but (R) is false.

Vinyl halide [$(CH_3)_3C—CH{=}CH—Cl$] do not undergo nucleophilic substitution reaction.

This is due to the formation of highly unstable carbocation $(CH_3)_3C—CH{=}\overset{\oplus}{C}H$; which cannot be delocalised by the $\pi$-electrons, also C—Cl has partial double bond character which is difficult to break.

**51.** *(11)* For *s*-orbital $l = 0, m_l = 0$

For *p*-orbital $l = 1, m_l = -1, 0, +1$

For *d*-orbital $l = 2, m_l = -2, -1, 0, +1, +2$

$\therefore$ For every orbital, 2 electrons will contribute for $m_l = 0$.

$Fe = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^6$ [Atomic no. 26]

$Fe^{3+} = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^5$

For $m = 0$, No. of electrons = ⇅ (1s) ⇅ (2s) ⇅ (2p) ⇅ (3s) ⇅ (3p) 1 (3d)

Hence, electrons with $m = 0$ are 11.

**52.** *(1)* Equivalents of $CO_2$ produced

$$= \frac{I \times n \times t}{96500}$$

$$= \frac{0.5 \times 0.9 \times 96.45 \times 60}{96500}$$

$$= \frac{2604.15}{96500} \simeq 0.027$$

Moles of $CO_2$ $(n = 2)$ produced $\approx 0.027$

Moles of $C_2H_6$ $(n = 1)$ produced $= \dfrac{0.027}{2} = 0.0135$

Total moles of gases produced = 0.0405

$$V_{gas} = \frac{nRT}{p} = \frac{0.0405 \times 0.0821 \times 300}{\frac{760}{760}} = 0.997\,\text{L} \approx 1\text{ L}$$

**53.** *(2)* 1 mole of the gas at STP = 22.4 L

5.6 L at STP = 16 g

Hence, 22.4 L at STP = $\frac{16 \times 22.4}{5.6} = 64$ g

Thus, molecular mass of gas = 64 g

Given, gas is $SO_x$.

$$32 + 16x = 64 \Rightarrow x = 2$$

Thus, gas is $SO_2$.

**54.** *(4624)* $t_{1/2} = 40$ minutes

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{40} = 0.0173$$

Number of atoms present initially = $\frac{\text{Initial rate of decay}}{\lambda}$

$$= \frac{80}{0.0173} = 4624.2 \simeq 4624$$

**55.** *(169)* Solubility of $PbCl_2$ = 4.5 g $L^{-1}$ = $\frac{4.5}{278}$ mol $L^{-1}$

$$PbCl_2 \rightleftharpoons \underset{x}{Pb^{2+}} + \underset{2x}{2Cl^-}$$

$$K_{sp} = 4x^3 = 4 \times \left(\frac{4.5}{278}\right)^3$$

$$= 1.69 \times 10^{-5} = 169 \times 10^{-7}$$

**56.** *(0)* The E.C. of Zn = $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^{10}, 4s^2$

So, electronic configuration of $Zn^+$ will be

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^1$$

Valence electron is in *s*-orbital, so azimuthal quantum number (*l*) for valence shell electron is (0).

**57.** *(29)* Molecular weight of graphite = 12 g/mol

Heat of combustion per mol for graphite = $-3.45 \times 10^2$ kJ

Heat generated for 1 g of graphite

$$= -\frac{3.45 \times 10^2}{12} = -0.2875 \times 10^2 \text{ kJ}$$

$$= 28.75\text{ kJ} \approx 29\text{ kJ}$$

The negative sign shows that the heat is evolved.

**58.** (3) Electronic configuration of Cr = $[Ar]\,3d^5 4s^1$

Electronic configuration of $Cr^{3+}$ = $[Ar]\,3d^3$

$Cr^{3+}$ has 3 unpaired electrons.

Magnetic moment ($\mu$) = $\sqrt{n(n+2)}$

$$= \sqrt{3(3+2)} = \sqrt{15} = 3.87$$

Electronic configuration of Mn = $[Ar]\,3d^5 4s^2$

Electronic configuration of $Mn^{2+}$ = $[Ar]\,3d^5$

$Mn^{2+}$ has 5 unpaired electrons.

Magnetic moment ($\mu$) = $\sqrt{5(5+2)} = \sqrt{35} = 5.92$

So, $\frac{\mu_{Mn^{2+}}}{\mu_{Cr^{3+}}} = \frac{5.92}{3.87} = 1.53 : 1$

The sum of ratio of $\mu_{Mn^{2+}}$ and $\mu_{Cr^{3+}}$ is

$$1.53 + 1 = 2.53 \approx 3 \text{ (nearest integer)}$$

**59.** *(1)* $\Delta T_b$ = molality $\times K_b$

$$\Delta T_b = \frac{w}{m \times W} \times 1000 \times K_b$$

$$K_b = \frac{\Delta T_b \times m \times W}{w \times 1000}$$

Given, $\Delta T_b = 0.1°$ C, $m = 180$, $W = 100$, $w = 1.8$

$$K_b = \frac{180 \times 0.1 \times 100}{1000 \times 1.8} = 1.0\,°\text{C m}^{-1}$$

**60.** *(3)* Only equation (B) does not involve reduction.

$$\underset{(+4)}{TiO_2} \longrightarrow \underset{(+4)}{TiCl_4}$$

# Mathematics

**61.** *(b)* Let $I = \int_0^1 \frac{xe^{x^2}}{x^2+1}\,dx$

Put $x^2 = t \Rightarrow 2x\,dx = dt$

At $x = 0, t = 0$

$x = 1, t = 1$

$$\therefore \quad I = \frac{1}{2}\int_0^1 \frac{e^t\,dt}{t+1} \Rightarrow I = \frac{1}{2}A$$

**62.** *(c)* Here, $z = 1 + ai$

$\Rightarrow \quad z^2 = 1 - a^2 + 2ai$

So, $z^2 \cdot z = \{(1-a^2) + 2ai\}\{1 + ai\}$

$$= (1-a^2) + 2ai + (1-a^2)\,ai - 2a^2$$

$\because z^3$ is real.

$\therefore \quad 2a + (1-a^2)\,a = 0$

$\Rightarrow \quad a(3-a^2) = 0 \Rightarrow a = \sqrt{3} \qquad [\because a > 0]$

Now, $1 + z + z^2 + \ldots\ldots + z^{11}$

$$= \frac{(1+\sqrt{3}i)^{12} - 1}{1+\sqrt{3}i - 1} = \frac{(1+\sqrt{3}i)^{12} - 1}{\sqrt{3}i}$$

$$\because \quad (1+\sqrt{3}\,i)^{12} = 2^{12}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{12}$$

$$= 2^{12}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{12}$$

$$= 2^{12}\,(\cos 4\pi + i\sin 4\pi) = 2^{12}$$

$\because 1+z+z^2+......+z^{11} = \frac{2^{12}-1}{\sqrt{3}i} = \frac{4095}{\sqrt{3}i}$

$= -\frac{4095}{3}\sqrt{3}\,i = -1365\sqrt{3}\,i$

**63.** *(c)* $g(x) = \lim_{x\to\infty} \frac{f(x)+x^n h(x)}{1+x^n}$

If $|x| = 1$, then $g(x) = \frac{f(x)+h(x)}{2}$ ...(i)

If $|x| < 1$, then $\lim_{n\to\infty} x^n = 0$

$\therefore \quad g(x) = f(x)$ ...(ii)

If $|x| > 1$, then $\lim_{n\to\infty} x^{-n} = 0$

$g(x) = \lim_{n\to\infty} \frac{x^{-n} f(x)+h(x)}{x^{-n}+1} = h(x)$ ...(iii)

The function $g(x)$ shall be continuous everywhere, if $f(x)$ and $h(x)$ are continuous everywhere and if $g(x)$ is continuous at $x = \pm 1$.

Now, from Eqs. (i), (ii) and (iii), we get

$f(1) = \frac{f(1)+h(1)}{2} = h(1)$

$\Rightarrow \quad f(1) = h(1)$

and $f(-1) = \frac{f(-1)+h(-1)}{2} = h(-1)$

$\Rightarrow f(-1) = h(-1)$

Statement I and Statement II both are true.

**64.** *(b)* $\because (1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$

On differentiating above expansion two times, we get

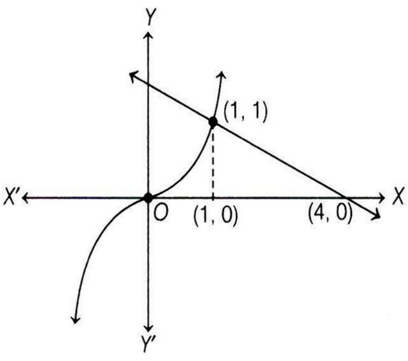
$n(n-1)(1+x)^{n-2} = 1\cdot 2C_2 + 2\cdot 3C_3x + ... + (n-1)nC_n\cdot x^{n-2}$

On putting $x = 1$ in above equation, we get

$1\cdot 2C_2 + 2\cdot 3C_3 + ... + (n-1)nC_n$

$= n(n-1)2^{n-2}$

**65.** *(c)*




Required area $= \int_0^4 y\,dx = \int_0^1 x^3 dx + \int_1^4 \frac{4-x}{3} dx$

$= \left(\frac{x^4}{4}\right)_0^1 + \frac{1}{3}\left(4x - \frac{x^2}{2}\right)_1^4$

$= \frac{1}{4} + \frac{1}{3}\left[(16-8) - \left(4-\frac{1}{2}\right)\right] = \frac{1}{4} + \frac{3}{2} = \frac{7}{4}$

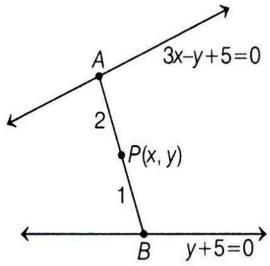
$= 1.75$

**66.** *(d)* Given, equation of lines

$3x - y + 5 = 0$ ...(i)

$y + 5 = 0$ ...(ii)

Let any point on line (i) be $A(\alpha, 3\alpha+5)$ and on line (ii) be $B(\beta, -5)$.




Using section formula,

$x = \frac{\alpha+2\beta}{3} \Rightarrow 3x = \alpha + 2\beta$ ...(iii)

and $y = \frac{-10+3\alpha+5}{3}$

$\Rightarrow \quad 3y = 3\alpha - 5$ ...(iv)

From Eqs. (iii) and (iv), on eliminating $\alpha$ and $\beta$, we get

$\alpha = \frac{3y+5}{3},\ \beta = \frac{9x-3y-5}{6}$

$\therefore \quad l^2 = AB^2$

$= (\alpha-\beta)^2 + (3\alpha+10)^2$

$= \frac{1}{4}(3x-3y-5)^2 + (3y+15)^2$

**67.** *(c)* Given lines, $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7} = \lambda$ (say) ...(i)

and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \mu$ (say) ...(ii)

From Eqs. (i) and (ii), we get

$\lambda + 4 = 2\mu + 1$

$\Rightarrow \quad \lambda - 2\mu = -3$ ...(iii)

and $\quad -3 - 4\lambda = -1 - 3\mu$

$\Rightarrow \quad 4\lambda - 3\mu = -2$ ....(iv)

From Eqs. (iii) and (iv), we get

$\lambda = 1$ and $\mu = 2$

$\therefore$ Point of intersection $(4+1, -3-4, -1+7)$

i.e. $(5, -7, 6)$.

Distance of $(5, -7, 6)$ from $(1, -4, 7) = \sqrt{16+9+1}$

$= \sqrt{26}$ units

**68.** *(d)* $h'(x) = h(x) \Rightarrow \frac{h'(x)}{h(x)} = 1$

On integrating both sides w.r.t $x$, we get

$$\log h(x) = \log A + x$$

$\Rightarrow$ $h(x) = Ae^x$

$\because$ $h(0) = 1$

$\Rightarrow$ $A = 1$

$\therefore$ $h(x) = e^x$

and $g(x) = 2x - e^x$

$\therefore$ $I = \int_0^2 e^x(2x - e^x)dx$

$$= \int_0^2 (2xe^x - e^{2x})dx$$

$$= 2[xe^x - e^x]_0^2 - \left[\frac{e^{2x}}{2}\right]_0^2$$

$$= 2[(2e^2 - e^2) - (0 - 1)] - \left[\frac{e^4 - 1}{2}\right]$$

$$= 2[e^2 + 1] - \frac{(e^2 - 1)(e^2 + 1)}{2}$$

$$= \frac{(e^2 + 1)(4 - e^2 + 1)}{2}$$

$$= \frac{(e^2 + 1)(5 - e^2)}{2}$$

**69.** *(b)* Given, $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2}$

On taking $\sin^{-1} x = \alpha, \sin^{-1} y = \beta$ and $\sin^{-1} z = \gamma$, we get

$\therefore$ $\alpha + \beta + \gamma = \frac{\pi}{2}$

$$\alpha + \beta = \frac{\pi}{2} - \gamma$$

$$\cos(\alpha + \beta) = \cos\left(\frac{\pi}{2} - \gamma\right)$$

$\cos \alpha \cos \beta - \sin \alpha \sin \beta = \sin \gamma$ ...(i)

and we have, $\sin \alpha = x, \sin \beta = y$ and $\sin \gamma = z$

$\therefore$ $\cos \alpha = \sqrt{1 - x^2}, \cos \beta = \sqrt{1 - y^2}$

$\therefore$ From Eq. (i), we get

$$\sqrt{1 - x^2} \cdot \sqrt{1 - y^2} - xy = z$$

$$\sqrt{1 - x^2} \cdot \sqrt{1 - y^2} = (z + xy)$$

On squaring both sides, we get

$$x^2 + y^2 + z^2 + 2xyz = 1$$

**70.** *(b)* $\because \mathbf{a} \cdot \mathbf{c} = |\mathbf{a}||\mathbf{c}|\cos \theta$

and given that $\theta = \cos^{-1}\left(\frac{1}{4}\right)$

$\Rightarrow$ $\cos \theta = \frac{1}{4}$

$\therefore$ $\mathbf{a} \cdot \mathbf{c} = |\mathbf{a}||\mathbf{c}| \times \frac{1}{4} = \frac{1}{4}$ $[\because |\mathbf{a}| = |\mathbf{c}| = 1]$

On taking dot product by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ with $\mathbf{b} - 2\mathbf{c} = \lambda \mathbf{a}$, we have

$$\mathbf{a} \cdot \mathbf{b} - 2(\mathbf{a} \cdot \mathbf{c}) = \lambda(\mathbf{a} \cdot \mathbf{a})$$

$\Rightarrow$ $\mathbf{a} \cdot \mathbf{b} = \lambda + \frac{1}{2}$ ...(i)

Similarly, $\mathbf{b} \cdot \mathbf{c} = 8 - \frac{\lambda^2}{2} - \frac{\lambda}{4}$ ...(ii)

and $\mathbf{b} \cdot \mathbf{c} - 2 = \lambda(\mathbf{a} \cdot \mathbf{c})$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$8 - \frac{\lambda^2}{2} - \frac{\lambda}{4} - 2 = \lambda\left(\frac{1}{4}\right) \Rightarrow (\lambda - 3)(\lambda + 4) = 0$$

$\because$ $\lambda > 0 \Rightarrow \lambda = 3$

**71.** *(d)* $f'(x) = \begin{cases} 3(3 + x)^2, & -3 < x \le -1 \\ 2x^{\frac{-1}{3}}, & -1 < x < 2 \end{cases}$

Clearly, $f'(x)$ changes its sign at $x = -1$ from positive to negative and so $f(x)$ has local maxima at $x = -1$.

Also, $f'(0)$ does not exist but $f'(0^-) < 0$ and $f'(0^+) > 0$. It can only inferred that $f(x)$ has a possibility of minimum at $x = 0$.

Hence, it has one local maxima at $x = -1$ and one local minima at $x = 0$.

Thus, there are total number of 2 local maxima and local minima.

**72.** *(b)* $abcde = 1050 = 2 \times 3 \times 5^2 \times 7$

We can assign 2, 3 or 7 to any of 5 variables $a, b, c$ $d$ and $e$.

We can assign $5^2$ to just one variable in 5 ways or can assign $5^2 = 5 \times 5$ to two variables in ${}^5C_2$ ways, then 5 can be assigned in ${}^5C_1 + {}^5C_2 = 5 + 10 = 15$ ways

Hence, required solutions $= 5 \times 5 \times 5 \times 15 = 1875$

Now, according to the question, we get

$$375\,k = 1875 \Rightarrow k = \frac{1875}{375} = 5$$

**73.** *(b)* Given, $\frac{dy}{dx} = \frac{y(x + y^3)}{x(y^3 - x)}$

$\Rightarrow$ $x(y^3 - x)dy = y(x + y^3)dx$

$\Rightarrow$ $(xy^3 - x^2)dy = (xy + y^4)dx$

$\Rightarrow$ $xydx + y^4dx - xy^3dy + x^2dy = 0$

$\Rightarrow x(ydx + xdy) + y^3(ydx - xdy) = 0$

$\Rightarrow xd(xy) - y^3\left(\frac{xdy - ydx}{x^2}\right) \cdot x^2 = 0$

$$\Rightarrow xd(xy) - x^2y^3 d\left(\frac{y}{x}\right) = 0 \Rightarrow \frac{d(xy)}{x^2y^2} = \frac{y}{x} d\left(\frac{y}{x}\right)$$

On integrating both sides, we get

$$\int \frac{d(xy)}{(xy)^2} = \int \frac{y}{x} d\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad -\frac{1}{xy} = \left(\frac{y}{x}\right)^2 \cdot \frac{1}{2} + C$$

$$\Rightarrow \quad y^3 + 2x + 2Cx^2 y = 0$$

At $(1, 1)$, $\quad 1 + 2 + 2C = 0 \Rightarrow C = -\frac{3}{2}$

Then, $\quad y^3 + 2x - 3x^2 y = 0$

**74.** *(c)* Given, $\sqrt{3}p\cos x + 2q\sin x = r$ ...(i)

It has two roots such that $\alpha + \beta = \frac{\pi}{3}$

$\therefore$ From Eq. (i), we get

$$\sqrt{3}p\cos\left(\frac{\pi}{3} - x\right) + 2q\sin\left(\frac{\pi}{3} - x\right) = r$$

$$\Rightarrow \sqrt{3}p\left(\cos x\cos\frac{\pi}{3} + \sin x\sin\frac{\pi}{3}\right) + 2q\left(\sin\frac{\pi}{3}\cos x - \cos\frac{\pi}{3}\sin x\right) = r$$

$$\Rightarrow \sqrt{3}p\left(\cos x\cdot\frac{1}{2} + \sin x\cdot\frac{\sqrt{3}}{2}\right) + 2q\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right) = r$$

$$\Rightarrow \left(\frac{\sqrt{3}}{2}p + \sqrt{3}q\right)\cos x + \left(\frac{3}{2}p - q\right)\sin x = r \quad \text{...(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$\left(\sqrt{3}q - \frac{\sqrt{3}}{2}p\right)\cos x + \left(\frac{3}{2}p - 3q\right)\sin x = 0$$

$$\Rightarrow \quad \sqrt{3}q - \frac{\sqrt{3}}{2}p = 0$$

$$\Rightarrow \quad \frac{q}{p} = \frac{1}{2}$$

$$\Rightarrow \quad q : p = 1 : 2$$

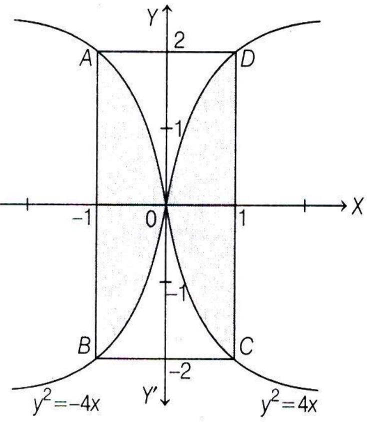
**Study Tactics**

Given, $\sqrt{3}p\cos x + 2q\sin x = r$ ...(i)

$\because$ Eq. (i) has two distinct real roots $\alpha$ and $\beta$, with $\alpha + \beta = \frac{\pi}{3}$.

So, replace $x$ by $\left(\frac{\pi}{3} - x\right)$ in given equation, solve both equations and find $p : q$.

**75.** *(b)* The common region is shown shaded. $AB$ and $CD$ are the latus rectum of the parabolas.




Thus, the integral points on or within the shaded region are

(1, 0), (1, 1), (1, 2), (1, –1), (1, –2), (–1, 0), (–1, 1), (–1, 2), (–1, –1), (–1, –2) and (0, 0).

Hence, there will be 11 such points.

**76.** *(a)* $f(x) = [16^x + 2^{2x+1} + 1] = [4^{2x} + 2\cdot 4^x + 1]$

$f(x) = [(4^x + 1)^2]$

$\Rightarrow (4^x + 1)^2 > 1 \forall x$

$\therefore [(4^x + 1)^2] =$ Integral values

which is greater than equal to one.

Hence, natural numbers are range.

**77.** *(c)* Let $\alpha$ be the common root.

$$\alpha^3 + 2\alpha^2 - \alpha + \lambda = 0 \Rightarrow \lambda = -\alpha^3 - 2\alpha^2 + \alpha$$

and $\quad \alpha^2 + 5\alpha - 3\lambda = 0$

$$\alpha^2 + 5\alpha - 3(-\alpha^3 - 2\alpha^2 + \alpha) = 0$$

$$\Rightarrow \quad 3\alpha^3 + 7\alpha^2 + 2\alpha = 0$$

$$\Rightarrow \quad \alpha(3\alpha^2 + 7\alpha + 2) = 0$$

$$\Rightarrow \alpha[3\alpha(\alpha + 2) + 1(\alpha + 2)] = 0$$

$$\Rightarrow \quad \alpha(3\alpha + 1)(\alpha + 2) = 0$$

$$\alpha = 0, -2, \frac{-1}{3}$$

$$\lambda = 0, -2, \frac{-14}{27}$$

**78.** *(a)* $\sum_{k=1}^{n} D_k = 56$

$$\Rightarrow \begin{vmatrix} \sum_{k=1}^{n} 1 & n & n \\ \sum_{k=1}^{n} 2k & n^2 + n + 1 & n^2 + n \\ \sum_{k=1}^{n} (2k - 1) & n^2 & n^2 + n + 1 \end{vmatrix} = 56$$

$$\Rightarrow \begin{vmatrix} n & n & n \\ n(n+1) & n^2+n+1 & n^2+n \\ n^2 & n^2 & n^2+n+1 \end{vmatrix} = 56$$

$\Rightarrow \quad n(n+1) = 56$

$\Rightarrow \quad n = 7$

**79.** *(d)* $\dfrac{i \cdot C_i}{C_i + C_{4-i}} = \dfrac{i \cdot {}^4C_i}{{}^4C_i + {}^4C_{4-i}} = \dfrac{i\,{}^4C_i}{2\,{}^4C_i} = \dfrac{i}{2}$

$$\therefore \sum_{i=1}^{4}\left(\frac{i \cdot C_i}{C_i + C_{4-i}}\right)^2 = \sum_{i=1}^{4}\left(\frac{i}{2}\right)^2$$

$$= \frac{1}{4}(1^2 + 2^2 + 3^2 + 4^2)$$

$$= \frac{1}{4}(30) = \frac{15}{2}$$

**80.** *(b)* $\because p^2 + q^2 + r^2 = 2$

$\Rightarrow \quad p = 0, q = \pm 1, r = \pm 1$

$p = \pm 1, q = 0, r = \pm 1$

$p = \pm 1, q = \pm 1, r = 0$

Total solutions = 12

Now, $|A| \neq 0$

$\Rightarrow \quad p + q + r \neq 0$

$\therefore$ Required solution = 6

$\therefore$ Probability of system has unique solution

$$= \frac{6}{12} = \frac{1}{2}$$

**81.** *(1)* Let $y = \sin hx = \dfrac{e^x - e^{-x}}{2}$

$\Rightarrow \quad e^{2x} - 1 = 2ye^x$

$\Rightarrow \quad e^{2x} - 2ye^x - 1 = 0$

$\Rightarrow \quad e^x = y \pm \sqrt{y^2 + 1}$

$\Rightarrow \quad g^{-1}(x) = \log(x + \sqrt{x^2 + 1}) \qquad [\because e^x > 0]$

$\because \quad f(x) = g^{-1}(x)$

$= \log(y + \sqrt{y^2 + 1})$

$$\Rightarrow f\left(\frac{e^{1002} - 1}{2e^{501}}\right) = \log\left(\frac{e^{1002} - 1}{2e^{501}} + \frac{e^{1002} + 1}{2e^{501}}\right)$$

$$= \log e^{501} = 501$$

$$\Rightarrow \frac{1}{501} f\left(\frac{e^{1002} - 1}{2e^{501}}\right) = 1$$

**82.** *(16)* Given, $M = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$

$$M^2 = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix}$$

$$\lambda M = \begin{pmatrix} \lambda & 2\lambda \\ 2\lambda & 3\lambda \end{pmatrix}$$

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\therefore \quad M^2 - \lambda M - I_2 = 0$

$$\Rightarrow \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix} - \begin{pmatrix} \lambda & 2\lambda \\ 2\lambda & 3\lambda \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 4-\lambda & 8-2\lambda \\ 8-2\lambda & 12-3\lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \lambda = 4$$

$\therefore \quad 2^{\lambda} = 2^4 = 16$

**Time Saver Tip**

$\because$ The characteristic equation is

$$|M - tI| = 0$$

$$\Rightarrow \begin{vmatrix} 1-t & 2 \\ 2 & 3-t \end{vmatrix} = 0$$

$$t^2 - 4t - 1 = 0$$

$\therefore \quad M^2 - 4M - I_2 = 0$

[using Cayley Hamilton theorem]

Thus, $\lambda = 4$

So, $2^{\lambda} = 2^4 = 16$

**83.** *(3)* If a line makes the angle $\alpha$, $\beta$ and $\gamma$ with $X$, $Y$ and $Z$-axes respectively, then

$$l^2 + m^2 + n^2 = 1$$

$$2l^2 + m^2 = 1 \text{ or } 2n^2 + m^2 = 1$$

$\Rightarrow \quad 2\cos^2\alpha = 1 - \cos^2\beta \qquad [\because l = \cos\alpha, m = \cos\beta]$

$\Rightarrow \quad 2\cos^2\alpha = \sin^2\beta$

$\Rightarrow \quad 2\cos^2\alpha = 3\sin^2\alpha \qquad [\sin^2\beta = 3\sin^2\alpha]$

$\therefore \quad 5\cos^2\alpha = 3$

**84.** *(5)* General term $T_{r+1}$ in the expansion of $(x + y)^n$ is given by

$$T_{r+1} = {}^nC_r\, x^{n-r} y^r$$

and in the expansion of $(x + y)^{n+3}$

$$T'_{r+1} = {}^{n+3}C_r x^{n+3-r}\, y^r$$

Given, $\dfrac{T_2}{T_3} = \dfrac{T'_3}{T'_4}$

$$\Rightarrow \frac{{}^nC_1 x^{n-1} y^1}{{}^nC_2 x^{n-2} y^2} = \frac{{}^{n+3}C_2\, x^{n+1} y^2}{{}^{n+3}C_3 x^n y^3}$$

$$\Rightarrow \frac{2n}{n(n-1)} = \frac{\frac{(n+3)(n+2)}{2}}{\frac{(n+3)(n+2)(n+1)}{6}}$$

$$\Rightarrow \frac{2}{n-1} = \frac{3}{n+1} \Rightarrow n = 5$$

**85.** *(2)* Let $I = \int_{-\pi/2}^{0} \sqrt{\cos x - \cos^3 x}\, dx$

$$= \int_{-\pi/2}^{0} \sqrt{\cos x(1-\cos^2 x)}\, dx$$

$$= \int_{-\pi/2}^{0} \sqrt{\cos x}\, |\sin x|\, dx$$

For $-\frac{\pi}{2} < x < 0$

$\Rightarrow \quad -1 \le \sin x \le 0 = -\int_{-\pi/2}^{0} \sqrt{\cos x} \sin x\, dx$

Let $\cos x = t \Rightarrow \sin x\, dx = -dt$

When $x = -\frac{\pi}{2}, t = 0$

When $x = 0, t = 1$

$\therefore \quad I = \int_0^1 t^{\frac{1}{2}}\, dt$

$$= \left( \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right)_0^1 = \frac{2}{3}$$

$\therefore \quad 3I = 2$

**86.** *(1)* $\because \sin(a_{n+1} - a_n) + \frac{\sin(a_n)\sin(a_{n+1})}{2^{n+1}} = 0$

$\Rightarrow \sin(a_{n+1})\cos(a_n) - \cos(a_{n+1})\sin(a_n) + \frac{\sin(a_n)\sin(a_{n+1})}{2^{n+1}} = 0$

On dividing by $\sin(a_n)\sin(a_{n+1})$ to both sides, we get

$$\cot a_{n+1} - \cot a_n = \frac{1}{2^{n+1}}$$

$\Rightarrow \quad \cot a_2 - \cot a_1 = \frac{1}{2^2}$

$\cot a_3 - \cot a_2 = \frac{1}{2^3}$

$\cot a_4 - \cot a_3 = \frac{1}{2^4}$ and so on

But $\quad a_1 = \tan^{-1} 2$

$\Rightarrow \tan a_1 = 2$

$\Rightarrow \cot a_1 = \frac{1}{2}$

$\therefore \cot a_n = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^n}$

$$\lim_{n\to\infty} \cot a_n = \lim_{n\to\infty} \left( \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^n} \right)$$

$$= \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$$

**87.** *(2)* $z = -\frac{i}{2} + \frac{\sqrt{3}}{2} = +i\omega$

where, $\omega$ is cube root of units.

$$z^{95} = (i\omega)^{95} = -i\omega^2$$

$$(z^{95} + i^{71})^{106} = (-i\omega^2 + i^{71})^{106}$$

$$= (-i\omega^2 - i)^{106}$$

$$= (-i)^{106}(1+\omega^2)^{106}$$

$$= 1(-\omega)^{106} \qquad [\because 1 + \omega + \omega^2 = 0]$$

$$= -\omega$$

Since, $(z^{95} + i^{71})^{106} = z^k$

$\Rightarrow \quad z^k = -\omega$

$\Rightarrow \quad (i\omega)^k = -\omega$

$\Rightarrow \quad i^k \omega^{k-1} = -1$

$\therefore \quad k = 2, 6, 10, \ldots$

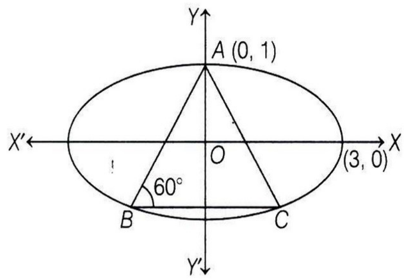
and $k - 1 = 3, 6, 9, \ldots$

$\therefore \quad k = 10$ is the required least positive integer.

Thus, $\frac{k}{5} = 2$

**88.** *(81)* Given, equation of ellipse is

$$\frac{x^2}{9} + \frac{y^2}{1} = 1 \qquad \ldots(i)$$




$\therefore$ Equation of side $AB$ is

$$y - 1 = (\tan 60°)(x - 0)$$

$\Rightarrow \quad y = \sqrt{3}x + 1 \qquad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$$\frac{x^2}{9} + (\sqrt{3}x + 1)^2 = 1$$

$\Rightarrow x^2 + 9(3x^2 + 1 + 2\sqrt{3}x) = 9$

$\Rightarrow \quad 28x^2 + 18\sqrt{3}x + 9 = 9$

$\Rightarrow \quad 28x^2 + 18\sqrt{3}x = 0$

$\Rightarrow \quad 14x^2 + 9\sqrt{3}x = 0$

$\Rightarrow \quad x(14x + 9\sqrt{3}) = 0$

$\Rightarrow \quad x = 0$ or $x = -\frac{9\sqrt{3}}{14}$

On putting $x = -\frac{9\sqrt{3}}{14}$ in Eq. (ii), we get

$$y = \sqrt{3}\left(-\frac{9\sqrt{3}}{14}\right) + 1$$

$$= \frac{-27}{14} + 1 = \frac{-13}{14}$$

Hence, $B \equiv \left(\frac{-9\sqrt{3}}{14}, \frac{-13}{14}\right)$

$\therefore \quad AB = \sqrt{\left(-\frac{9\sqrt{3}}{14}\right)^2 + \left(\frac{-13}{14} - 1\right)^2} = \frac{9\sqrt{3}}{7}$ units

$\Rightarrow$ Perimeter of $\Delta ABC = \frac{3 \times 9\sqrt{3}}{7} = \frac{27}{7}\sqrt{3}$ units $= k$

$\therefore \; 7\sqrt{3}\,k = 81$

**89.** *(2)* Given, $n = 20$, $\sigma = 2$ and $\bar{x} = 10$

$\therefore \quad \Sigma x = n\bar{x} = 20 \times 10 = 200$

Incorrect $\Sigma x = 200$

Also, $\frac{1}{n}\Sigma x^2 - (\bar{x})^2 = \sigma^2$

$\Rightarrow \quad \frac{\Sigma x^2}{20} = 4 + 100$

$\Rightarrow \quad \Sigma x^2 = 2080$

If wrong observation 10 is omitted

Correct $\Sigma x = 200 - 10 = 190$

Correct $\Sigma x^2 = 2080 - 10^2 = 1980$

and $\quad n = 20 - 1 = 19$

Correct mean $\bar{x} = \frac{190}{19} = 10$

Correct standard deviation $= \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

$$= \sqrt{\frac{1980}{19} - (10)^2}$$

$$= 2.05$$

$$\approx 2$$

**90.** *(1)* $B$ does not follow $A \Rightarrow$ Book $B$ comes before Book $A$

$C$ does not follow $B \Rightarrow$ Book $C$ comes before Book $B$

$D$ does not follow $C \Rightarrow$ Book $D$ comes before Book $C$

Thus, 1st place $\rightarrow$ Book $D$

2nd place $\rightarrow$ Book $C$

3rd place $\rightarrow$ Book $B$

4th place $\rightarrow$ Book $A$

Hence, only one arrangement is possible.